EE-612:
Lecture 8
MOSFET IV: Part 2

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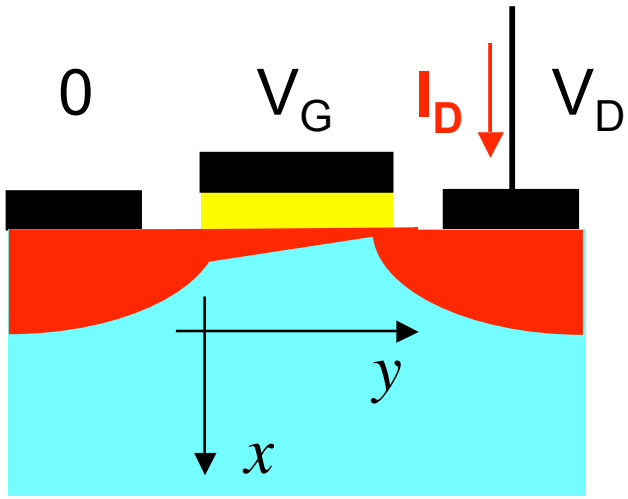
outline

1) Brief review
2) Bulk charge theory
3) Discussion

I-V formulation



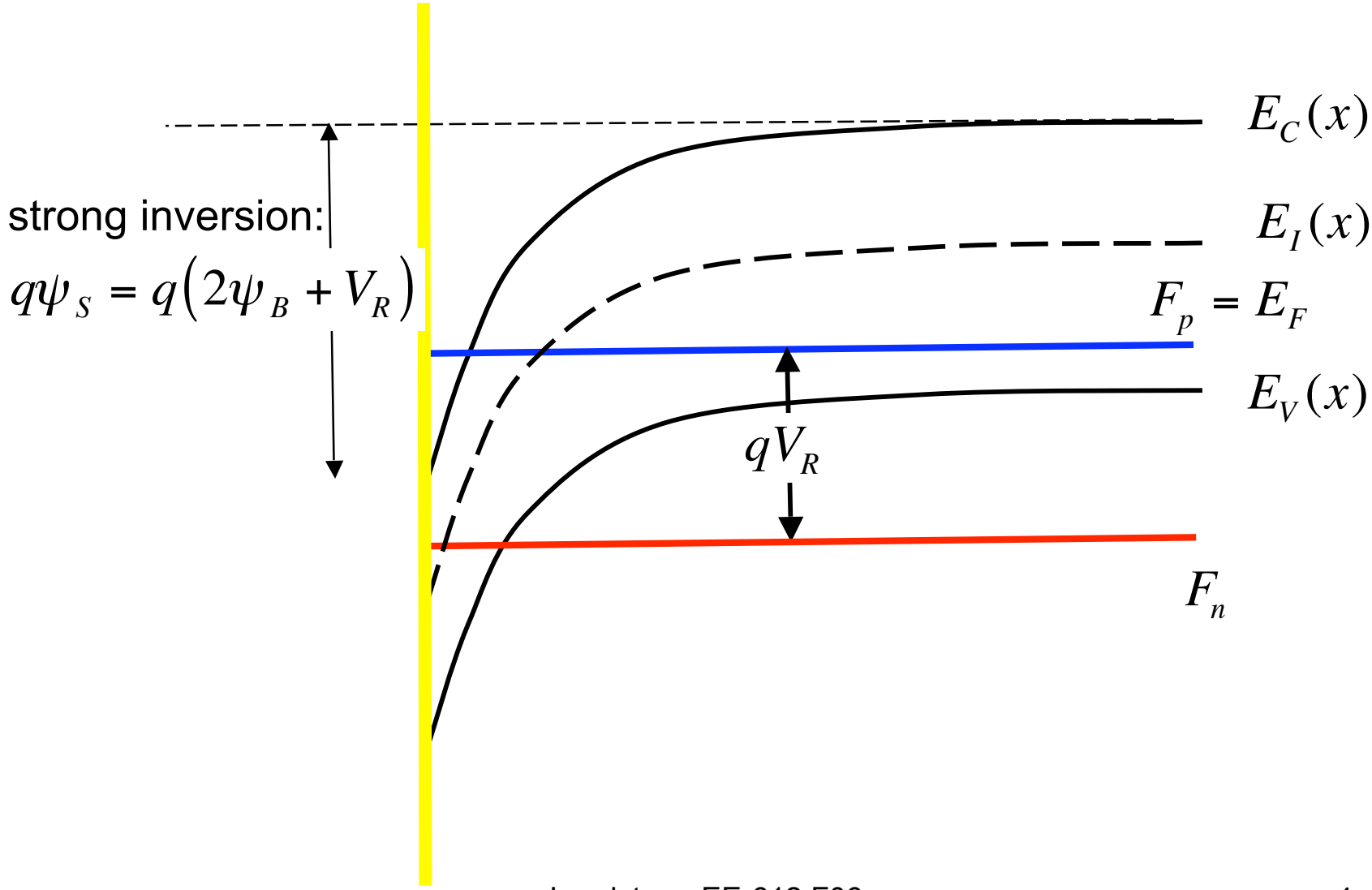
$$I_D = W\,Q_i(y)\,\upsilon_y(y)$$

$$I_D = -W\,Q_i(y)\,\mu_{eff}\frac{dV}{dy}$$

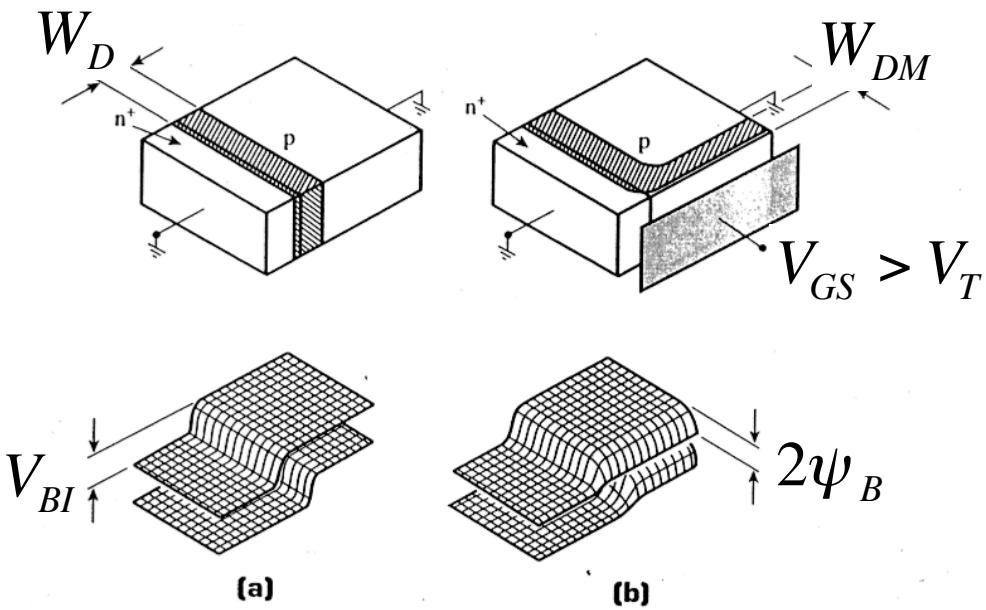
$$I_D dy = -W\,Q_i(V)\,\mu_{eff}\,dV$$

$$I_D = -\frac{W}{L}\mu_{eff}\int_0^{V_{DS}} Q_i(V)\,dV$$

effect of a reverse bias



effect of a reverse bias

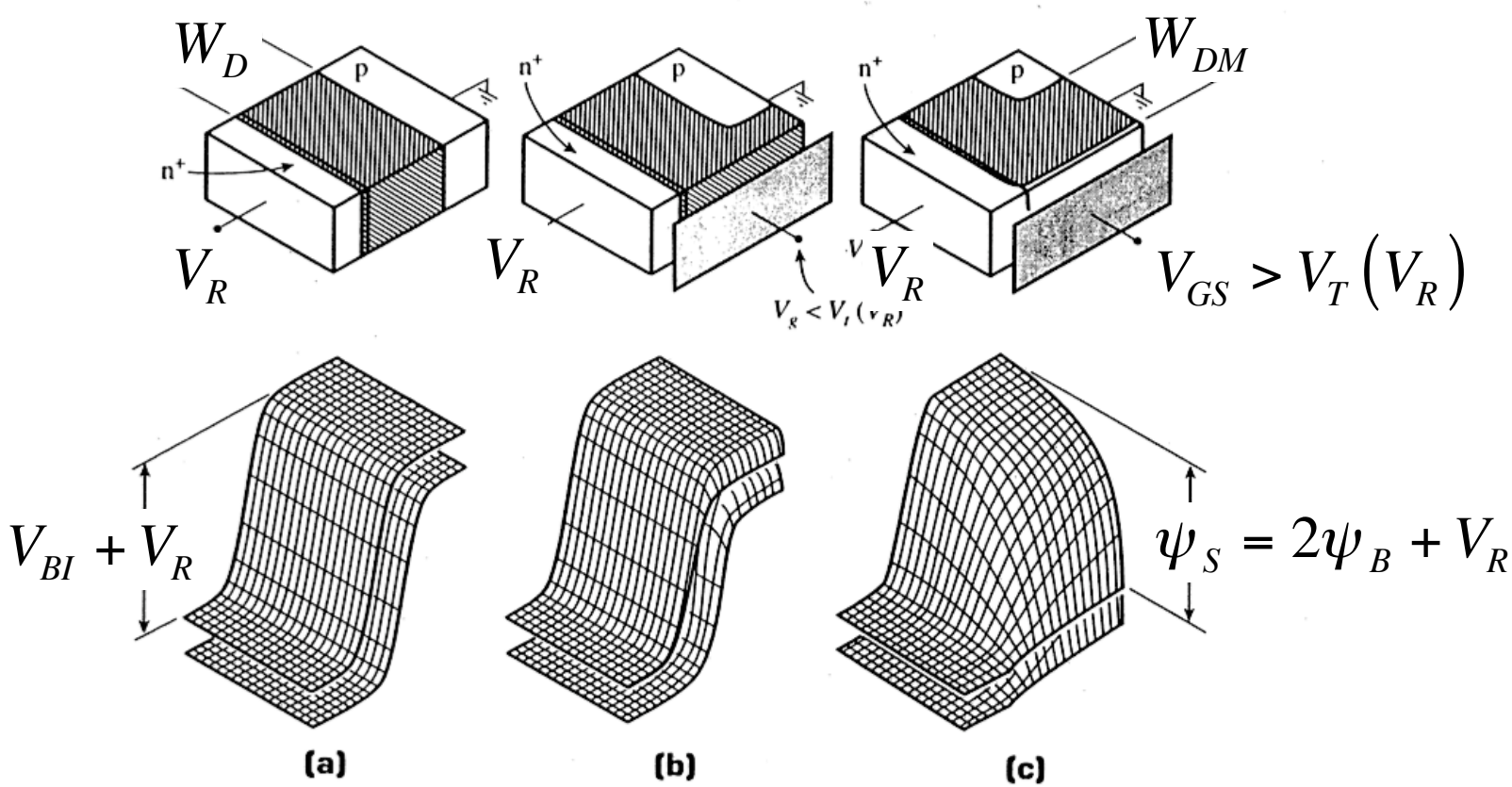


Gated doped or p-MOS with adjacent n^+ region

a) gate biased at flat-band
b) gate biased in inversion

A. Grove, *Physics of Semiconductor Devices*, 1967.

effect of a reverse bias

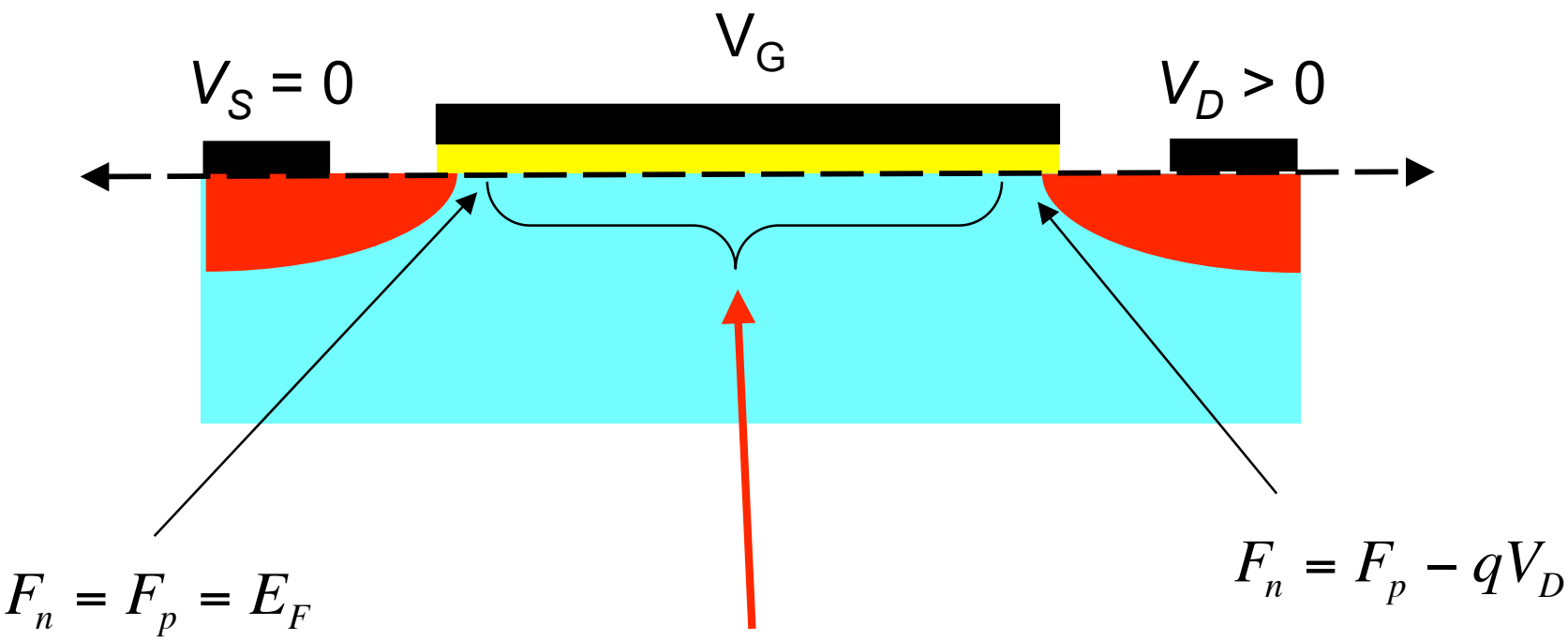


Gated doped or p-MOS with adjacent, reverse-biased n^+ region

a) gate biased at flat-band
b) gate biased in depletion
b) gate biased in inversion

A. Grove, *Physics of Semiconductor Devices*, 1967.

the MOSFET



F_n increasingly negative from source to drain
(reverse bias increases from source to drain)

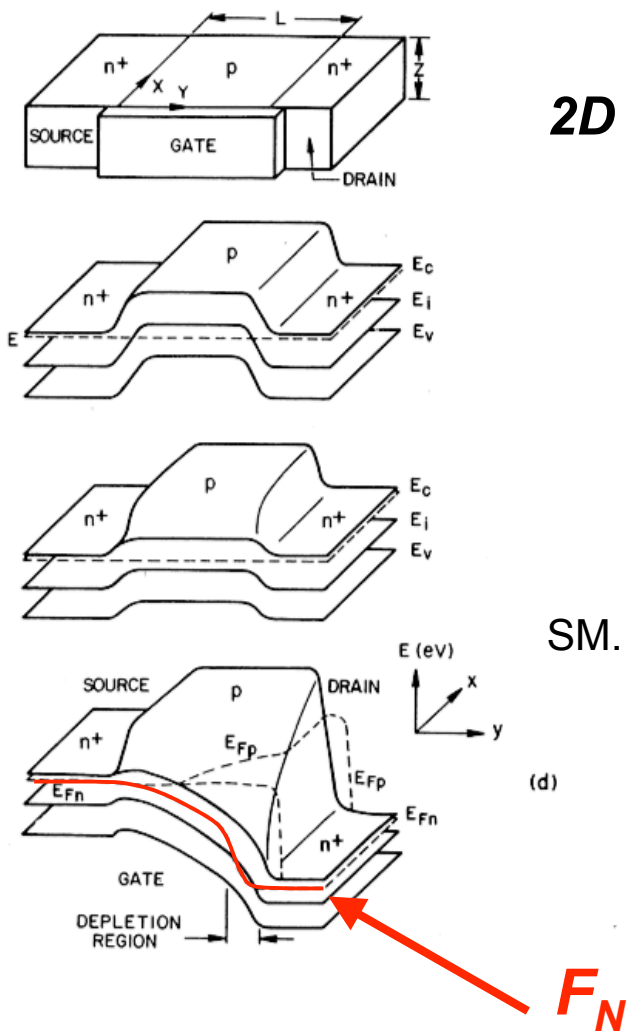
the MOSFET

(a)

(b)

(c)

(d)



2D e-band diagram for an n-MOSFET

a) device

b) equilibrium (flat band)

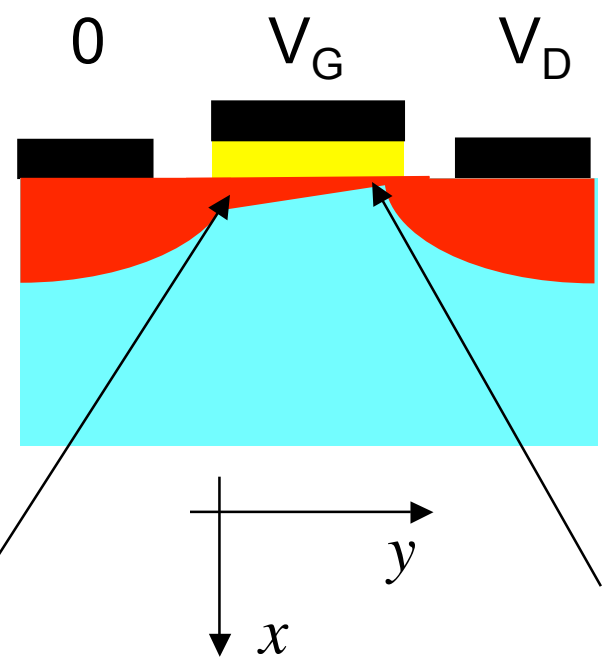
c) equilibrium ($\psi_S > 0$)

d) non-equilibrium with V_G and V_D >0 applied

SM. Sze, *Physics of Semiconductor Devices*, 1981 and Pao and Sah.

ψ_S vs. y in a MOSFET

$$V_{GS} - V_T > 0$$



$$\psi_S = 2\psi_B$$

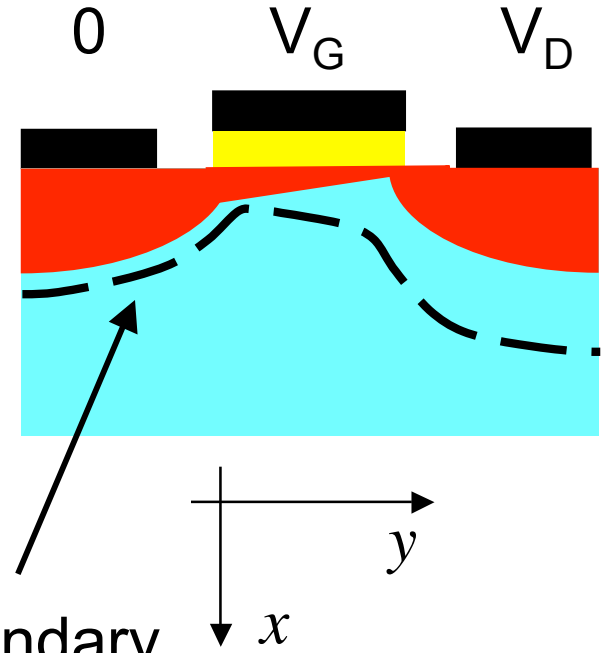
$$\psi_S = 2\psi_B + V_D$$

$$Q_D = -qN_A W_{dm} = -\sqrt{2q\varepsilon_{Si} N_A (2\psi_B)}$$

$$Q_D = -\sqrt{2q\varepsilon_{Si} N_A (2\psi_B + V_D)}$$

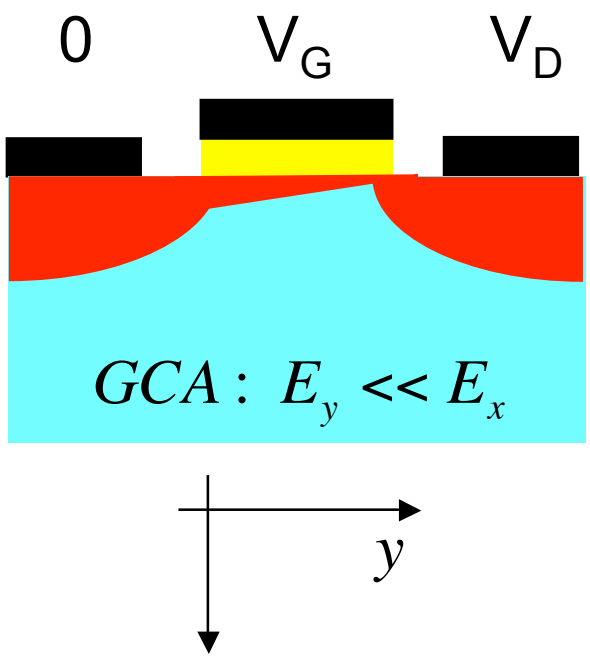
variation of bulk charge

$V_{GS} - V_T > 0$



depletion layer boundary

square law theory



need $Q_i(y)$ in the channel

$$MOS-C:$$

$$Q_i = -C_G\left(V_G - V_T\right)$$

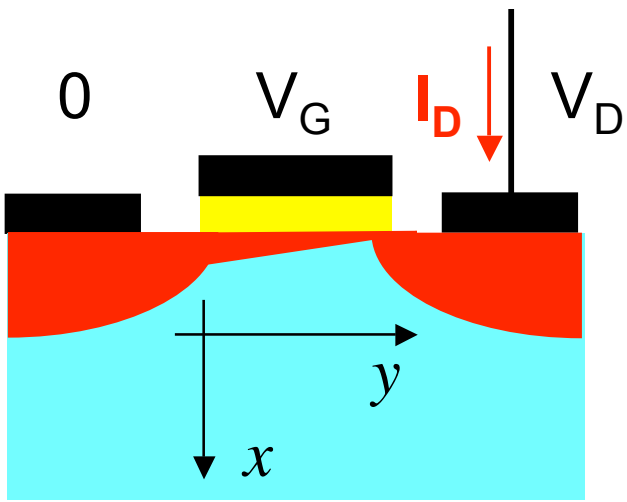
$$MOSFET:$$

$$Q_i(y) = -C_G\left[V_G \underbrace{- V_T - V(y)}_{V_T'(y)}\right]$$

outline

1) Brief review
2) **Bulk charge theory**
3) Discussion

I-V formulation



$$I_D = W\, Q_i(y)\, \upsilon_y(y)$$

$$I_D = -\frac{W}{L}\mu_{eff}\int_0^{V_{DS}} Q_i\left[V(y)\right]dV$$

$$Q_i(y) = -C_G\left(V_G - V_T'(y)\right)$$

local V_T along the channel

no reverse bias:

$$V_T = V_{FB} + 2\psi_B + \sqrt{2q\varepsilon_{Si}N_A(2\psi_B)}\Big/C_{OX}$$

with reverse bias:

$$V_T'(y) = V_{FB} + 2\psi_B + V(y) + \sqrt{2q\varepsilon_{Si}N_A\left(2\psi_B + V(y)\right)}\Big/C_{OX}$$

bulk charge

V_T of the MOSFET

$$V_T'(y) = V_{FB} + 2\psi_B + V(y) + \sqrt{2q\varepsilon_{Si}N_A\left(2\psi_B + V(y)\right)}\Big/C_{OX}$$

V_T for the MOSFET:

$$V_T = V_T'(y=0) = V_{FB} + 2\psi_B + \sqrt{2q\varepsilon_{Si}N_A(2\psi_B)}\Big/C_{OX}$$

IV relation

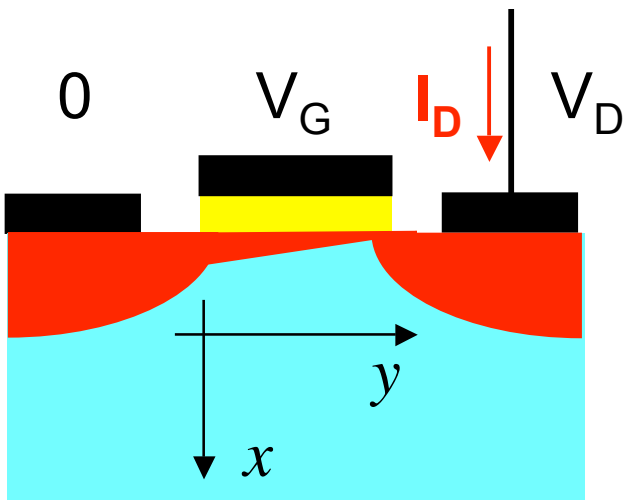
$$I_D = -\frac{W}{L}\mu_{eff}\int_0^{V_{DS}} Q_i(V)\,dV \qquad (1)$$

$$Q_i(y) = -C_G\left(V_G - V_{FB} - 2\psi_B - V(y) - \frac{\sqrt{2q\varepsilon_{Si}N_A\left(2\psi_B + V(y)\right)}}{C_{OX}}\right) \qquad (2)$$

Insert (2) in (1) and integrate, find eqn. (3.18) of Taur and Ning, but 3/2 powers are inconvenient.

(See Pierret, for a more extended discussion of the bulk charge theory.)

I-V formulation



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IV relation

$$I_D = -\frac{W}{L}\mu_{eff}\int_0^{V_{DS}} Q_i(V)\,dV \qquad (1)$$

$$Q_i(y) = -C_G\left(V_G - V_{FB} - 2\psi_B - V(y) - \frac{\sqrt{2q\varepsilon_{Si}N_A\left(2\psi_B + V(y)\right)}}{C_{OX}}\right) \qquad (2)$$

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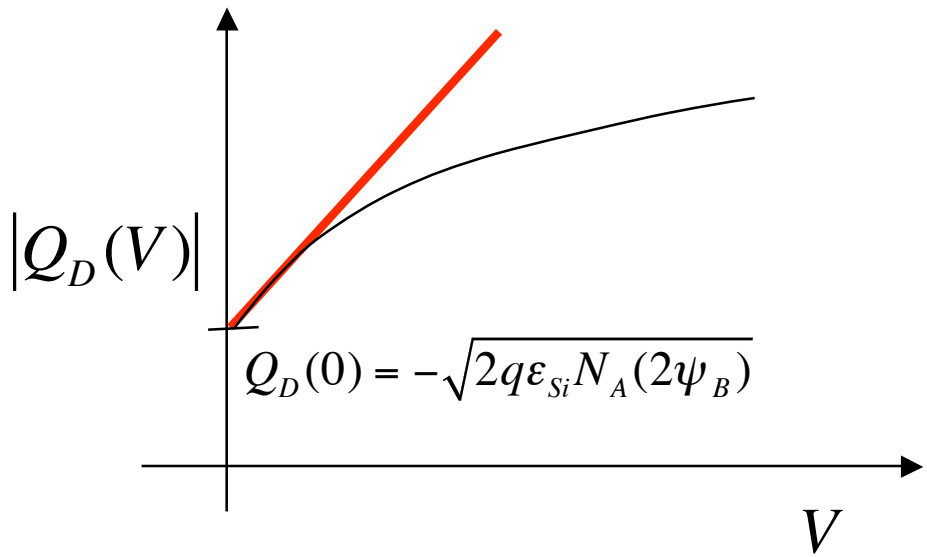
approximate Q_D

$$Q_D(V) = -\sqrt{2q\varepsilon_{Si}N_A(2\psi_B + V)}$$



$$Q_D(V) = Q_D(0) + \left.\frac{dQ_D}{dV}\right|_{V=0} V + \ldots$$

$$\frac{dQ_D}{dV} = -\frac{\varepsilon_{Si}}{W_{DM}} = -C_{DM}$$

$$Q_i(V) = -C_G\left(V_G - V_{FB} - 2\psi_B - V + \frac{Q_D(2\psi_B)}{C_{OX}} - \frac{C_{DM}}{C_{OX}}V\right)$$

IV relation

$$I_D = -\frac{W}{L}\mu_{eff}\int_0^{V_{DS}} Q_i(V)\,dV \qquad (1)$$

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approximate Q_D

$$Q_i(V) = -C_G\left(V_G - V_{FB} - 2\psi_B - V + \frac{Q_D(2\psi_B)}{C_{OX}} - \frac{C_{DM}}{C_{OX}}V\right)$$

$$Q_i(V) = -C_G\left(\underbrace{V_G - V_{FB} - 2\psi_B + \frac{Q_D(2\psi_B)}{C_{OX}}}_{-V_T} \underbrace{- V - \frac{C_{DM}}{C_{OX}}V}_{-\left(1 + C_{DM}/C_{OX}\right)V}\right)$$

$$\boxed{\begin{aligned} Q_i(y) &= -C_G\left(V_G - V_T - mV\right) \\ m &= \left(1 + C_{DM}/C_{OX}\right) \end{aligned}}$$

meaning of *m*

$$m = \left(1 + C_{DM}/C_{OX}\right)$$

'body effect coefficient'

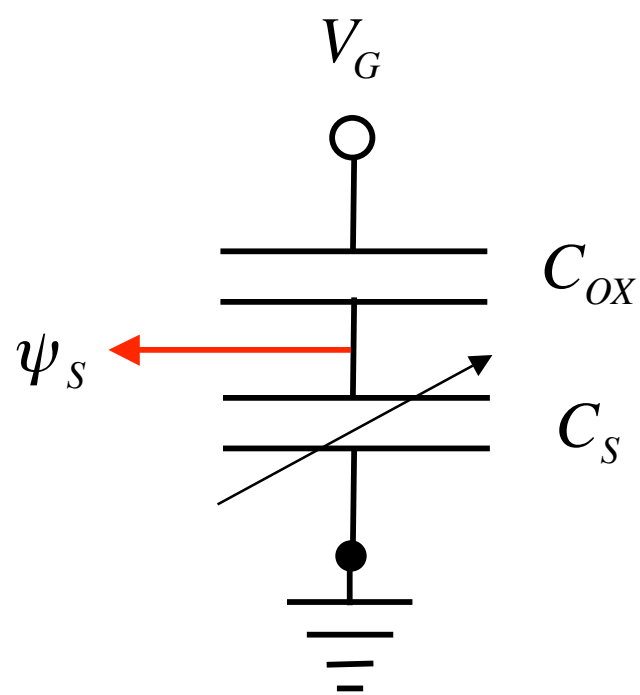
$$m = \left(1 + 3t_{OX}/W_{DM}\right)$$

also:

$$V_T = V_{FB} + (2m - 1)2\psi_B$$

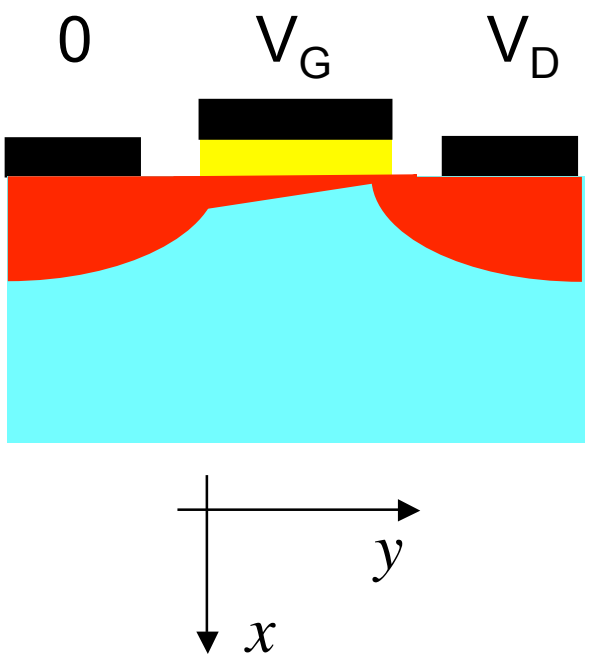
in practice:

$$1.1 \le m \le 1.4$$



$$\Delta\psi_S = \frac{C_{OX}}{C_{OX} + C_{DM}}\Delta V_G = \frac{\Delta V_G}{m}$$

IV relation

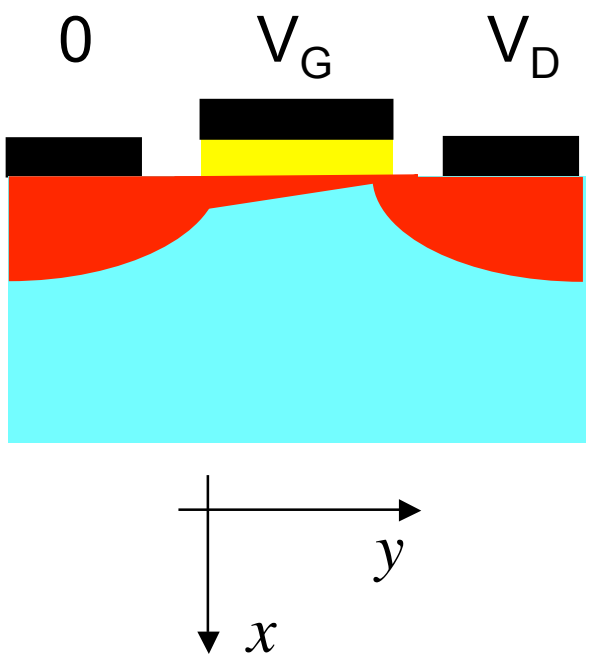


$$I_D = -\frac{W}{L}\mu_{eff}\int_0^{V_{DS}} Q_i[V]\,dV$$

$$I_D = \mu_{eff} C_G \frac{W}{L}\int_0^{V_D}\left[V_G - V_T - mV\right]dV$$

$$I_D = \mu_{eff} C_G \frac{W}{L}\left[\left(V_{GS} - V_T\right)V_{DS} - \frac{m}{2}V_{DS}^2\right]$$

pinch-off



$$Q_i(L) = -C_G\left[V_G - V_T - mV_D\right]$$

when $V_D = \left(V_G - V_T\right)/m$,

then $Q_i(L) = 0$

$E_y >> E_X$ GCA fails!

$$I_D = \mu_{eff} C_G \frac{W}{L}\left[\left(V_{GS} - V_T\right)V_{DS} - \frac{m}{2}V_{DS}^2\right]$$

$V_{GS} > V_T$

$V_{DS} < \left(V_{GS} - V_T\right)/m$

meaning of *m*

$$m = \left(1 + C_{DM}/C_{OX}\right)$$

'body effect coefficient'

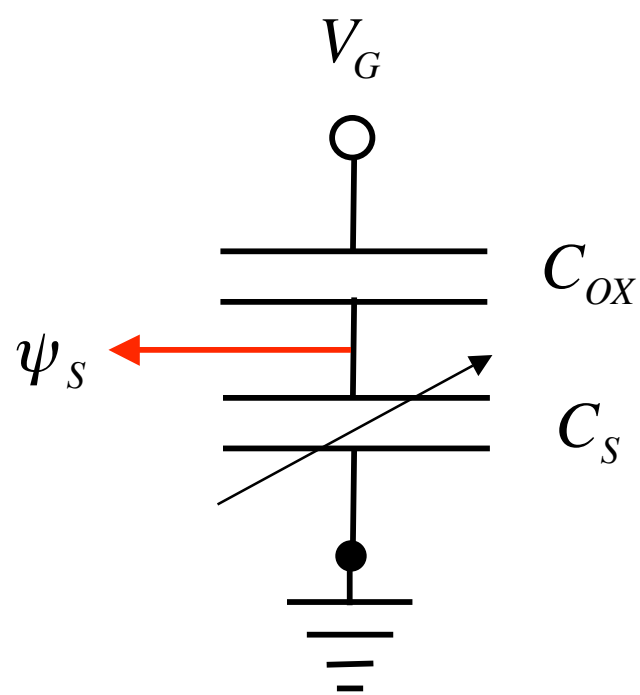
$$m = \left(1 + 3t_{OX}/W_{DM}\right)$$

also:

$$V_T = V_{FB} + (2m - 1)2\psi_B$$

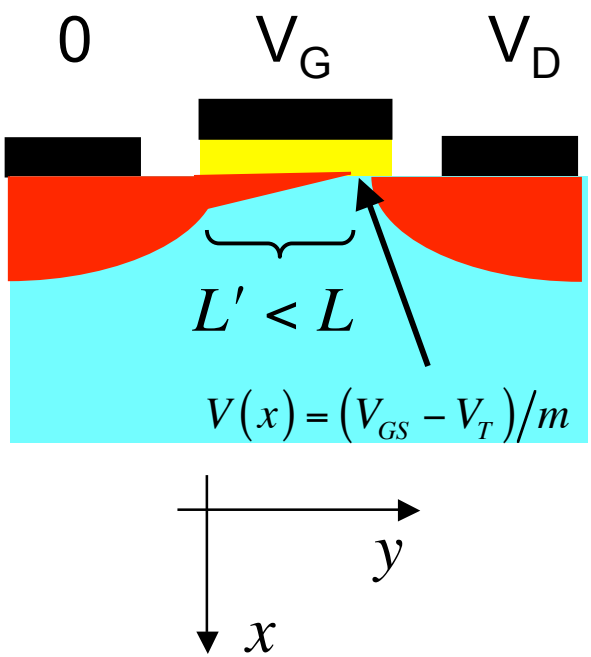
in practice:

$$1.1 \le m \le 1.4$$



$$\Delta\psi_S = \frac{C_{OX}}{C_{OX} + C_{DM}}\Delta V_G = \frac{\Delta V_G}{m}$$

beyond pinch-off, $V_{DS} > V_{DSAT}$



channel is pinched-off near the drain but current still flows.

$$I_D = -\frac{W}{L}\mu_{eff}\int_0^{V_{DSAT}} Q_i[V]dV$$

$$I_D \approx I_D\left(V_{DS} = (V_{GS} - V_T)/m\right)$$

$$I_D = \mu_{eff} C_G \frac{W}{2L'}\frac{(V_{GS} - V_T)^2}{m}$$

$$V_{GS} > V_T$$

$$V_{DS} > (V_{GS} - V_T)/m$$

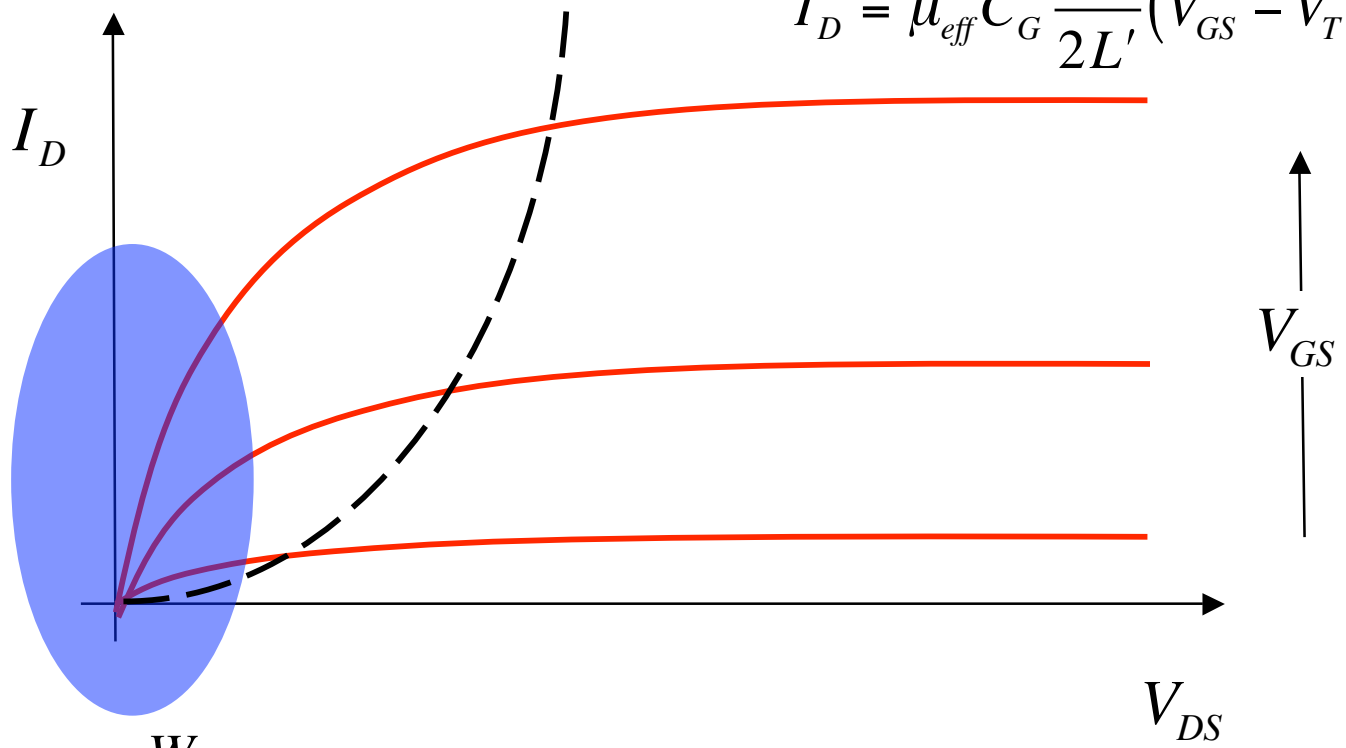
outline

1) Brief review
2) Bulk charge theory
3) **Discussion**

IV summary

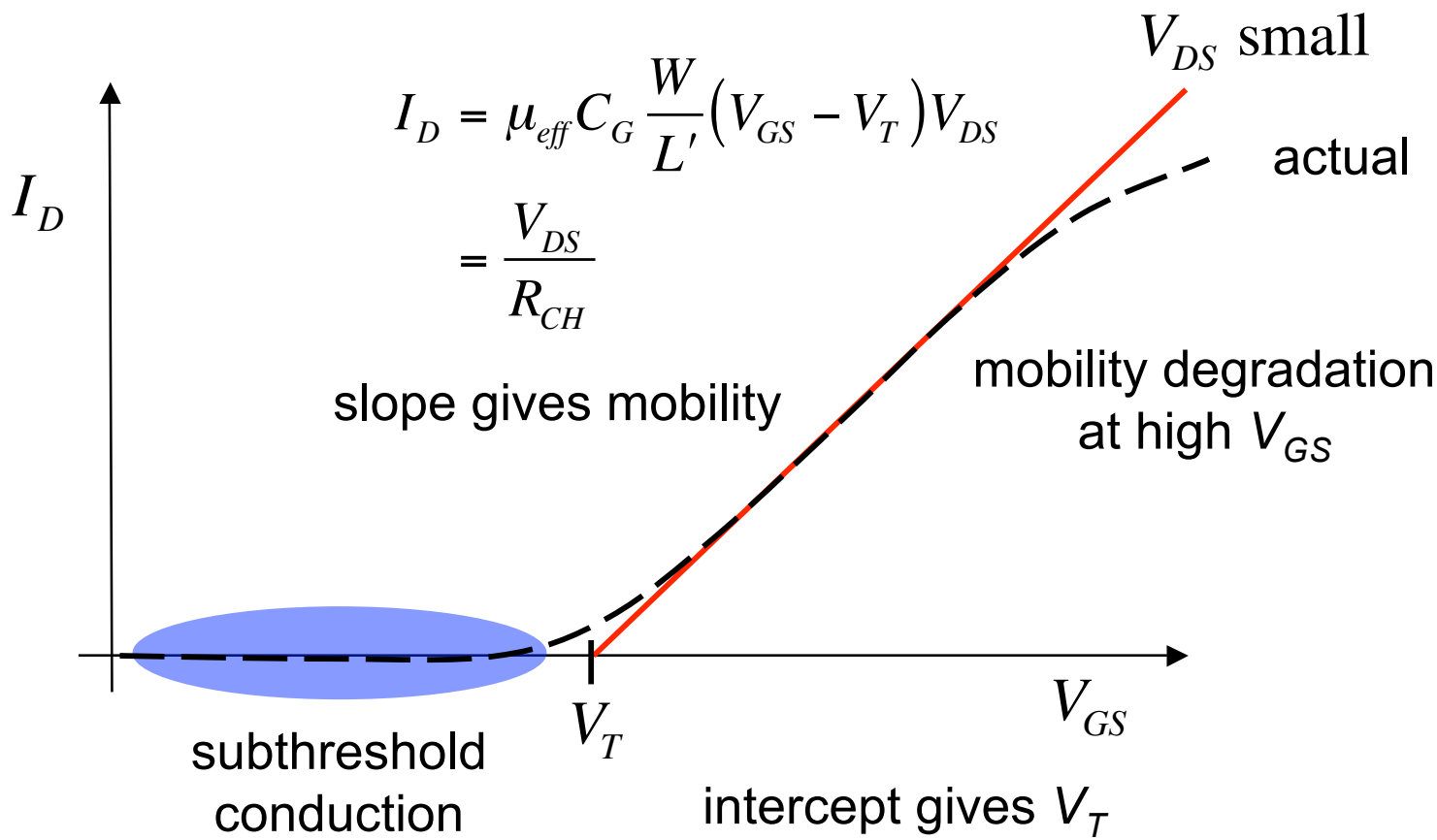
$$V_{DSAT} = \left(V_{GS} - V_T\right)/m$$

$$I_D = \mu_{eff} C_G \frac{W}{2L'}\left(V_{GS} - V_T\right)^2 / m$$



$$I_D = \mu_{eff} C_G \frac{W}{L'}\left(V_{GS} - V_T\right)V_{DS}$$

linear region (low V_{DS})



$$I_D = \mu_{eff} C_G \frac{W}{L'}\left(V_{GS} - V_T\right)V_{DS}$$

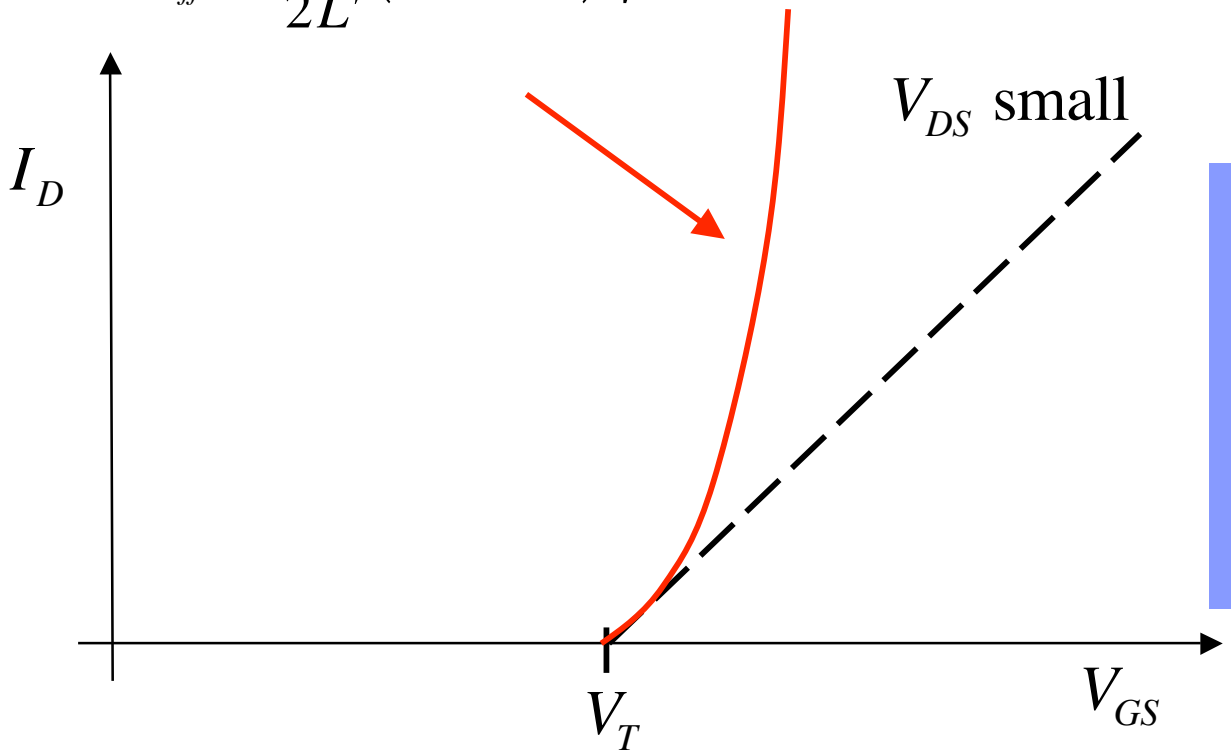
$$= \frac{V_{DS}}{R_{CH}}$$

slope gives mobility

mobility degradation at high V_{GS}

saturated region (high V_{DS})

$$I_{DSAT} = \mu_{eff} C_G \frac{W}{2L'} \left(V_{GS} - V_T\right)^2 / m$$



in practice:

$$I_{DSAT} \propto \left(V_{GS} - V_T\right)^{\alpha}$$

$$\alpha \approx 1.3$$

$$V_T(\text{SAT}) < V_T(\text{LIN})$$

MOSFET IV approaches

$$I_D = -\frac{W}{L}\mu_{eff}\int_0^{V_{DS}} Q_i(V)\,dV$$

1) "exact" (Pao-Sah or Pierret-Shields)

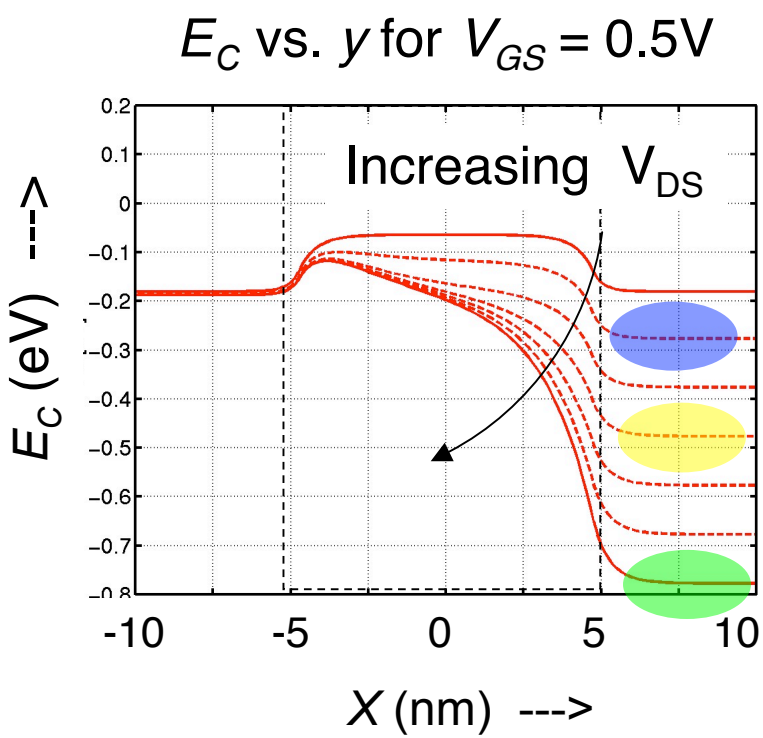
   see p. 117 Taur and Ning

2) Square Law $\quad Q_i(V) = -C_G\left[V_G - V_T - V)\right]$

3) Bulk Charge $\quad Q_i(V) = -C_G\left(V_G - V_{FB} - 2\psi_B - V - \frac{\sqrt{2q\varepsilon_{Si}N_A\left(2\psi_B + V\right)}}{C_{OX}}\right)$

4) Simplified Bulk Charge $\quad Q_i(V) = -C_G\left[V_G - V_T - mV)\right]$

physics of drain current saturation



1) Low V_{DS}:

$$Q_i(V) \approx -C_G\left[V_G - V_T\right]$$

$$E_y \approx V_{DS}/L$$

2) Larger V_{DS}:

$$Q_i(L) = -C_G\left[V_G - V_T - V(L)\right] < Q_i(0)$$

$$E_y(L) > E_y(0)$$

3) Larger V_{DS}:

$$Q_i(L) \approx 0$$

$$E_y(L) >> E_y(0)$$

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For a more detailed examination of MOSFET I-V theory, see: J.R. Brews, "A Charge Sheet Model of the MOSFET," *Solid-State Electronics*, Vol. 21, pp. 345-355, 1978.