ECE 495N EXAM I

CLOSED BOOK

Wednesday, Oct.1, 2008

NAME : SOLUTION

PUID # : ________________

Please show all work and write your answers clearly.

This exam should have seven pages.

Problem 1	[p. 2, 3]	8 points
Problem 2	[p. 4, 5]	8 points
Problem 3	[p. 6, 7]	9 points
Total		25 points

Problem 1: We have seen in class that the current-voltage (I-V) characteristics of a nanoscale device can be calculated from

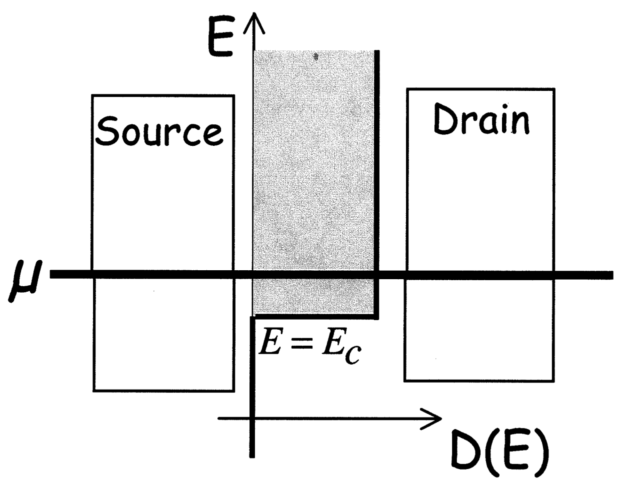
$$I = \frac{2q}{\hbar} \int dE\, D(E-U)\, \frac{\gamma_1 \gamma_2}{\gamma_1 + \gamma_2} [f_1(E) - f_2(E)]$$

where $f_1(E) = \dfrac{1}{e^{(E-\mu_1)/kT} + 1}$ and $f_2(E) = \dfrac{1}{e^{(E-\mu_2)/kT} + 1}$

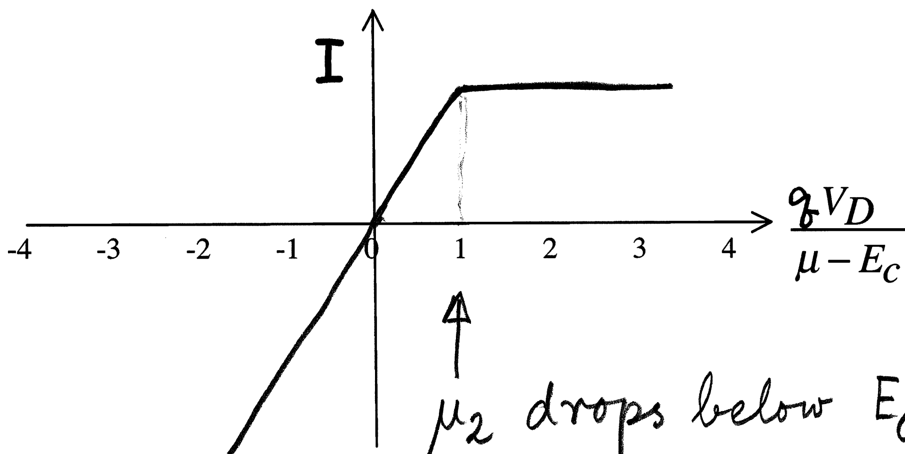
Also $U = U_L + U_0(N - N_0)$. Assume $U_0 = 0$ and the Laplace potential U_L to be a fraction α of the drain potential V_D (the source potential is assumed zero):

$$U_L = -\, q\, \alpha V_D,\ \alpha \text{ being a constant between 0 and 1.}$$

A channel has a density of states as shown, namely a constant non-zero value for $E \geq E_C$ and zero for $E < E_C$. Assume that the equilibrium electrochemical potential μ is located above E_C as shown. Sketch the current versus drain voltage assuming that the electrostatic potential of the channel (a) remains fixed with respect to the source ($\alpha = 0$) and (b) assumes a value halfway between the source and drain potentials ($\alpha = 0.5$),
Explain your reasoning clearly.

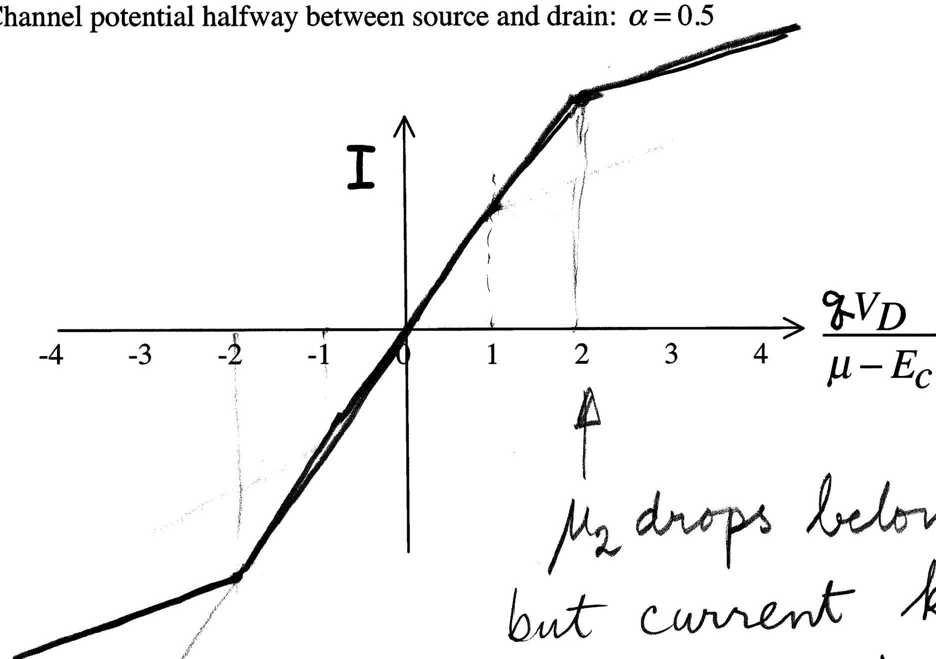


(a) Channel potential fixed with respect to source: $\alpha = 0$.



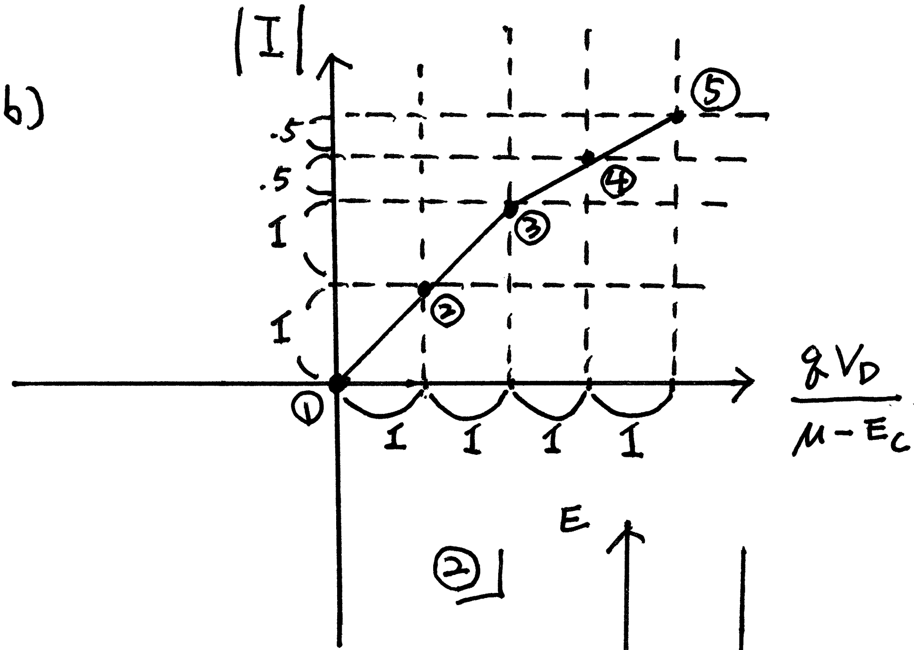
μ_2 drops below E_c
no further increase in I.

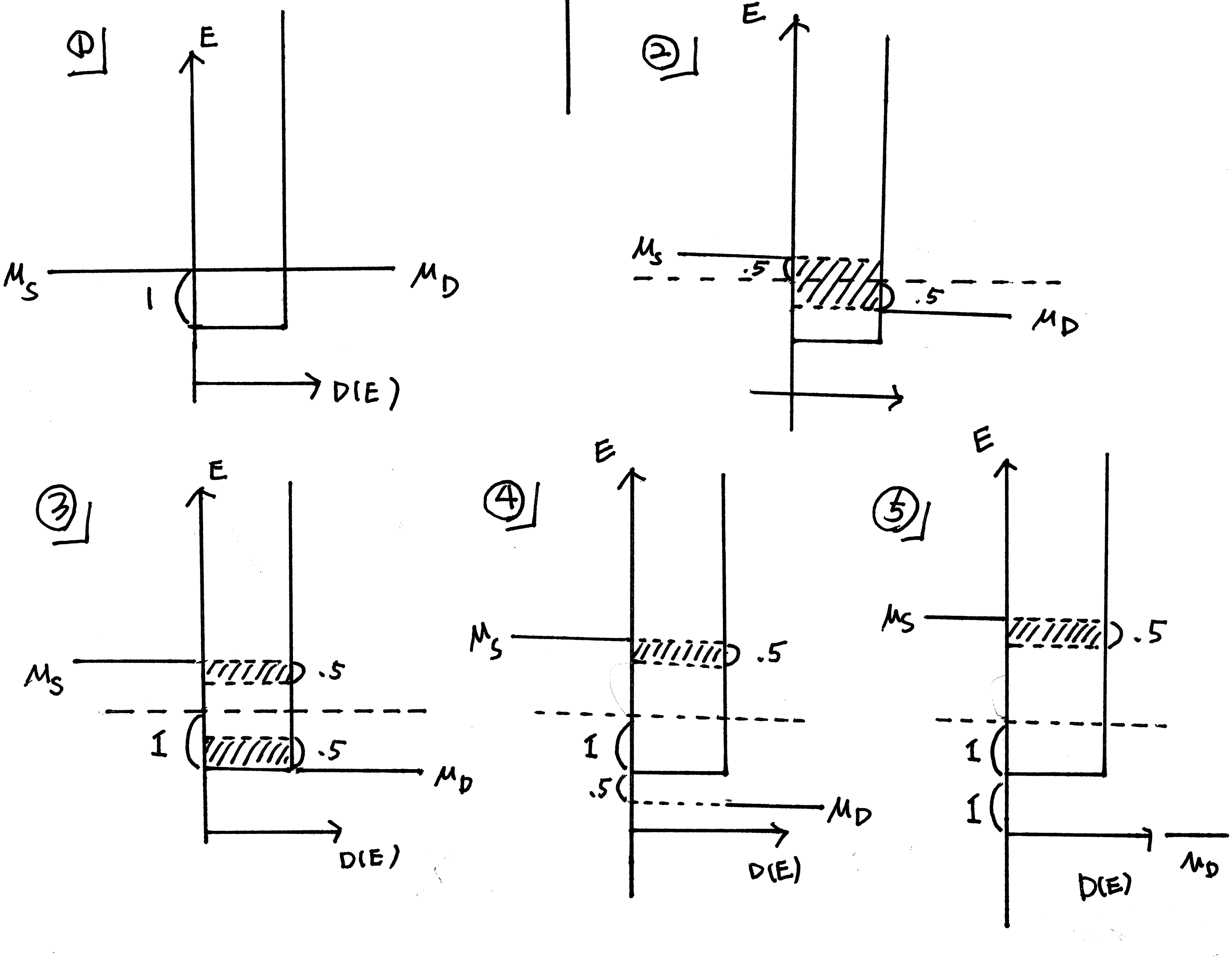
(b) Channel potential halfway between source and drain: $\alpha = 0.5$



μ_2 drops below E_c.
but current keeps increasing at half the rate because μ_1 keeps rising relative to channel.

Problem 1-(b)





: Additional current

: Equilibrium chemical potential

Problem 2: We have seen in class that free electrons in the absence of any external potential are described by (in one dimension)

$$i\hbar\frac{\partial\psi}{\partial t} = -\frac{\hbar^2}{2m}\frac{\partial^2\psi}{\partial x^2} \qquad (1)$$

whose solutions can be written in the form $\psi(x,t) = \underbrace{C}_{constant}\, e^{+ikx}\, e^{-iEt/\hbar}$ (2)

with E and k related by the dispersion relation: $E = \hbar^2 k^2/2m$ (3)

We have also seen that if the electrons are confined in a box of length L, the energy levels become discrete with the lowest energy given by $E_1 = \hbar^2\pi^2/2mL^2$ (4)

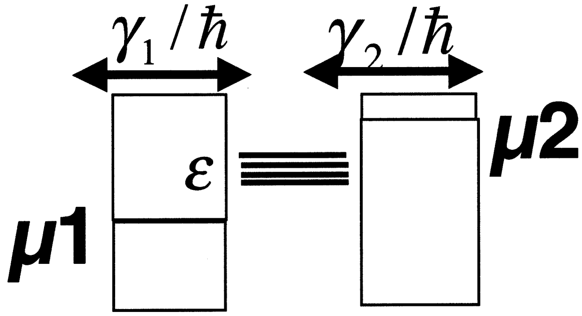
(a) Can you suggest a suitable differential equation to replace (1) if you wanted the dispersion relation to look like $E = Ak^4$ (3')
(A being a constant) instead of (3) ?

(b) If a system of electrons with a dispersion relation given by (3') were confined in a box of length L, how would the expression for the lowest energy given in (4) be modified?

(a) $i\hbar\dfrac{\partial\psi}{\partial t} = A\dfrac{\partial^4\psi}{\partial x^4}$

(b) $E_1 = A\cdot(\pi/L)^4$

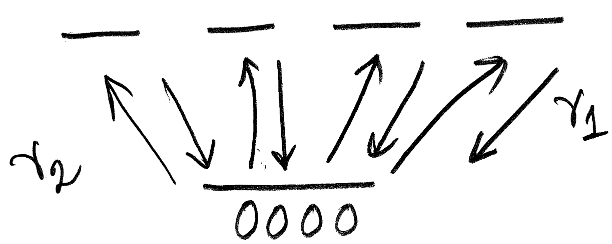
Problem 3:



A box has four degenerate energy levels all having energy ε. We know that for non-interacting electrons the maximum current under bias is

$$I = \frac{q}{\hbar}\frac{4\gamma_1\gamma_2}{\gamma_1+\gamma_2}$$

Use the multielectron picture to derive the correct expression for the maximum current if the electron-electron interaction energy is so high that no more than one electron can be inside the box at the same time.



$$\underbrace{P_{0000}}_{\equiv P_0} \cdot 4\gamma_2 = \underbrace{\left(P_{0001} + P_{0010} + P_{0100} + P_{1000}\right)}_{\equiv P_1} \cdot \gamma_1$$

$$\frac{P_1}{P_0} = \frac{4\gamma_2}{\gamma_1} \Rightarrow \frac{P_1}{\underbrace{P_0 + P_1}_{=1}} = \frac{4\gamma_2}{\gamma_1 + 4\gamma_2}$$

$$\boxed{I = \frac{q}{\hbar}\gamma_1 P_1 = \frac{q}{\hbar}\frac{4\gamma_1\gamma_2}{\gamma_1 + 4\gamma_2}}$$