NCN@Purdue Summer School, July 20-24, 2009

Graphene PN Junctions

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acknowledgments

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outline

1) Introduction

2) Electron optics in graphene

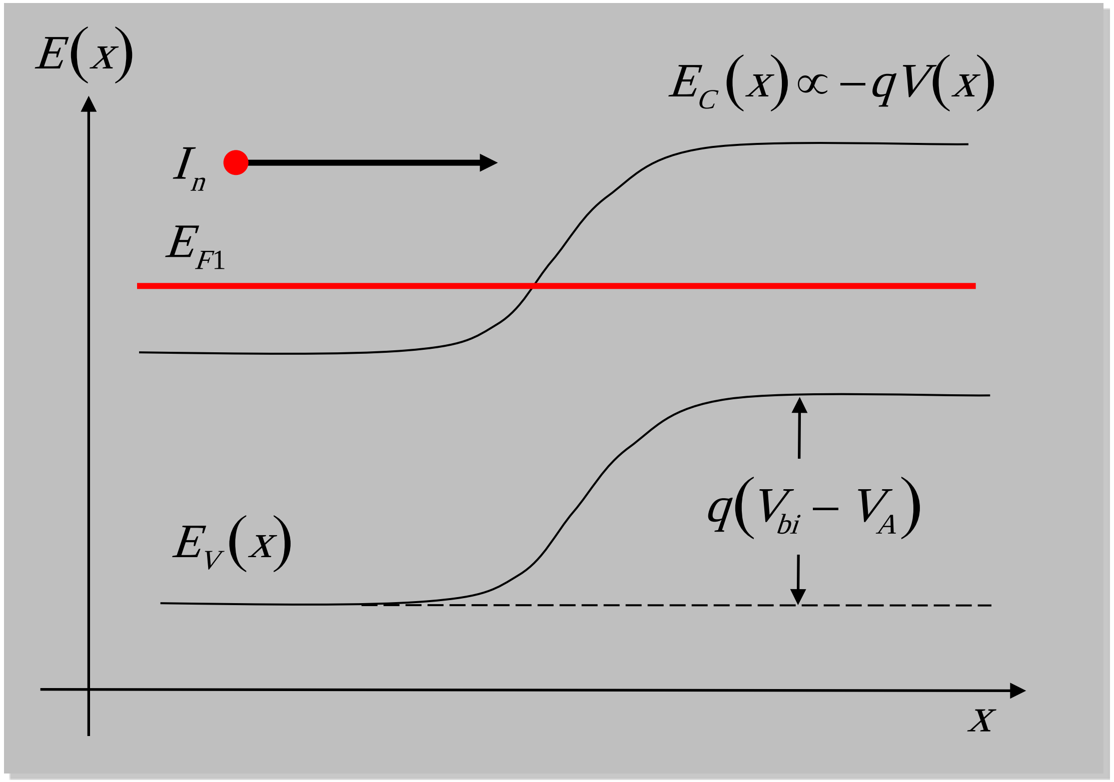
3) Transmission across NP junctions

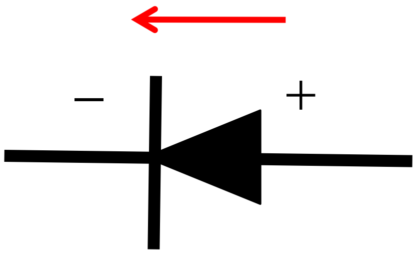
4) Conductance of PN and NN junctions

5) Discussion

6) Summary

PN junctions: semiconductors vs. graphene

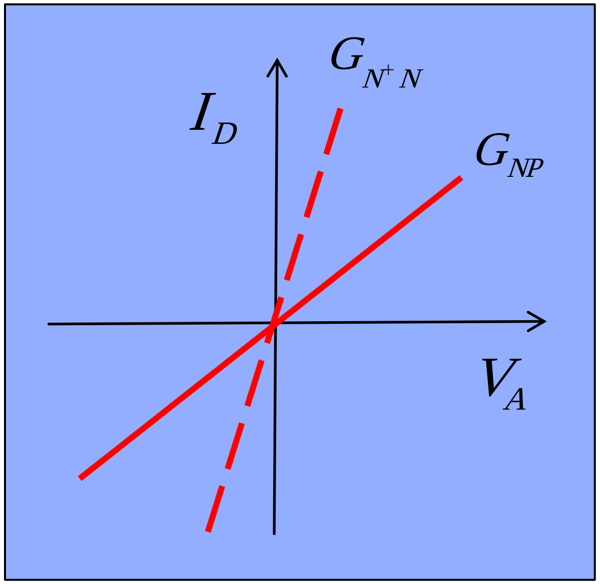




$$I = I_0 e^{qV_A/k_bT}$$

$$I_0 \propto n_i^2 \propto e^{-E_G/k_bT}$$

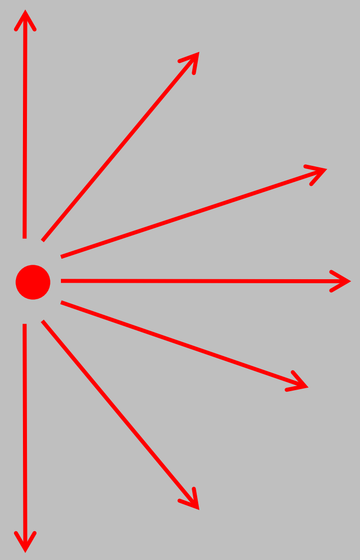
experimental observation



$$G_{NP} < G_{N^+N}$$

B. Huard, J.A. Sulpizo, N. Stander, K. Todd, B. Yang, and D. Goldhaber-Gordon, Transport measurements across a tunable potential barrier in graphene,” *Phys. Rev. Lett*., **98**, 236803, 2007.

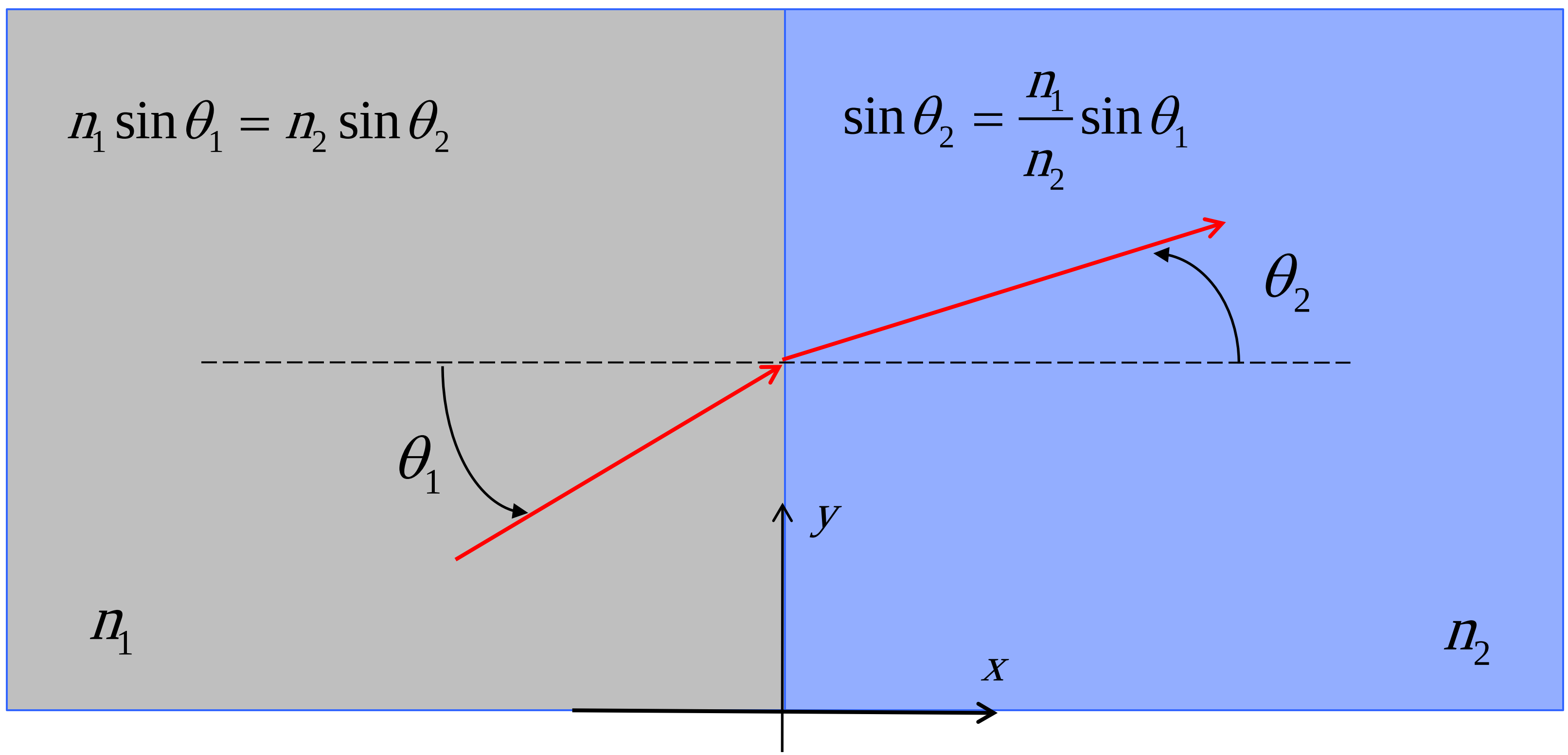
electron "optics" in graphene



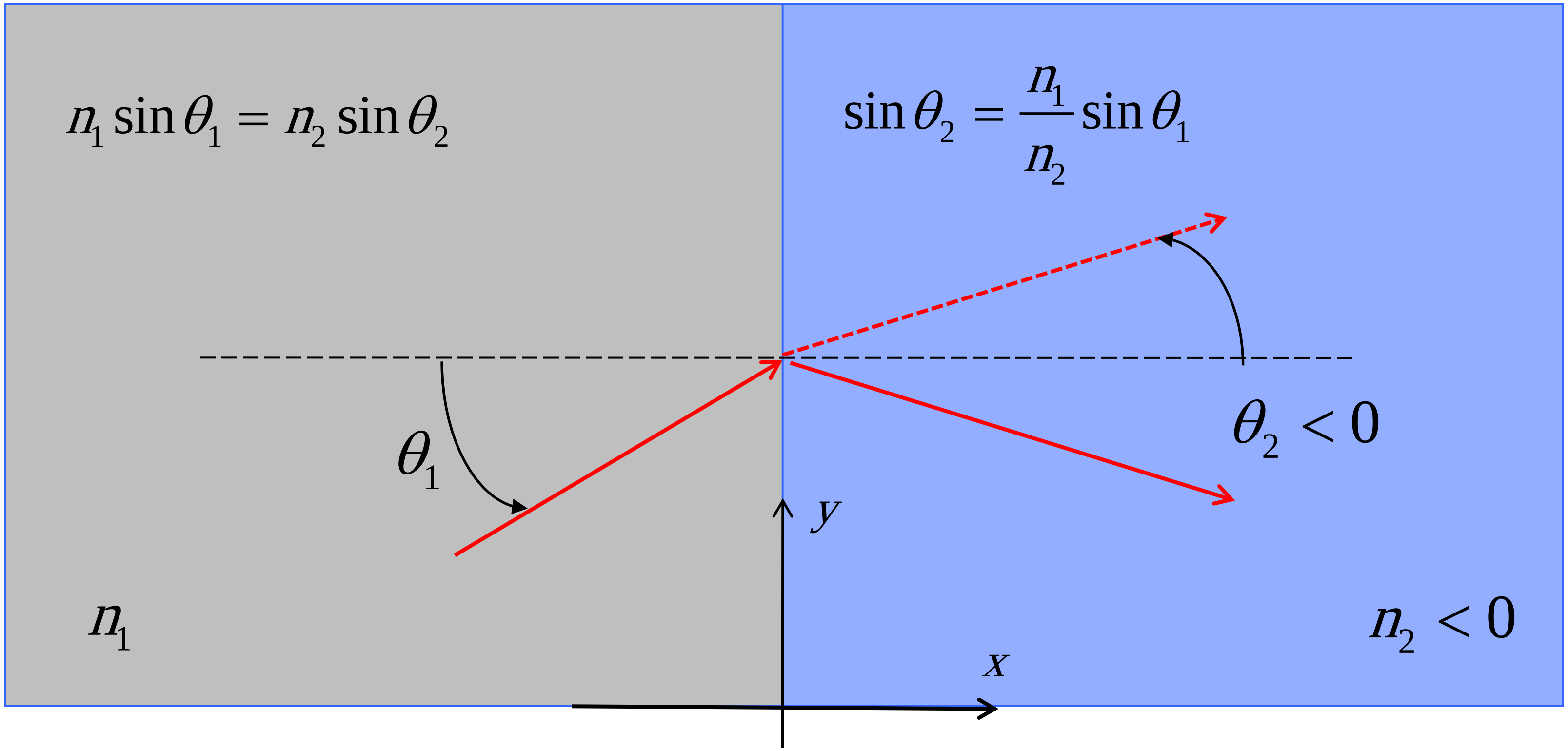
Semiclassical electron trajectories are analogous to the rays in geometrical optics.

If the mfp is long, one may be able to realize graphene analogues of optical devices.

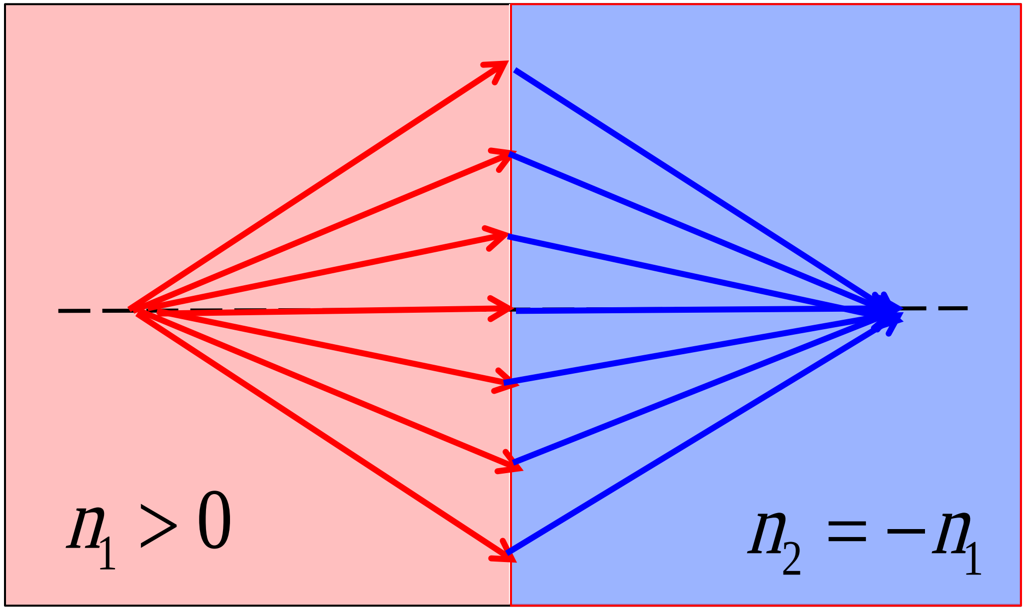
Snell's Law


$n_1 \sin\theta_1 = n_2 \sin\theta_2$
$\sin\theta_2 = \frac{n_1}{n_2}\sin\theta_1$
θ_2
θ_1
y
x
n_1
n_2

negative index of refraction



Veselago lens



theoretical prediction

The Focusing of Electron Flow and a Veselago Lens in Graphene *p-n* Junctions

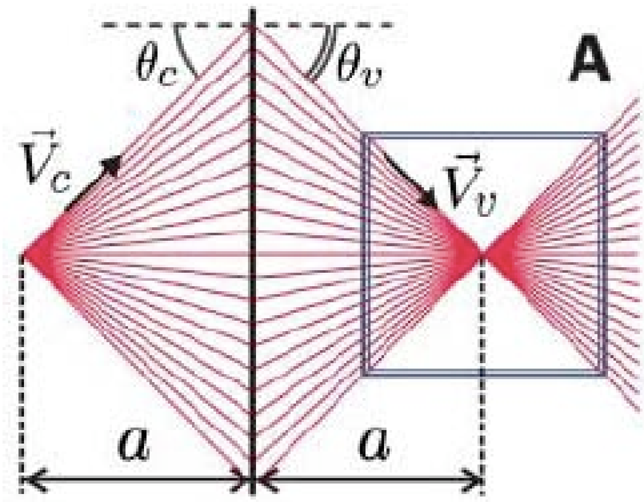
Vadim V. Cheianov,[1*] Vladimir Fal'ko,[1] B. L. Altshuler[2,3]

The focusing of electric current by a single *p-n* junction in graphene is theoretically predicted. Precise focusing may be achieved by fine-tuning the densities of carriers on the *n*- and *p*-sides of the junction to equal values. This finding may be useful for the engineering of electronic lenses and focused beam splitters using gate-controlled *n-p-n* junctions in graphene-based transistors.

There are many similarities between optics and electronics. Rays in geometrical optics are analogous to classical trajectories of electrons, whereas electron de Broglie waves can interfere. The electron microscope is one example of the technological implementation of this similarity. The analogy with optics may also hold considerable potential for semiconductor elec-

Science, **315**, 1252, March 2007

electron trajectories



N-type P-type

making graphene PN junctions

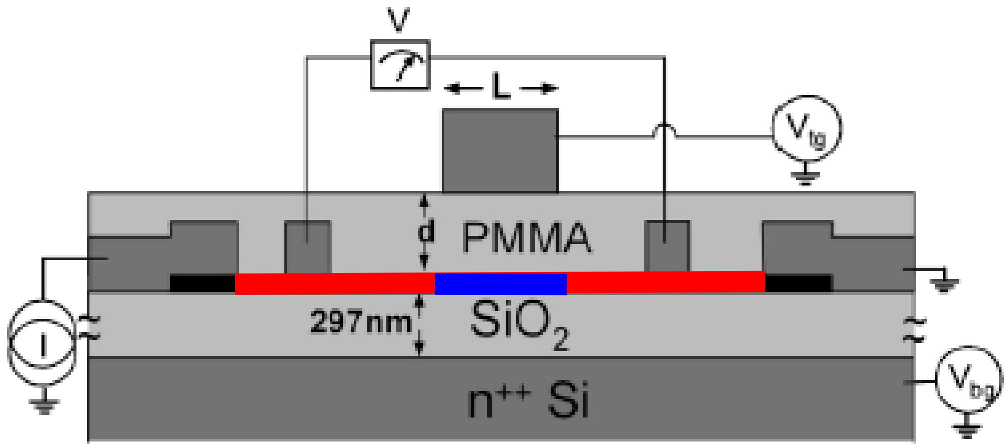


FIG. 1. Schematic diagram of a top-gated graphene device with a four-probe measurement setup. Graphene sheet is black, metal contacts and gates are dark gray.

From: N. Stander, B. Huard, and D. Goldhaber-Gordon, “Evidence for Klein Tunneling in Graphene p-n Junctions,” *PRL* **102**, 026807 (2009)

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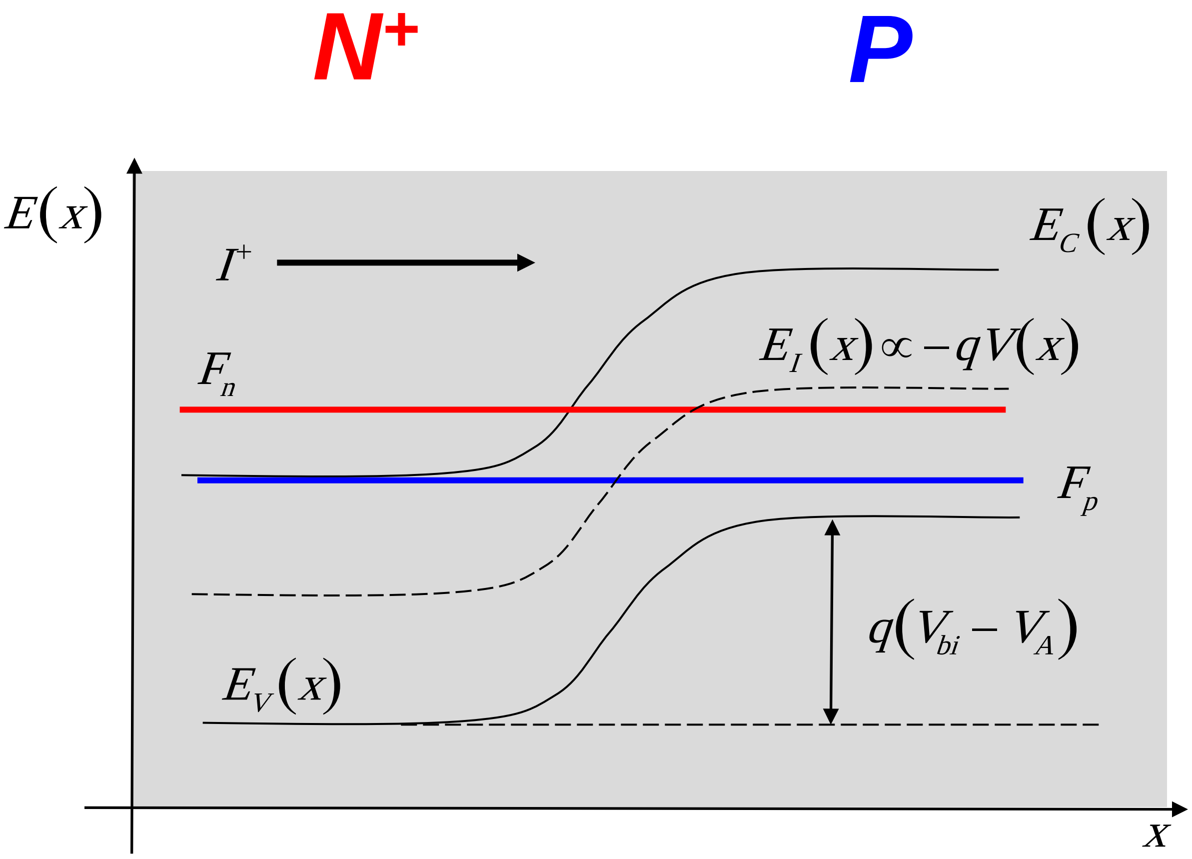
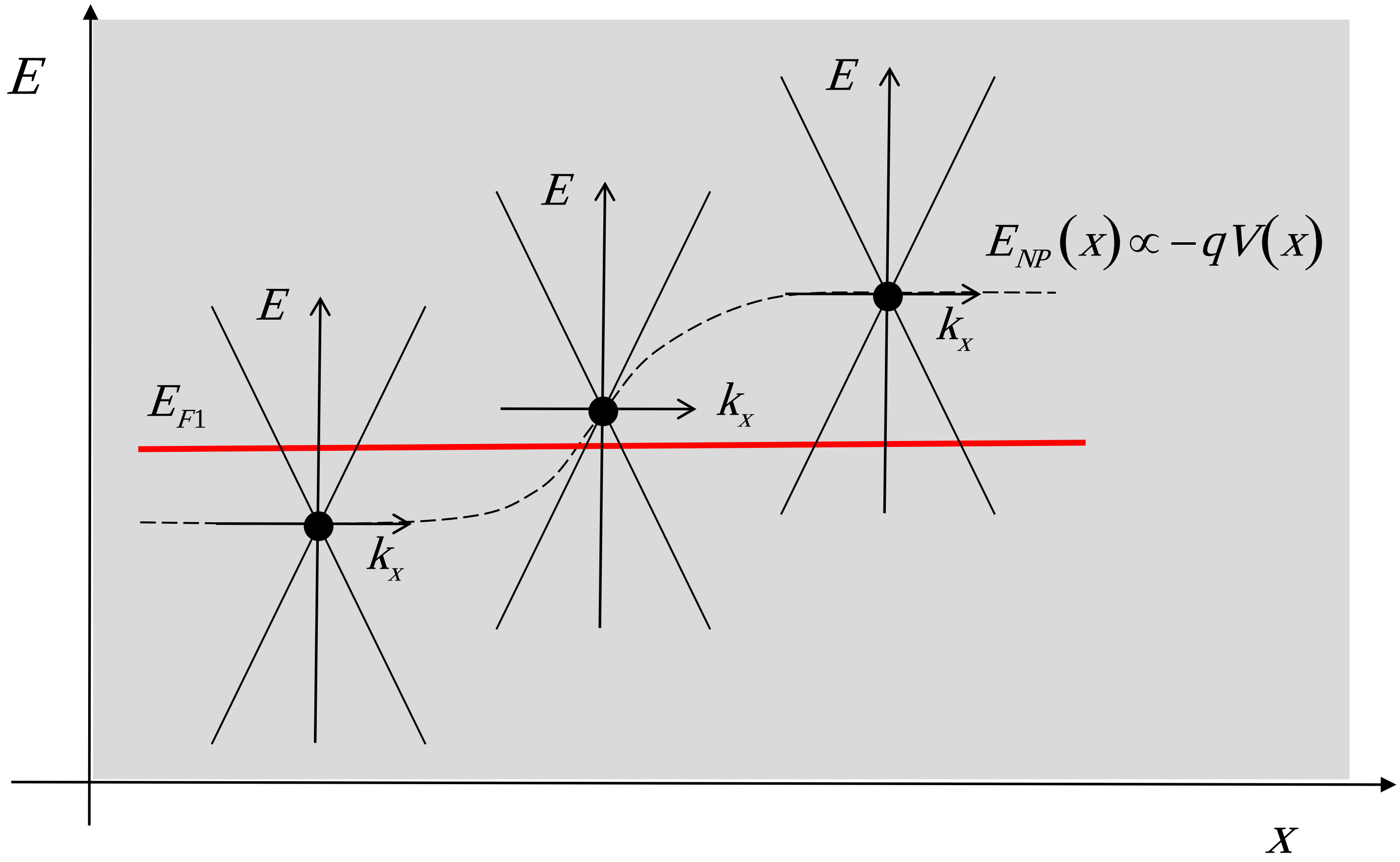
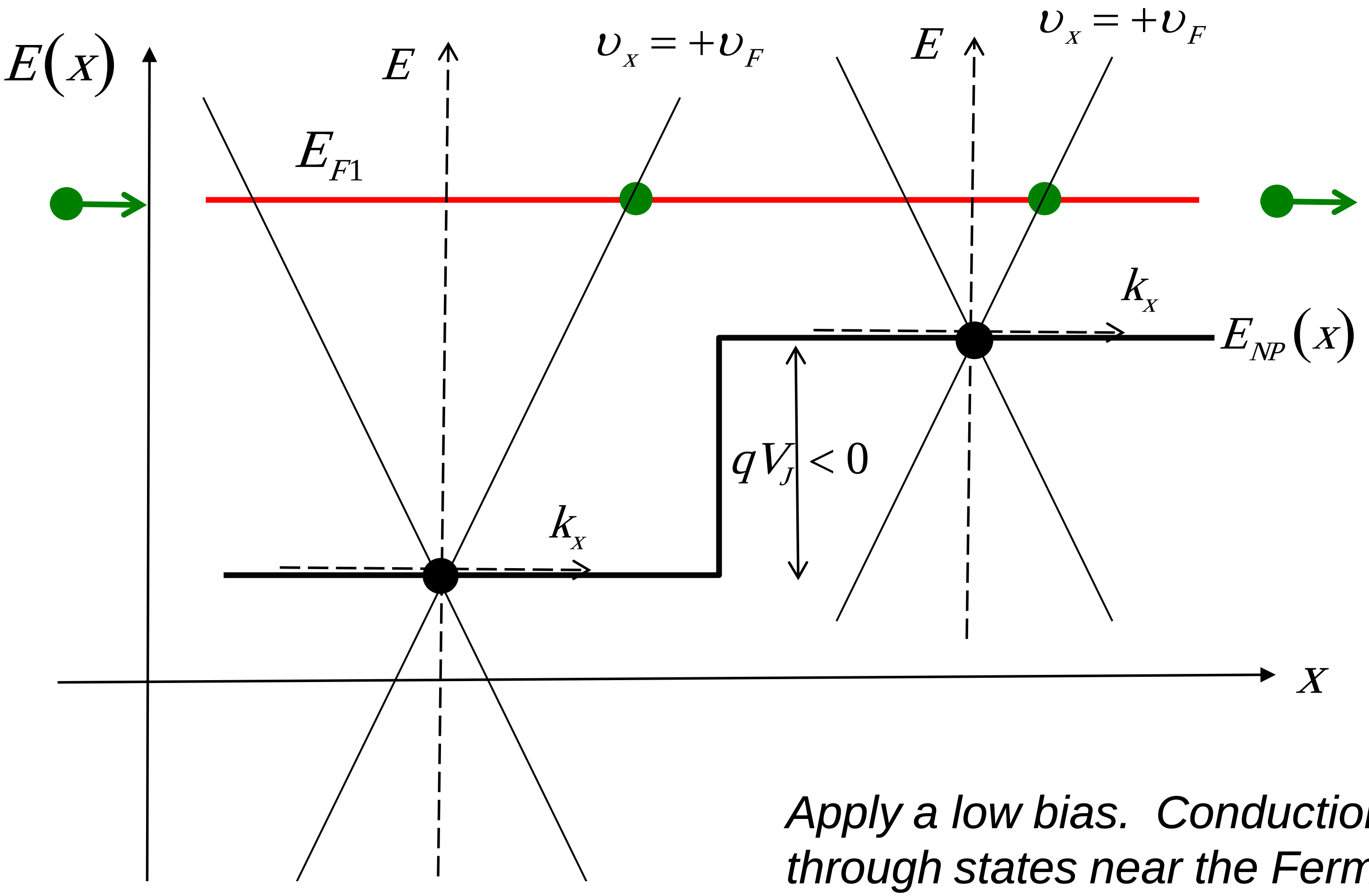
band diagrams: conventional PN junctions



band diagrams: graphene PN junctions

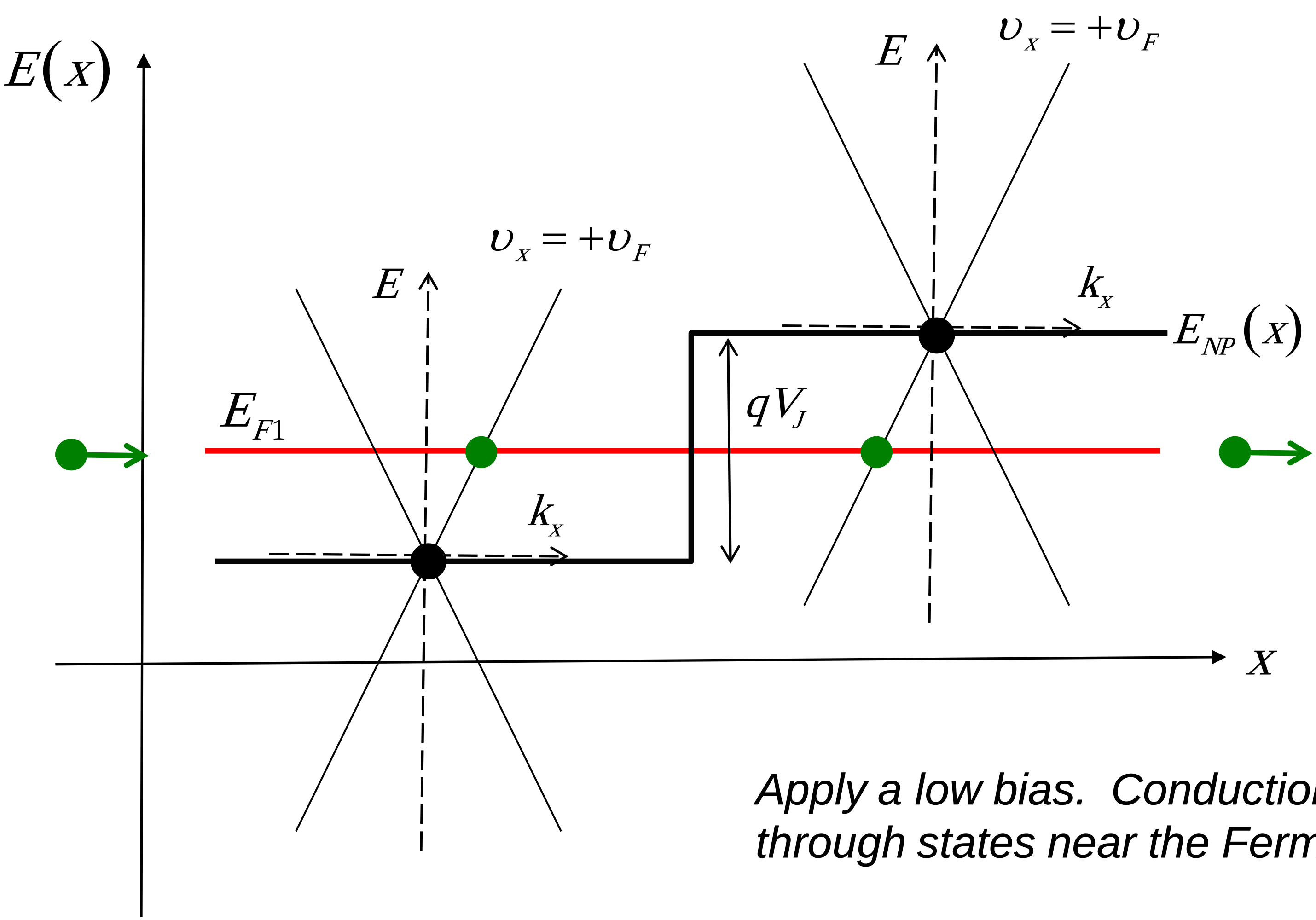


an abrupt graphene N^+N junction



Apply a low bias. Conduction occurs through states near the Fermi level.

an abrupt graphene PN junction

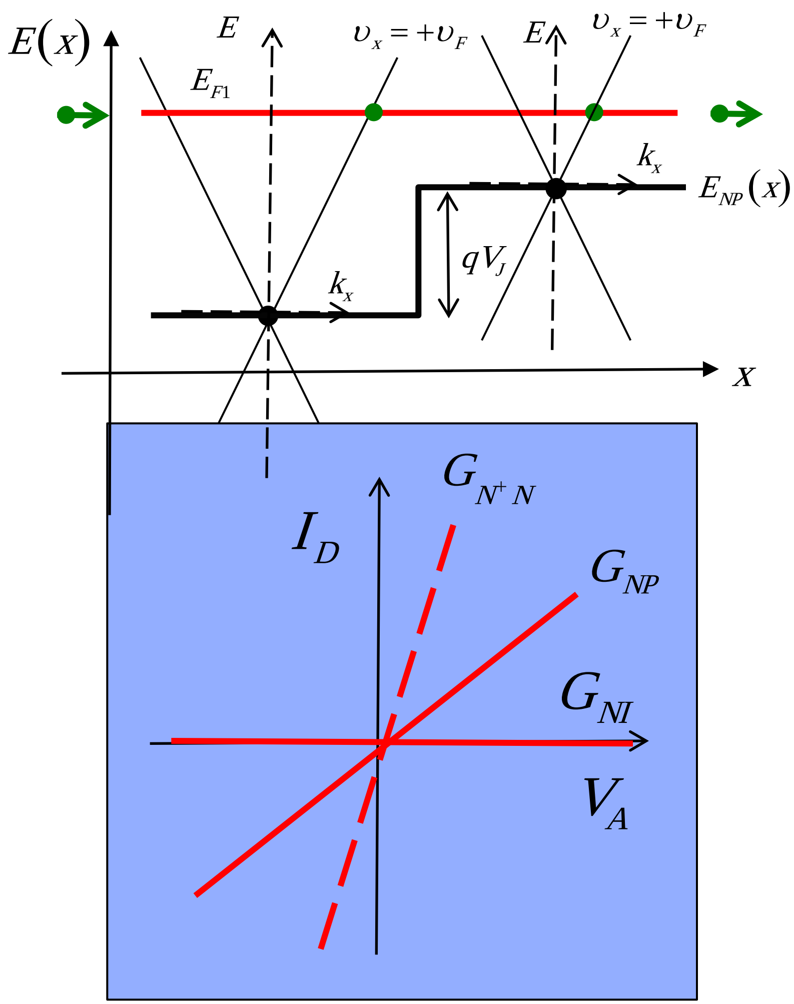


Apply a low bias. Conduction occurs through states near the Fermi level.

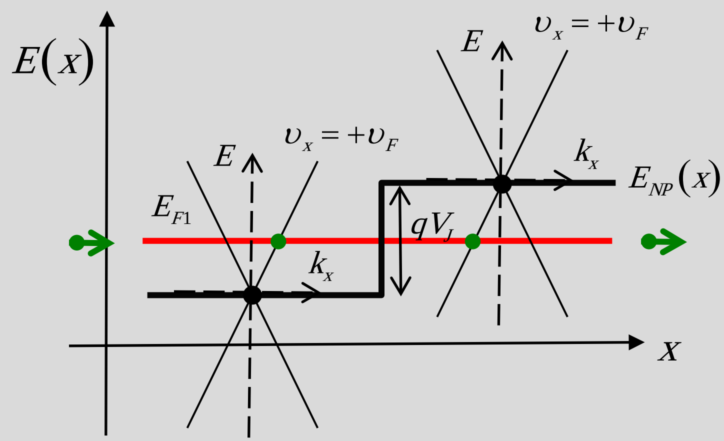
Note that $k_x - k_{x0}$ changes sign!

conductance of graphene junctions

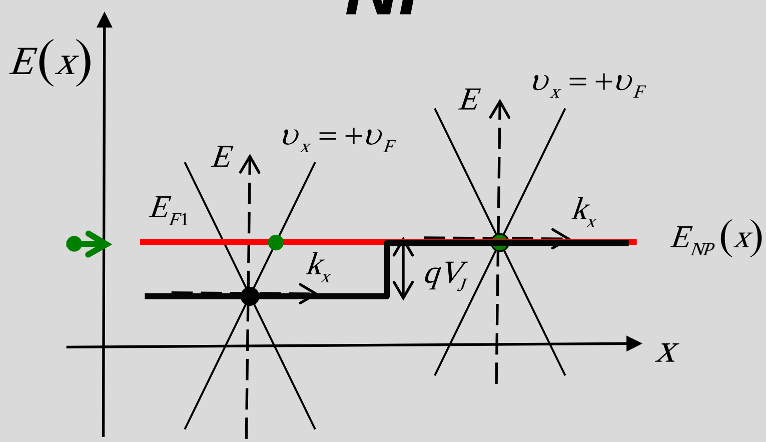
N+N



NP



NI



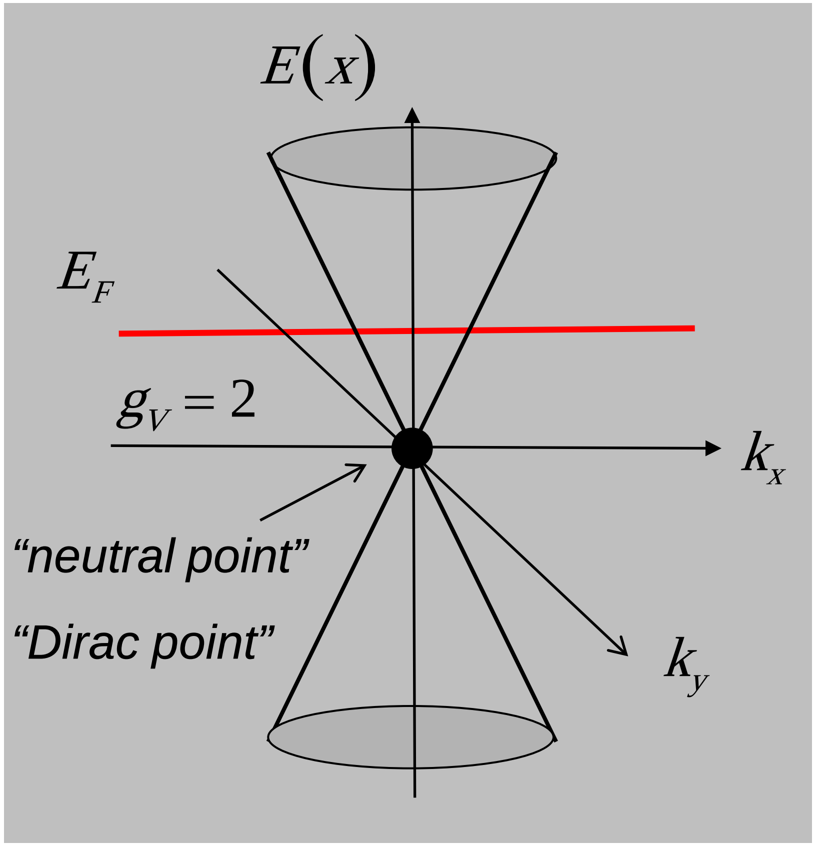
objectives

To understand:

1) Electron "optics" in NP junctions

2) The conductance of NP and NN junctions

about graphene



$$E(k) = \pm\hbar\upsilon_F k = \pm\hbar\upsilon_F\sqrt{k_x^2 + k_y^2}$$

$$\upsilon(k) = \frac{1}{\hbar}\frac{\partial E}{\partial k} = \upsilon_F$$

$$\upsilon_x(k) = \frac{1}{\hbar}\frac{\partial E}{\partial k_x} = \upsilon_F \cos\theta$$

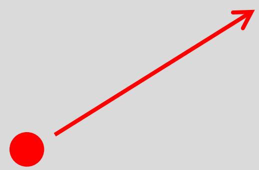
$$\upsilon_y(k) = \frac{1}{\hbar}\frac{\partial E}{\partial k_y} = \upsilon_F \sin\theta$$

$$\upsilon_F \approx 1\times 10^8 \text{ cm/s}$$

$$D(E) = 2|E|/\pi\hbar^2\upsilon_F^2$$

$$M(E) = W\,2|E|/\pi\hbar\upsilon_F$$

electron wavefunction in graphene

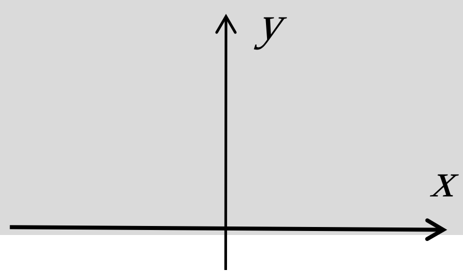


$$\psi\left(x,y\right)=\begin{pmatrix} 1 \\ se^{i\theta} \end{pmatrix} e^{i\left(k_x x+k_y y\right)}$$

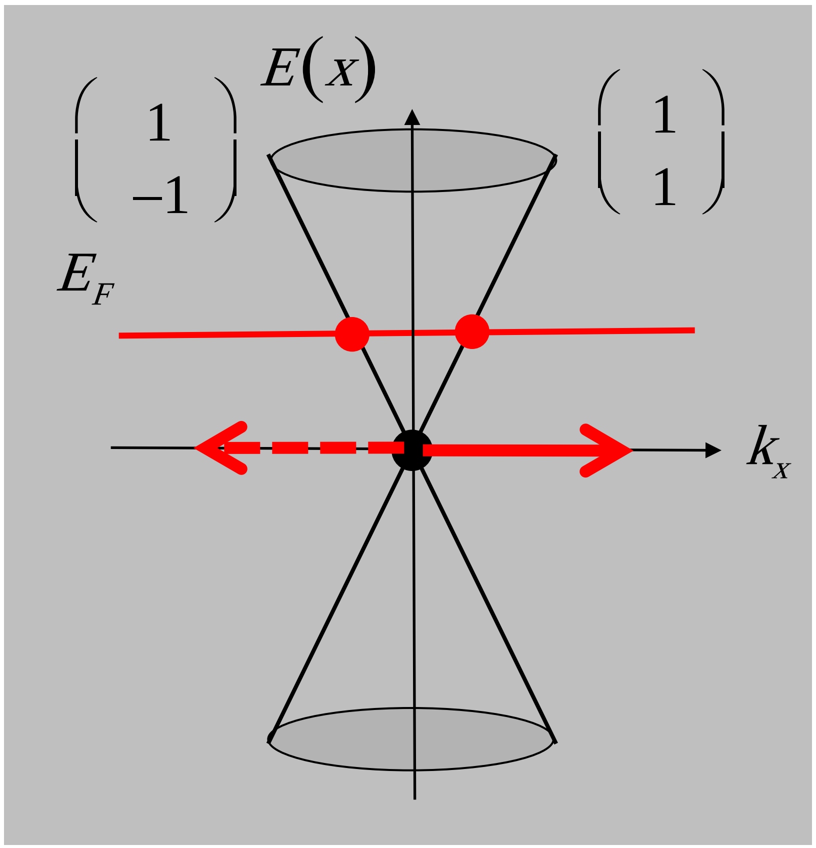
$$s=\operatorname{sgn}\left(E\right)$$

$$\theta=\arctan\left(k_y/k_x\right)$$

$$\psi\left(x,y\right)=\frac{1}{\sqrt{A}}e^{i\left(k_x x+k_y y\right)}$$

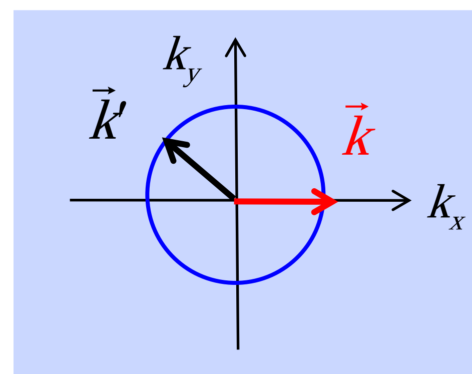


absence of backscattering



$$\psi(x,y) = \begin{pmatrix} 1 \\ s e^{i\theta} \end{pmatrix} e^{i(k_x x + k_y y)}$$

$$s = \mathrm{sgn}(E) \qquad \theta = \arctan(k_y / k_x)$$



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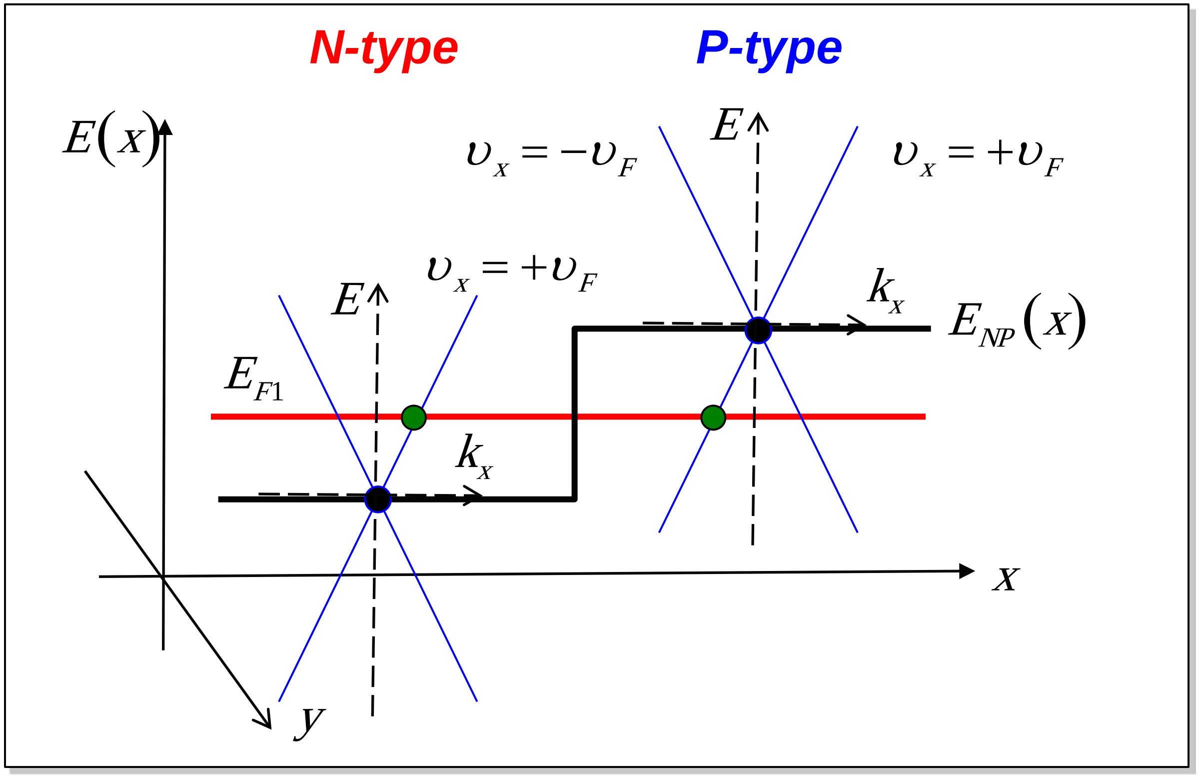
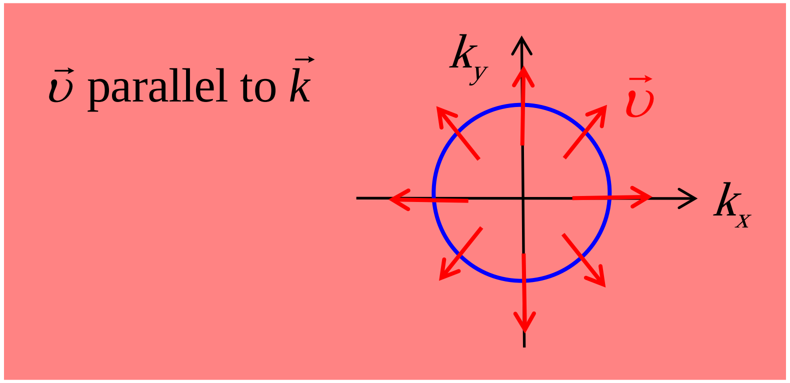
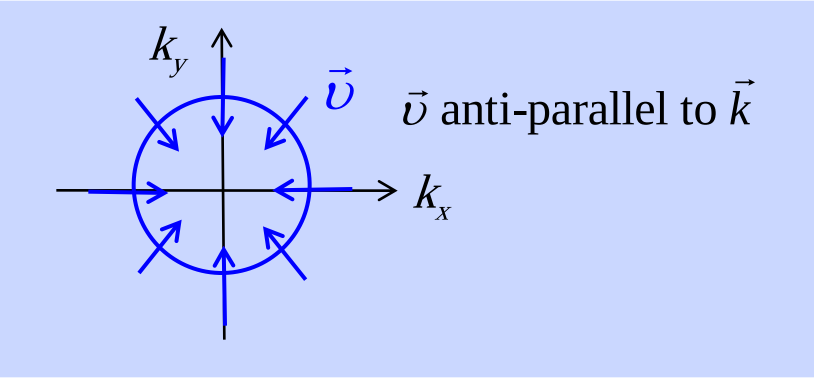
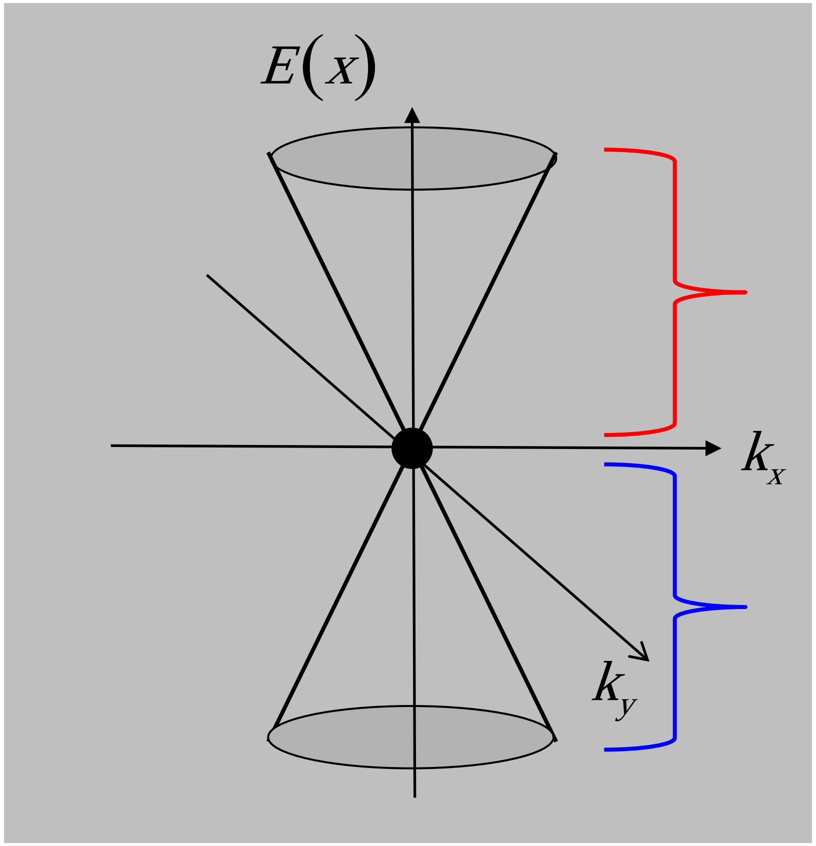
a graphene PN junction







group velocity and wavevector



$$\upsilon_g(k) = \frac{1}{\hbar}\frac{\partial E}{\partial k} = \upsilon_F \frac{\vec{k}}{k}$$

group velocity parallel to k

group velocity parallel to -k

$$\upsilon_g(k) = \frac{1}{\hbar}\frac{\partial E}{\partial k} = -\upsilon_F \frac{\vec{k}}{k}$$

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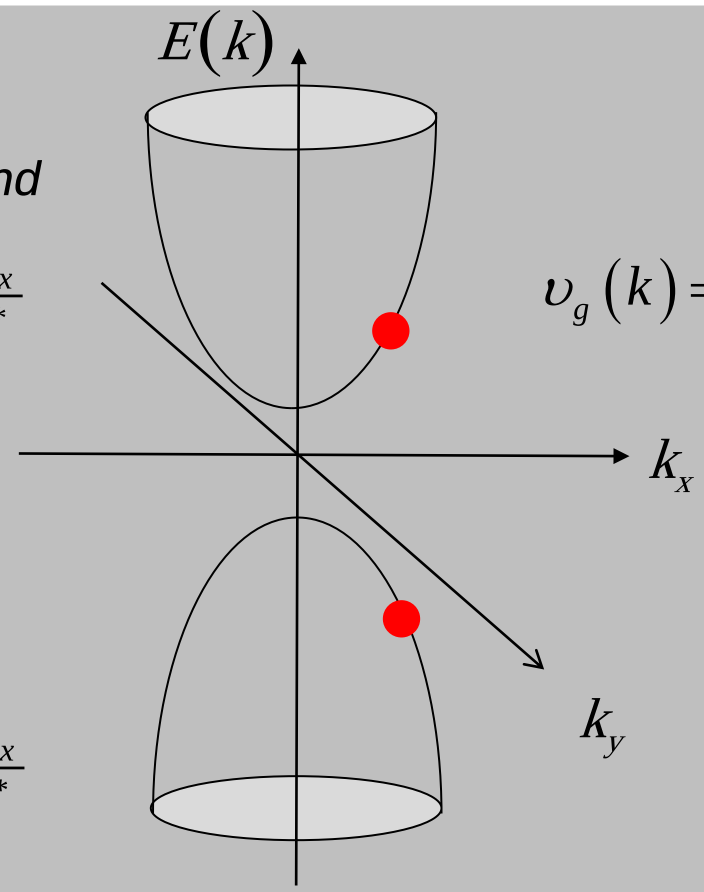
what happens for parabolic bands?

conduction band

$$\upsilon_g(k_x) = +\frac{\hbar k_x}{m^*}$$

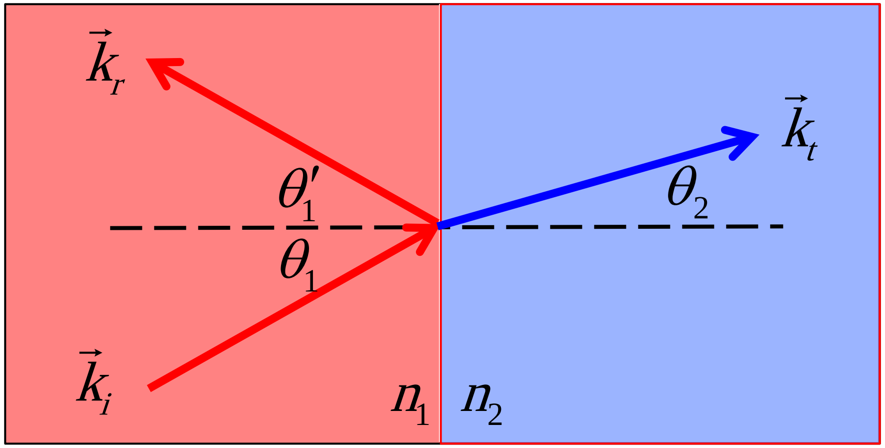
valence band

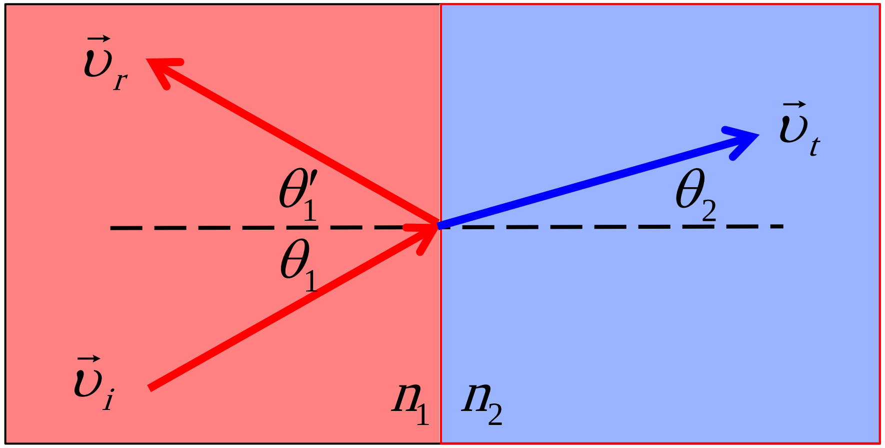
$$\upsilon_g(k_x) = -\frac{\hbar k_x}{m^*}$$



$$\upsilon_g(k) = \frac{1}{\hbar}\nabla_k E(k)$$

optics





Snell's Law

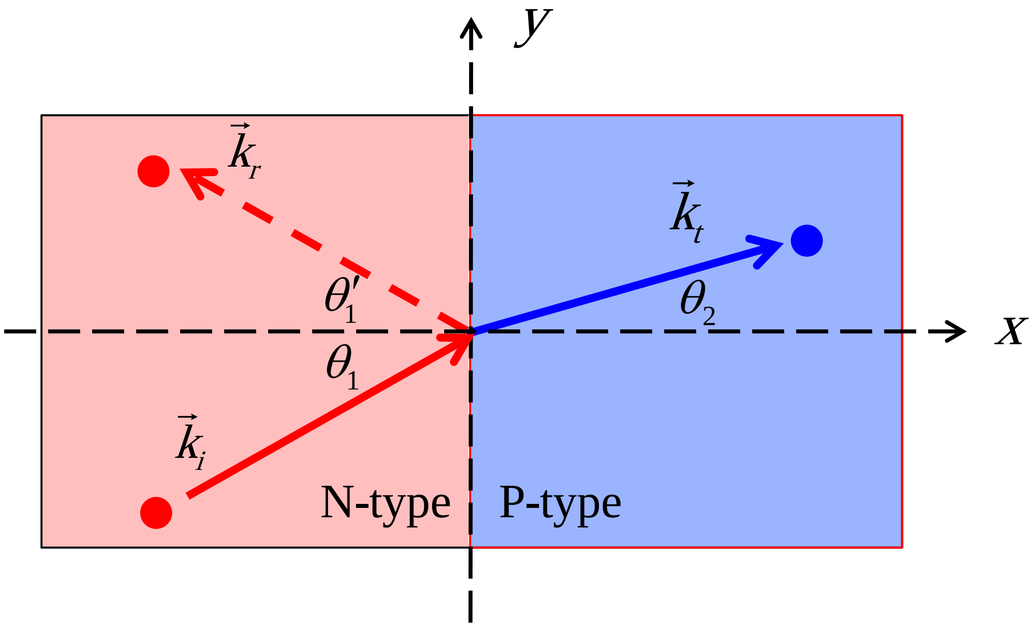
$$\theta_r = \theta_i$$

$$n_1 \sin\theta_1 = n_2 \sin\theta_2$$

group velocity parallel to wavevector

electron trajectories in graphene PN junctions

rays in geometrical optics are analogous to semiclassical electron trajectories



$$\vec{F}_e = \frac{d\left(\hbar \vec{k}\right)}{dt} = -q\vec{\mathcal{E}}$$

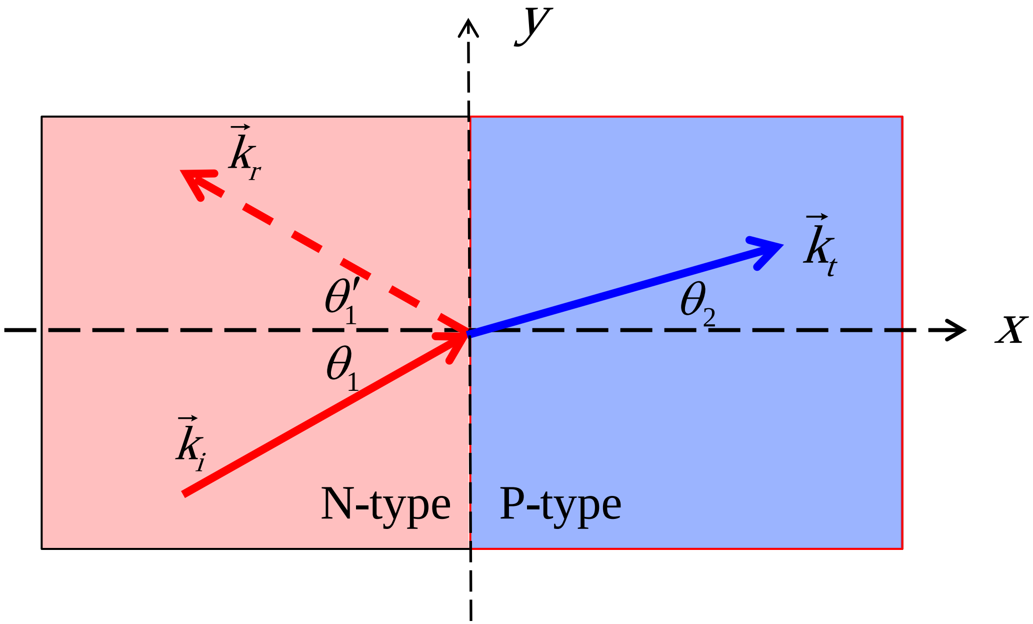
1) k_y *is conserved*

$$k_y^i = k_y^r = k_y^t$$

2) *Energy is conserved*

$$E_i = E_r = E_t$$

on the N-side…



$$E_F = \hbar \upsilon_F k_F$$

$$k_y^i = k_F \sin\theta_1 = k_y^i = k_F \sin\theta_1'$$

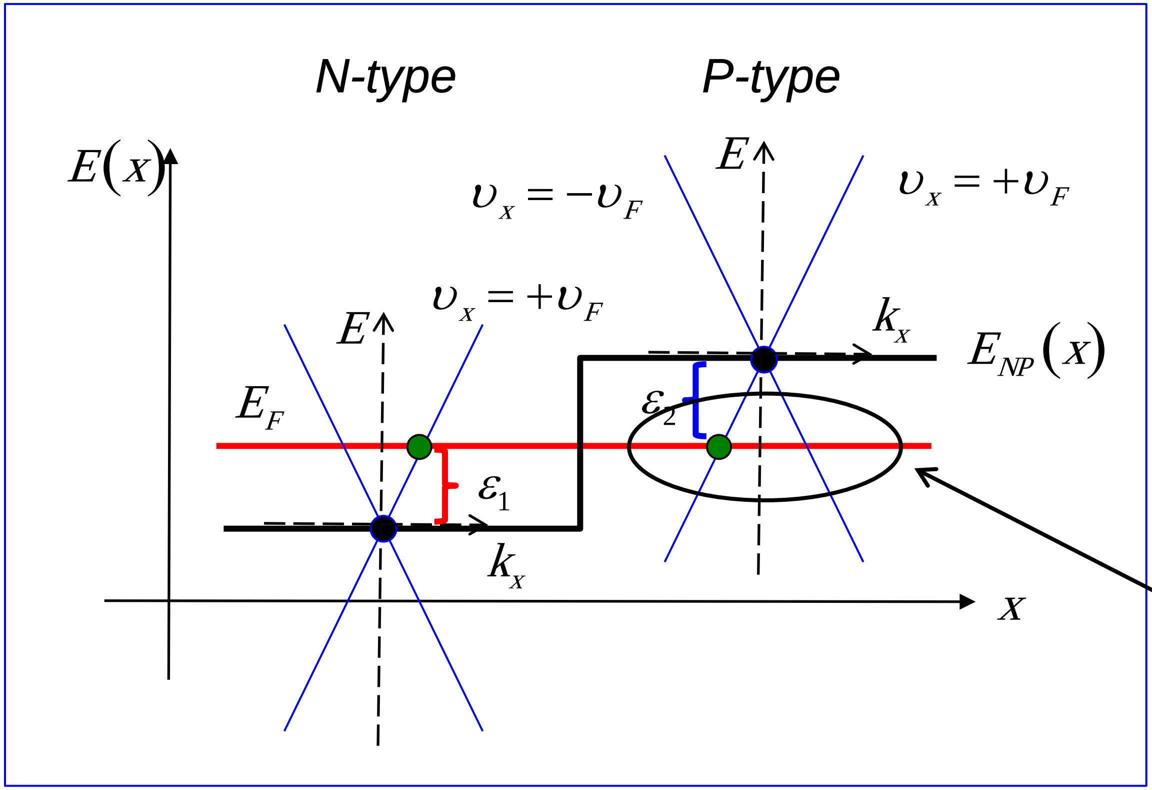
$$\theta_1 = \theta_1'$$

$$k_y^i = k_y^r = k_y^t$$

$$E_i = E_r = E_F$$

angle of incidence =
angle of reflection

a symmetrical PN junction



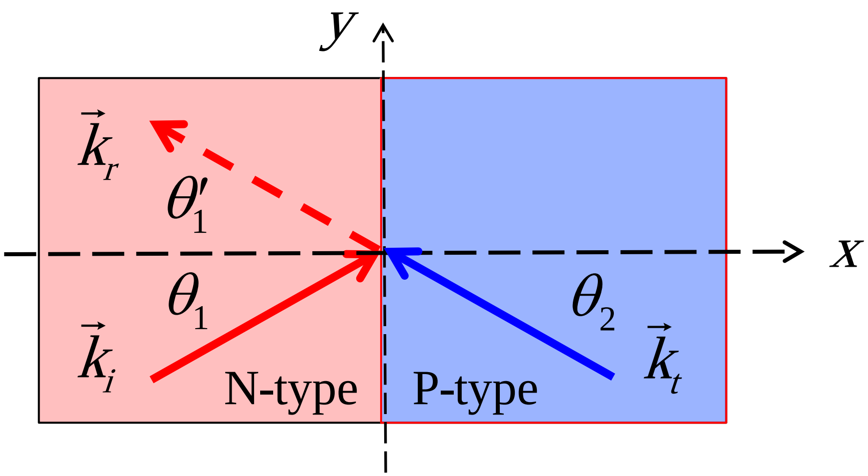
$$k_F = \frac{\left|E_F - E_{NP}(x)\right|}{\hbar \upsilon_F}$$

One choice for k_y, but two choices for k_x.

Symmetrical junction:

$$\left.\begin{aligned} E_F - E_{NP}(0) &= E_{NP}(L) - E_F \\ -qV_J &= 2\left[E_F - E_{NP}(0)\right] \end{aligned}\right\} \quad k_F^i = k_F^t$$

wavevectors

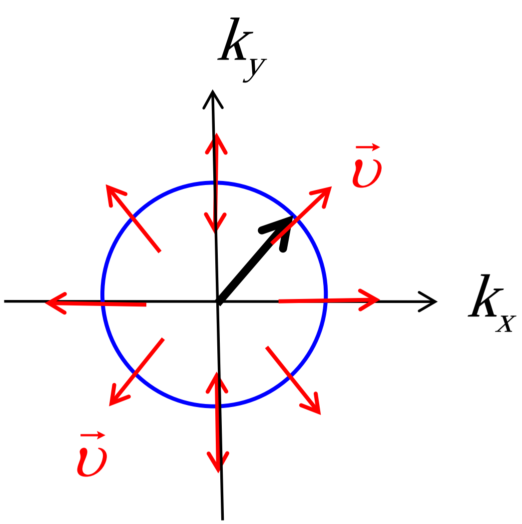


1) transverse momentum (k_y) is conserved
2) transmitted electron must have a positive *x* velocity

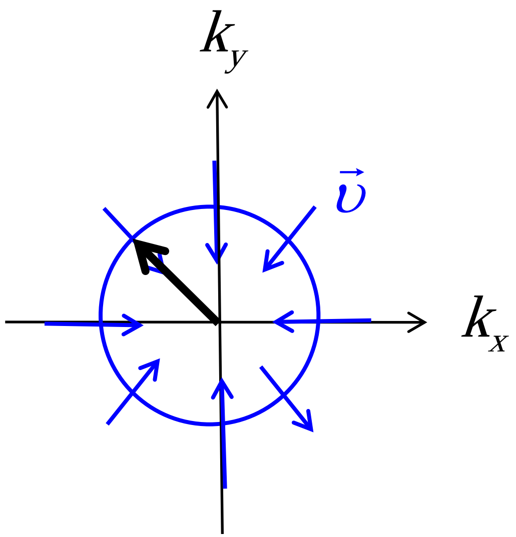
$$k_y^i = k_y^r = k_y^t$$

$$k_x^i = -k_x^r = -k_x^t$$

The sign of the tangential k-vector (k_y) stays the same, and the normal component (k_x) inverts.

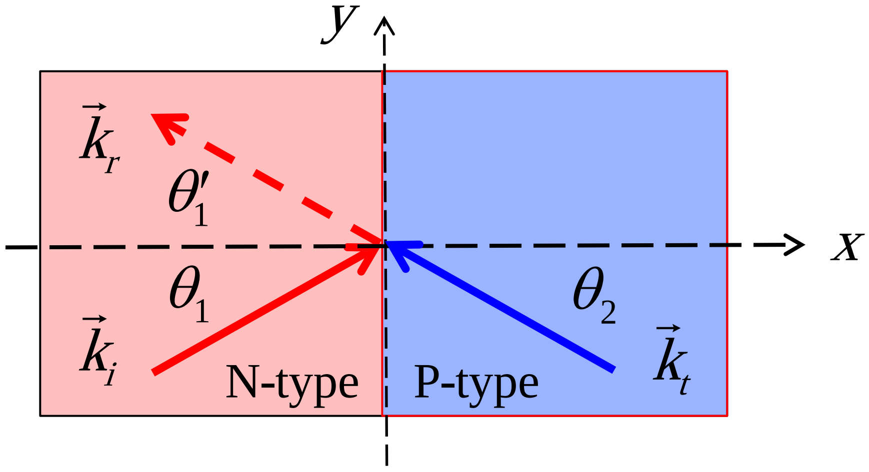


$\vec{\upsilon}$ parallel to $\vec{k}$



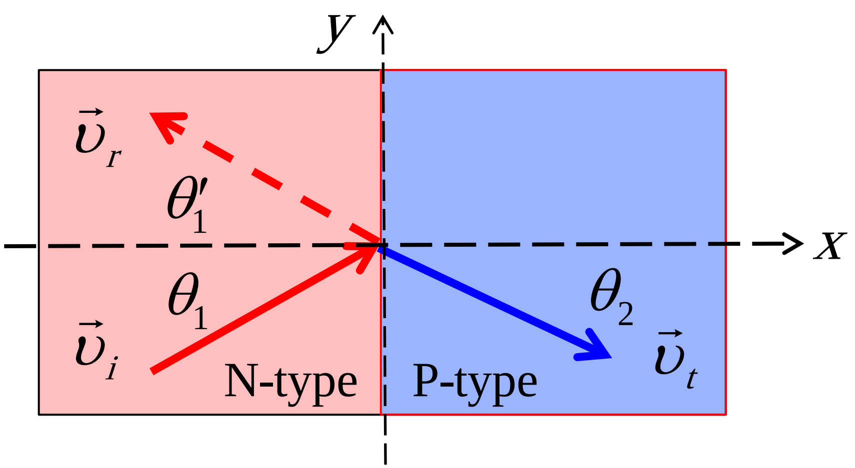
$\vec{\upsilon}$ anti-parallel to $\vec{k}$

wavevectors and velocities



$$k_y^i = k_y^r = k_y^t$$

$$k_x^i = -k_x^r = -k_x^t$$



$$\theta_1 = \theta_1'$$

$$\theta_2 = -\theta_1$$

$$n_1 \sin\theta_1 = n_2 \sin\theta_2$$

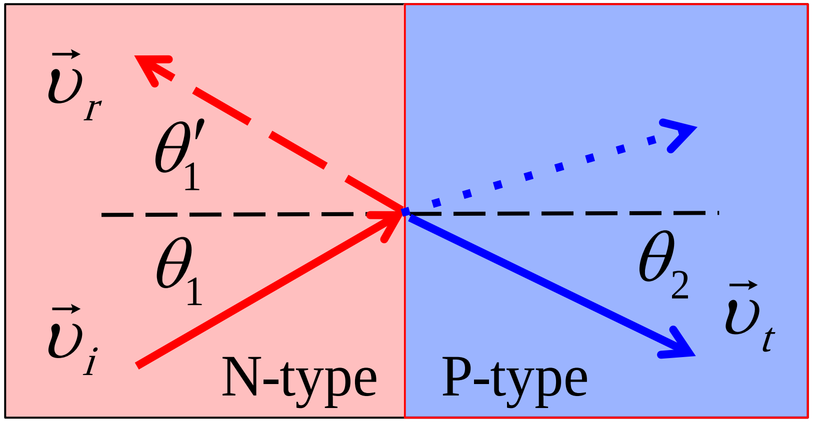
$$n_1 = -n_2$$

"negative index of refraction"

more generally

1) y-component of momentum conserved

2) energy conserved



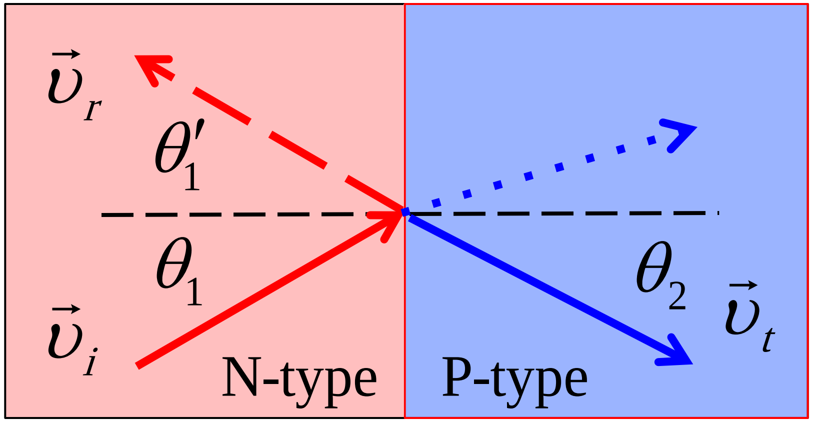
$$\theta_1 = \theta_1'$$

$$\varepsilon_1 \sin\theta_1 = \varepsilon_2 \sin\theta_2 \qquad \left(\varepsilon = E_F - E_{NP}\right)$$

critical angle for total internal reflection

$$\theta_C = \sin^{-1}\left(\left|\varepsilon_2/\varepsilon_1\right|\right) \qquad \left(\varepsilon_1 > \varepsilon_2\right)$$

reflection and transmission



We know the direction of the reflected and transmitted rays, but what are their magnitudes?

outline

1) Introduction
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3) **Transmission across NP junctions**
4) Conductance of PN and NN junctions
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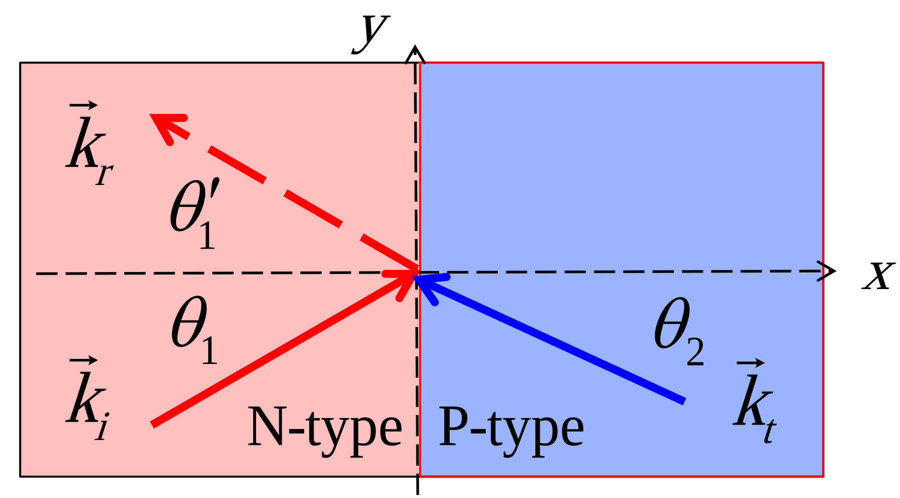
reflection and transmission

$$R(\theta_i) \propto |r|^2$$



$$T(\theta_i) \propto |t|^2$$

1) incident wave:

$$\psi_i(x,y) = \begin{pmatrix} 1 \\ se^{i\theta} \end{pmatrix} e^{i(k_x x + k_y y)}$$

2) reflected wave:

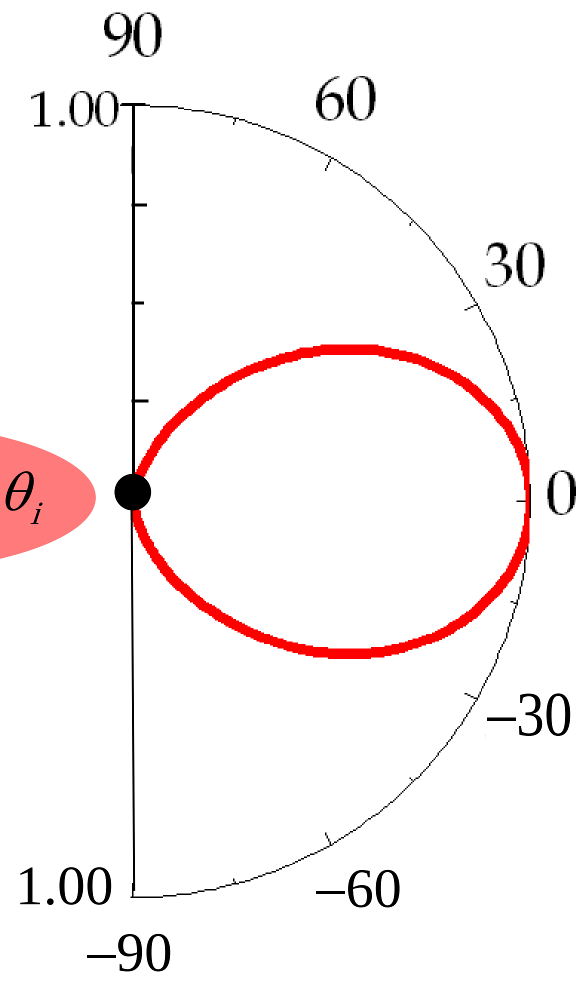
$$\psi_r(x,y) = r\begin{pmatrix} 1 \\ se^{i\theta} \end{pmatrix} e^{i(-k_x x + k_y y)}$$

3) transmitted wave:

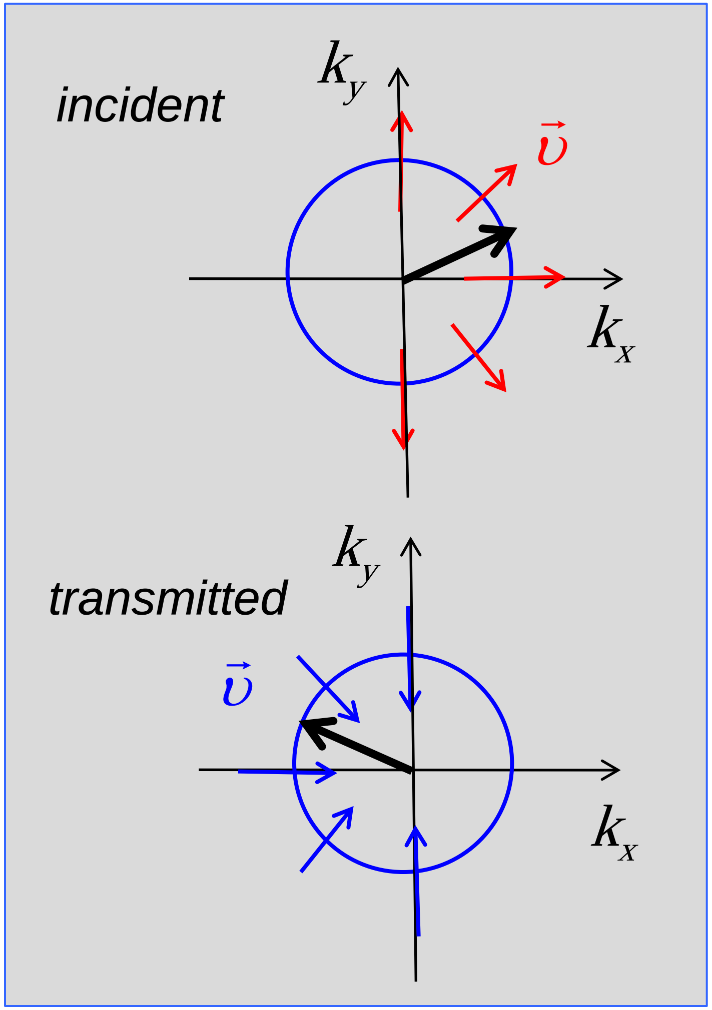
$$\psi(x,y) = t\begin{pmatrix} 1 \\ se^{i\theta} \end{pmatrix} e^{i(-k_x x + k_y y)}$$

$$\left(s = \mathrm{sgn}(E) \quad \theta = \arctan(k_y / k_x) \right)$$

transmission: abrupt, symmetrical NP junction



$$T(\theta_i) = \cos^2 \theta_i$$



perfect transmission for (= 0

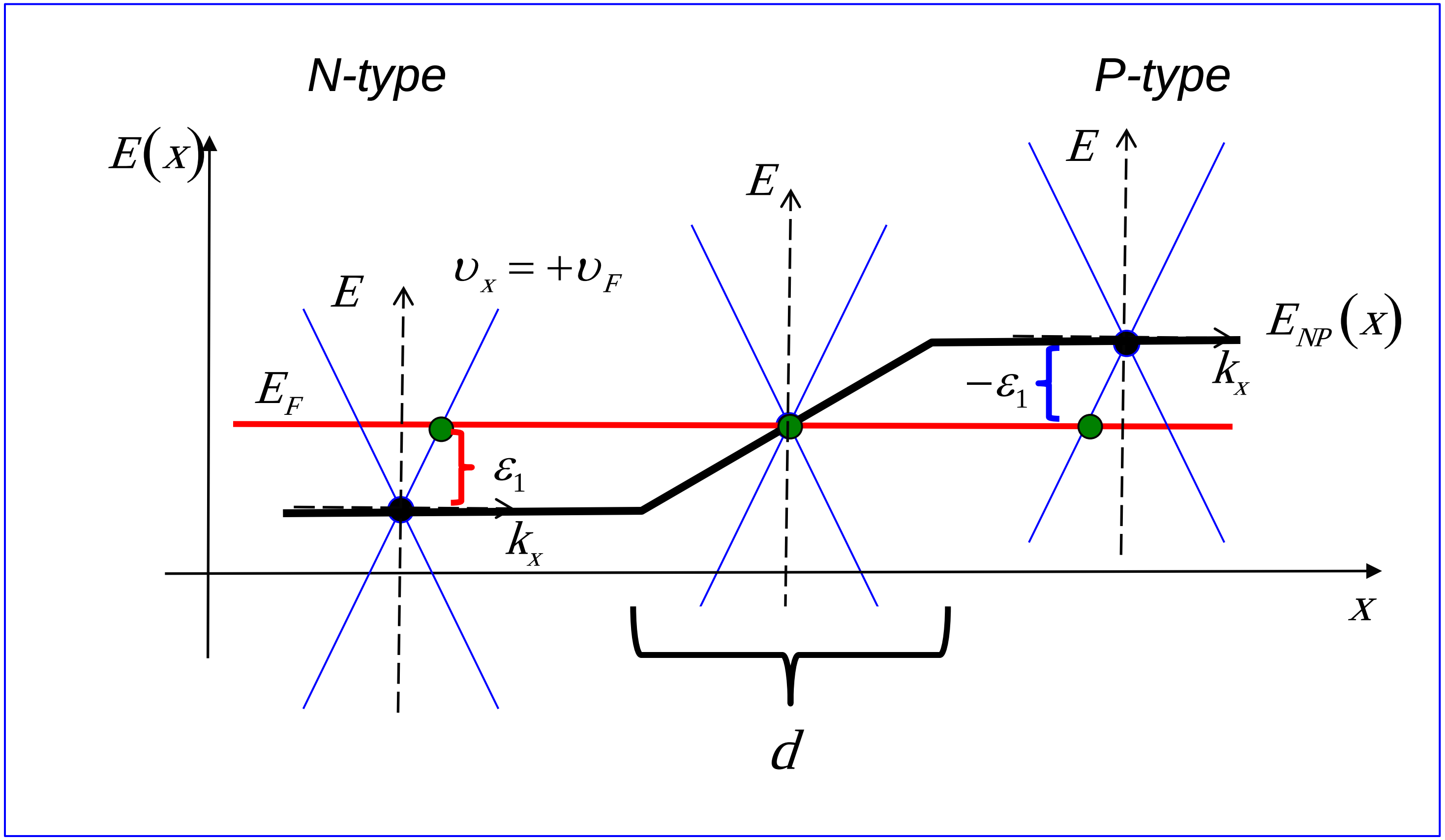
conductance of abrupt NN and NP junctions

$$T(\theta_i) = \cos^2 \theta_i$$

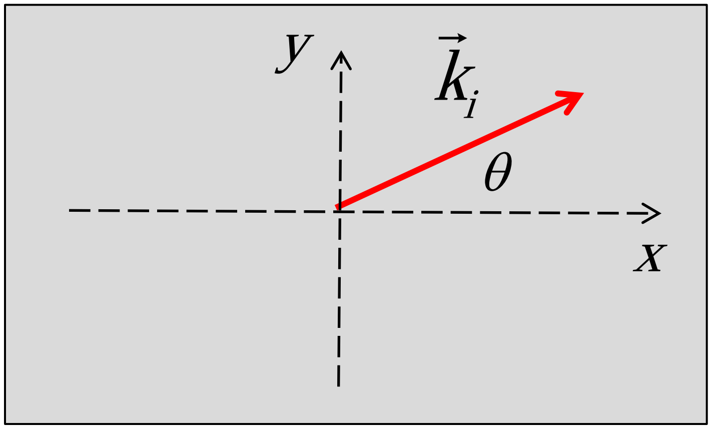
This transmission reduces the conductance of NP junctions compared to NN junctions, but not nearly enough to explain experimental observations.

a ***graded***, symmetrical PN junction

The Fermi level passes through the neutral point in the transition region of an NP junction. This **does not** occur in an NN or PP junction, and we will see that this lowers the conductance of an NP junction.

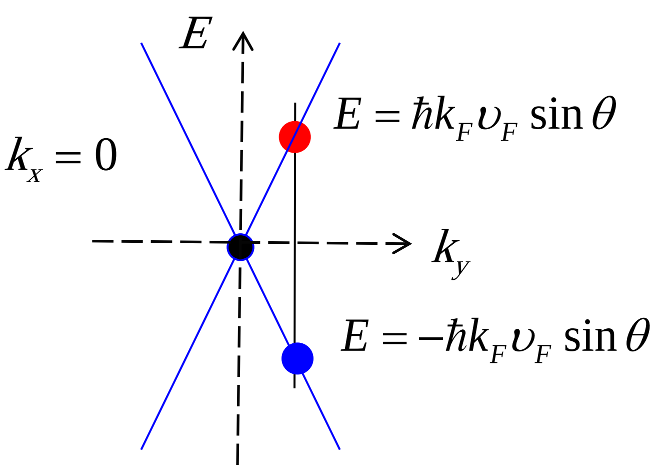


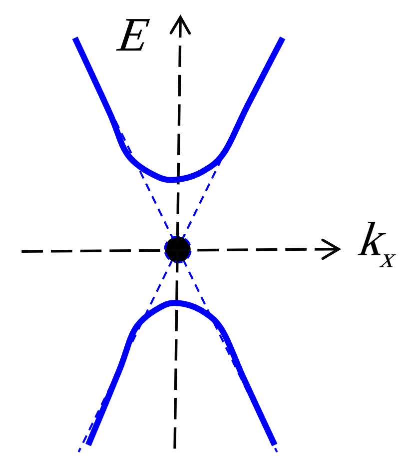
treat each ray (mode) separately



$$k_y = k_F \sin\theta$$

$$E = \hbar \upsilon_F \sqrt{k_x^2 + k_F^2 \sin^2\theta}$$

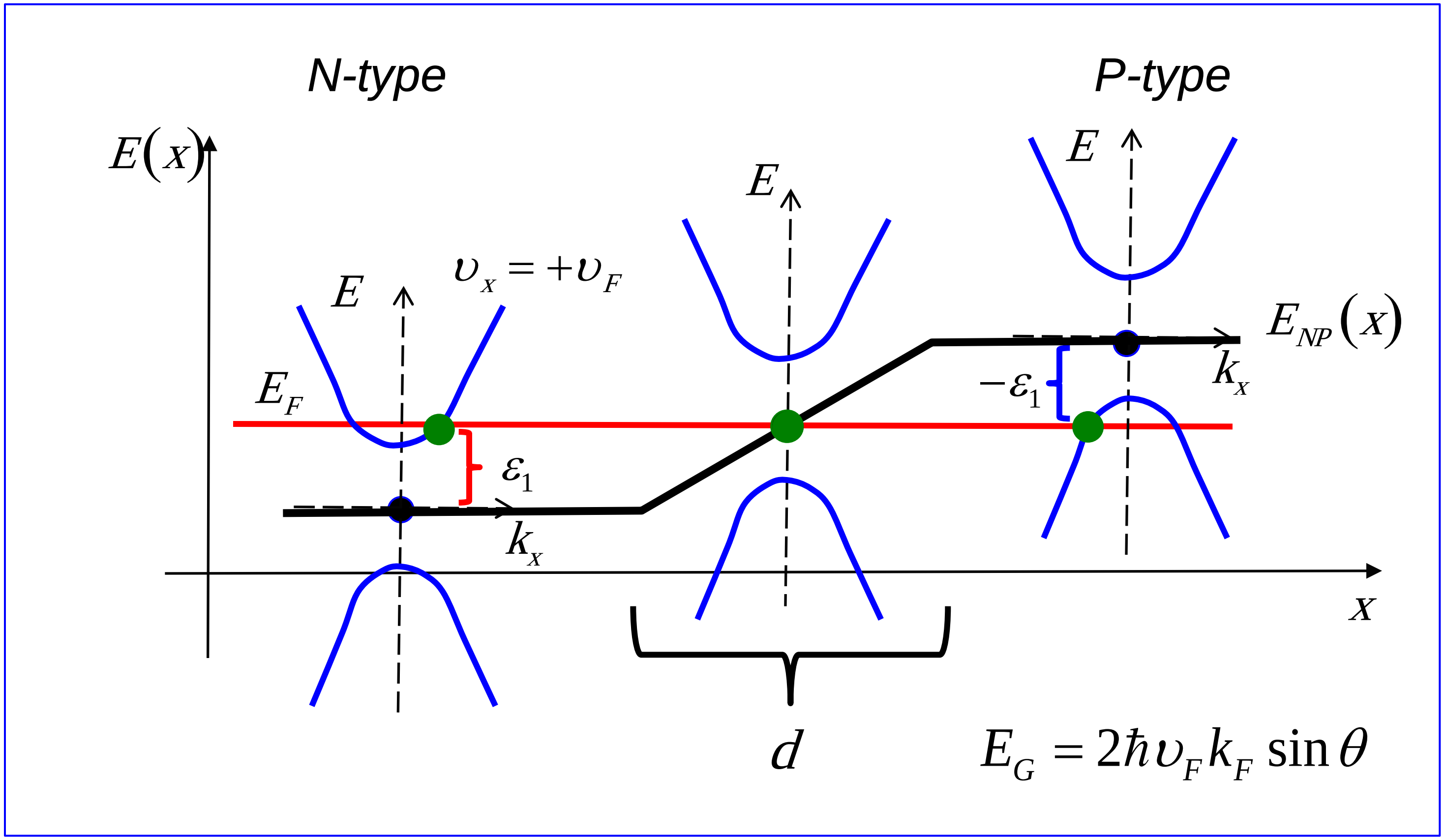




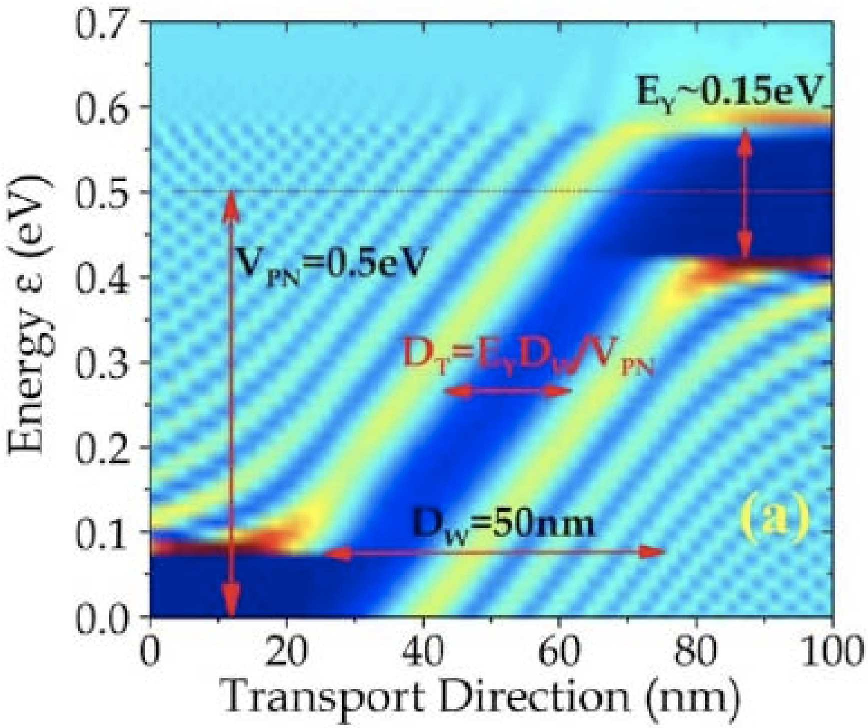
$$E_G = 2\hbar \upsilon_F k_F \sin\theta$$

for each k_y (transverse mode)

band to band tunneling (BTBT)

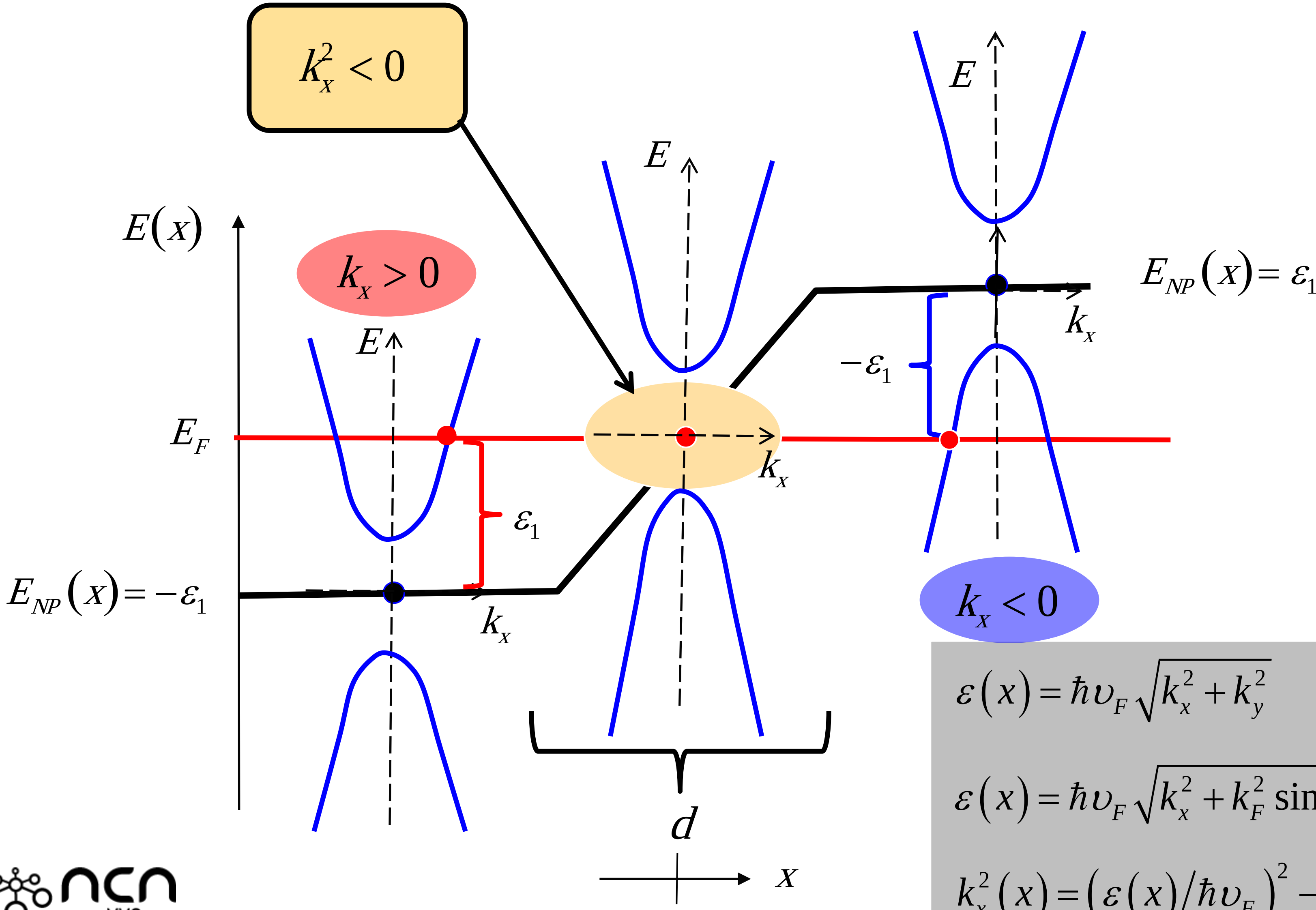


NEGF simulation



T. Low, et al., *IEEE TED*, **56**, 1292, 2009

propagation across a symmetrical NP junction



$$\varepsilon(x) = \hbar\upsilon_F \sqrt{k_x^2 + k_y^2}$$

$$\varepsilon(x) = \hbar\upsilon_F \sqrt{k_x^2 + k_F^2 \sin^2\theta}$$

$$k_x^2(x) = \left(\varepsilon(x)/\hbar\upsilon_F\right)^2 - k_F^2 \sin^2\theta$$

WKB tunneling

$$k_x^2(x) = \left(\varepsilon(x)/\hbar\upsilon_F\right)^2 - k_F^2 \sin^2\theta$$

For normal incidence, k_x is real
→ perfect transmission

$$\psi(x) \sim e^{ik_x x}$$

For any finite angle, there will a region in the junction where k_x is imaginary → evanescent

$$T \sim e^{-2i\int_{-l}^{+l} k_x(x)\,dx}$$

Slowly varying potential →
no reflections.

$$T(\theta) \sim e^{-\pi k_F d \sin^2\theta}$$

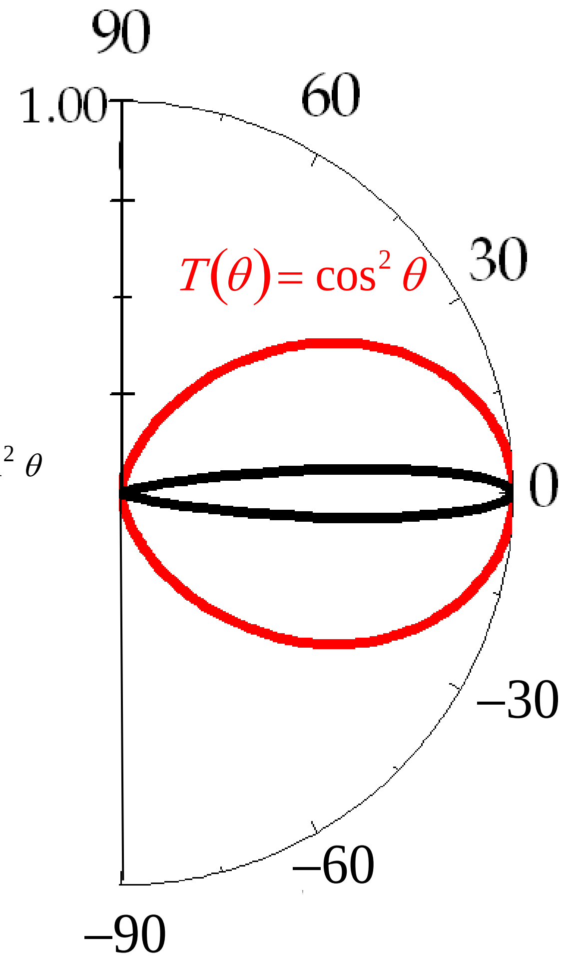
$$T \sim e^{-\pi E_G^2/(2q\hbar\upsilon_F \mathcal{E})}$$

$$E_G = 2\hbar\upsilon_F k_F \sin\theta$$

$$\mathcal{E} = \frac{2\varepsilon_1}{qd} = \frac{2\hbar\upsilon_F k_F}{qd}$$

transmission vs. angle

$$T(\theta) = e^{-\pi k_F d \sin^2\theta}$$



More generally, to include the reflections for abrupt junctions (small *d*):

$$T(\theta) : \cos^2\theta \, e^{-\pi k_F d \sin^2\theta}$$

outline

1) Introduction

2) Electron optics in graphene

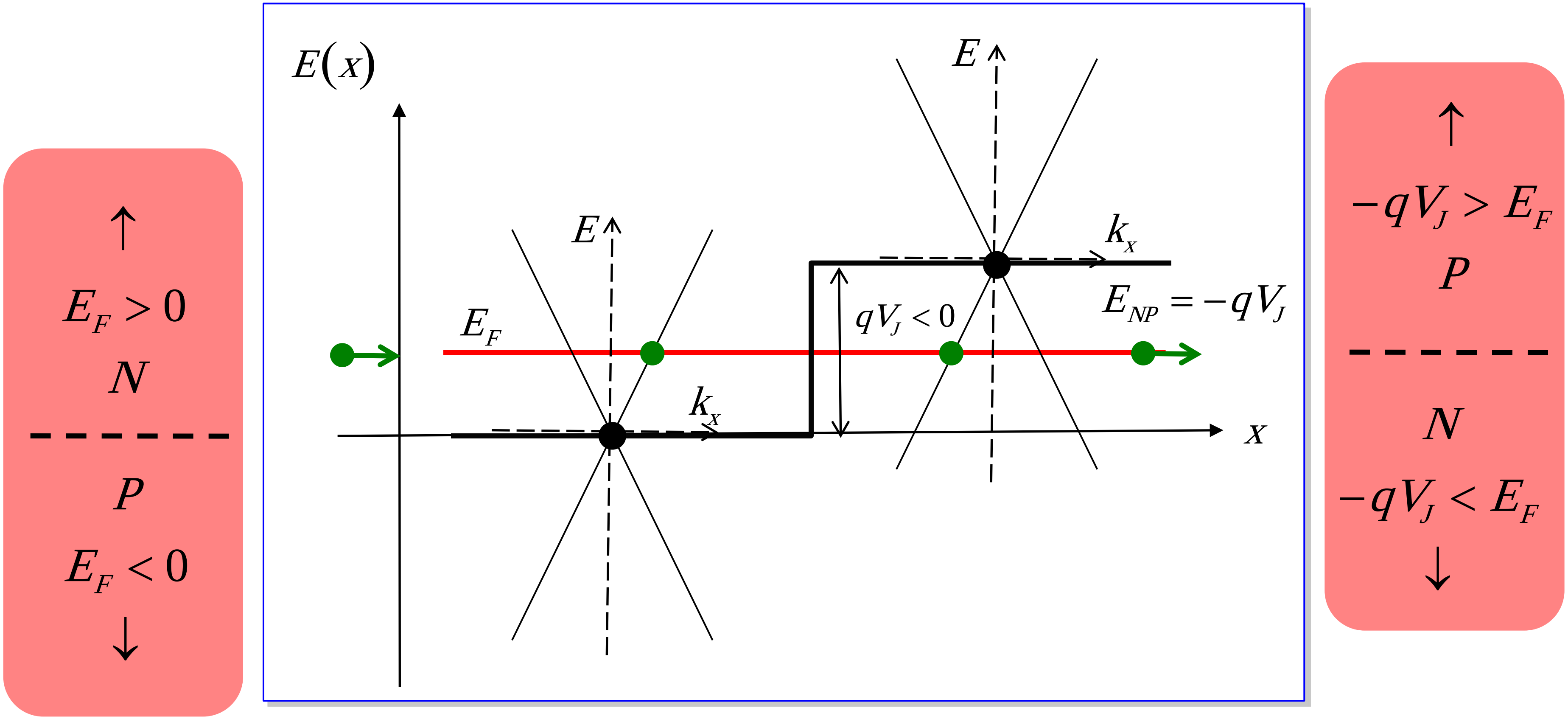
3) Transmission across NP junctions

4) Conductance of PN and NN junctions

5) Discussion

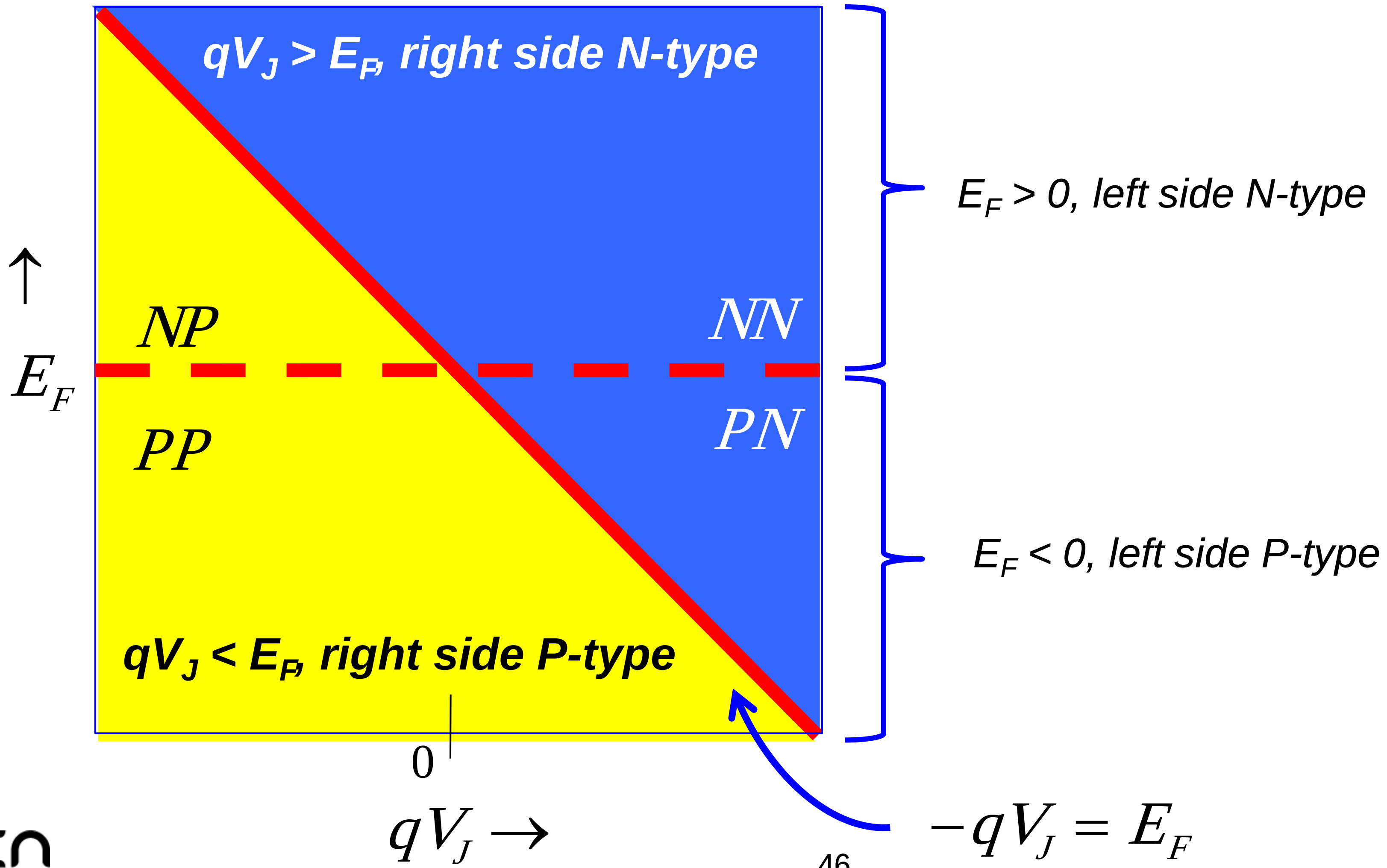
6) Summary

graphene junctions: NN, NP, PP, PN



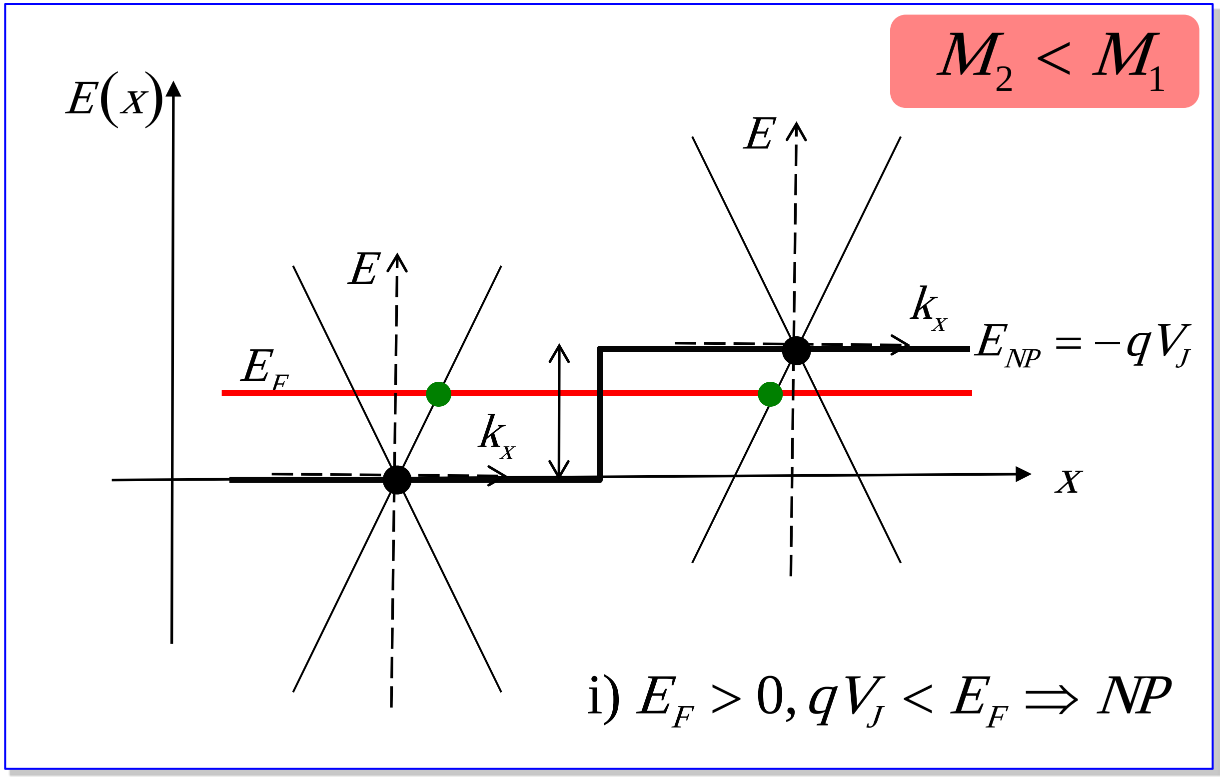
two independent variables: 1) E_F and 2) V_J

graphene junctions

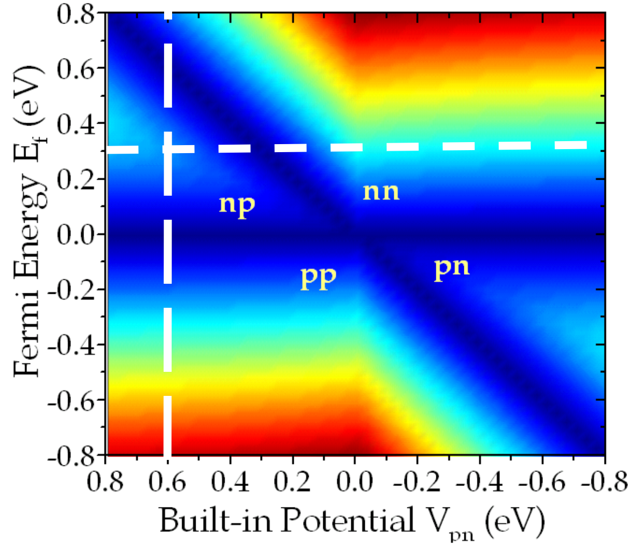


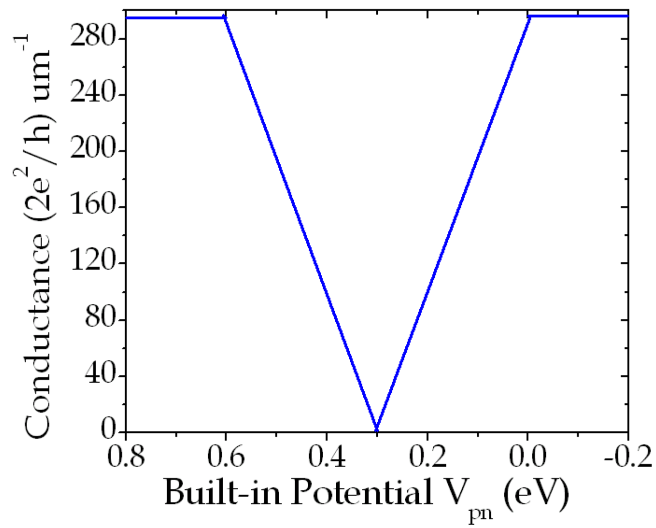
conductance of graphene junctions

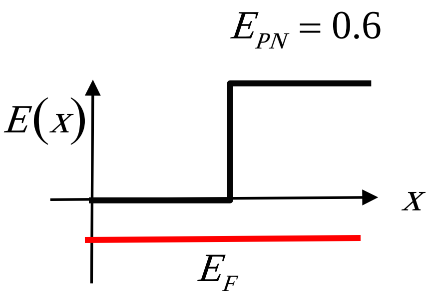
$$G = \frac{2q^2}{h} M(E_F) \qquad G/W = \frac{2q^2}{h}\frac{1}{W}\min(M_1, M_2)$$

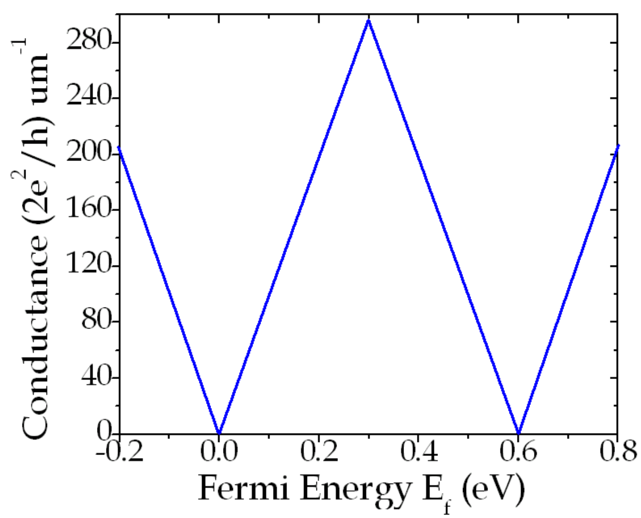


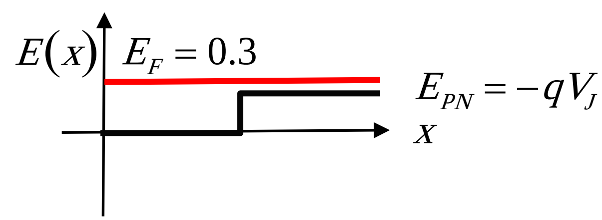
conductance vs. E_F and V_J





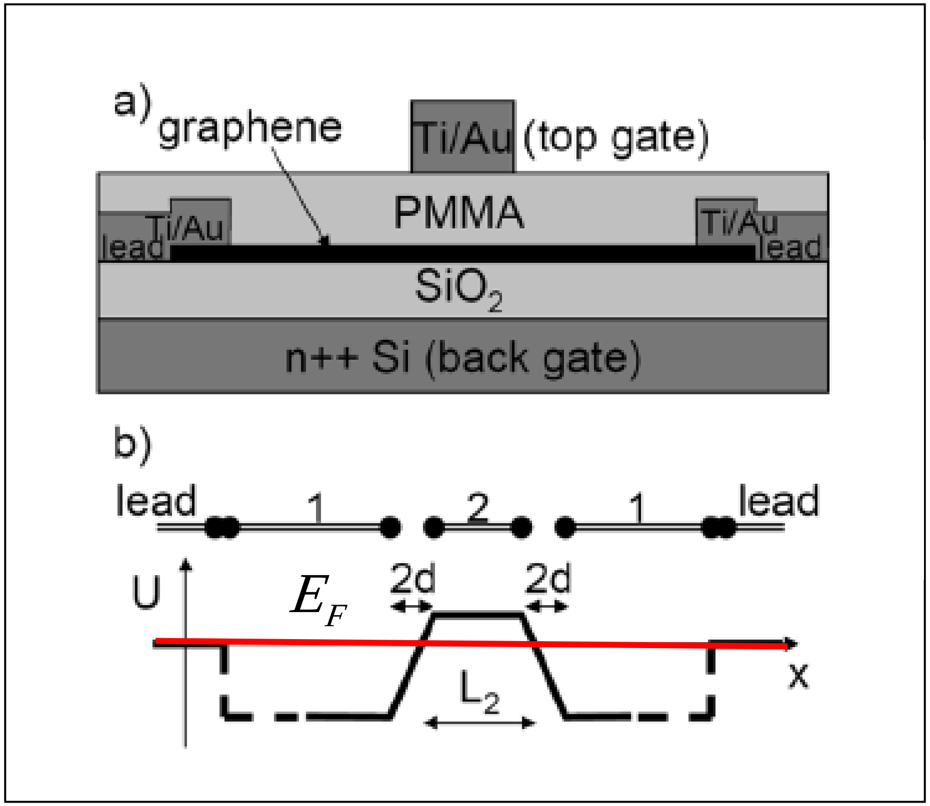






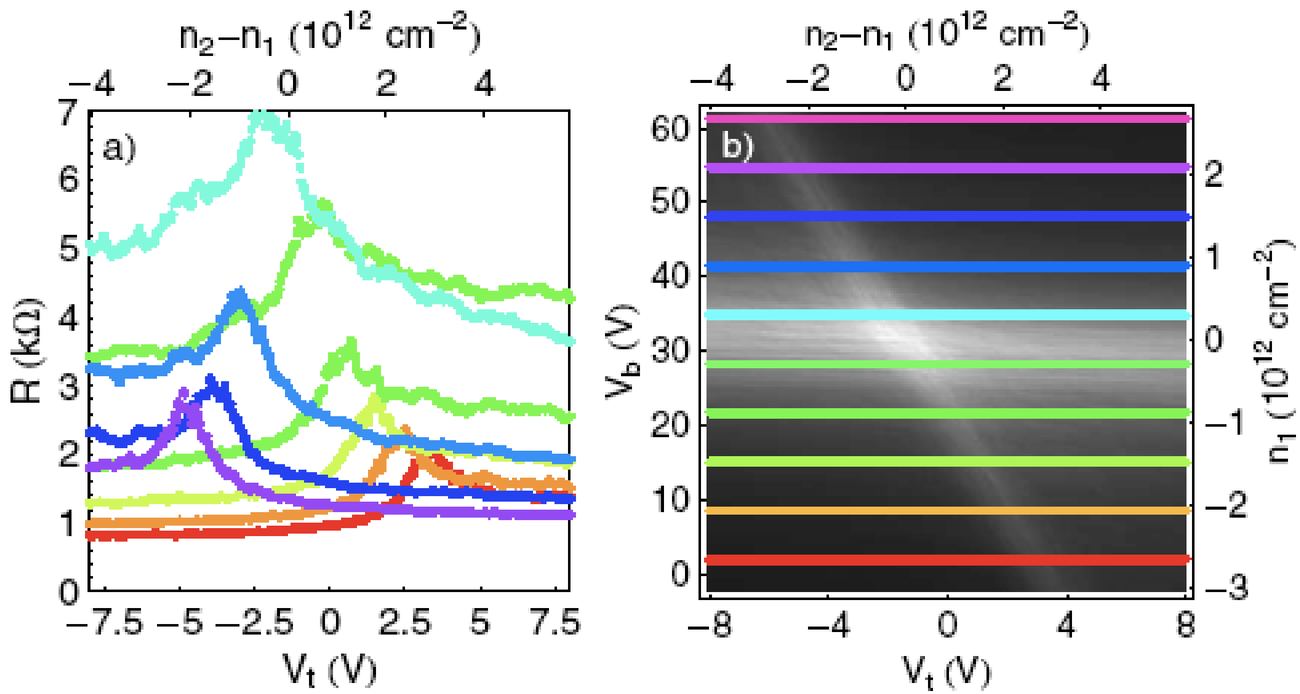
T. Low, et al., *IEEE TED*, **56**, 1292, 2009

measured transport across a tunable barrier



B. Huard, J. A. Sulpizio, N. Stander, K. Todd, B. Yang, and D. Goldhaber-Gordon, “Transport Measurements Across a Tunable Potential Barrier in Graphene, *Phys. Rev. Lett*., **98**, 236803, 2007.

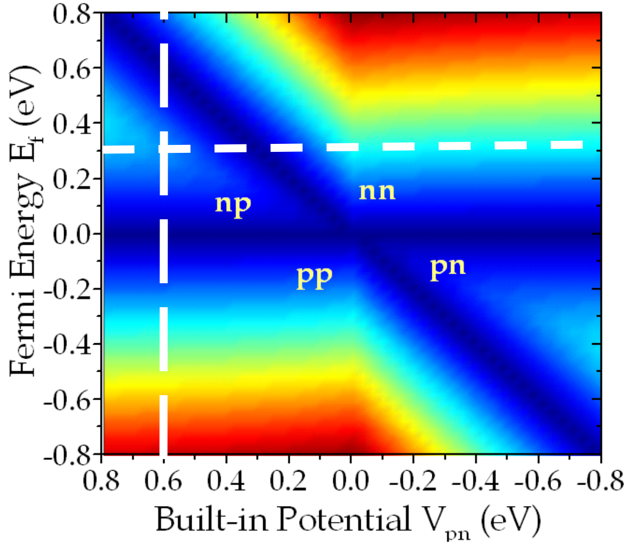
experimental resistance

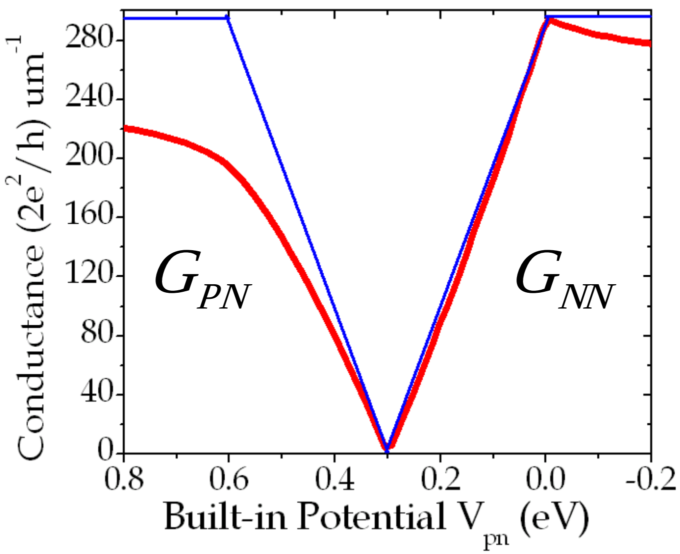


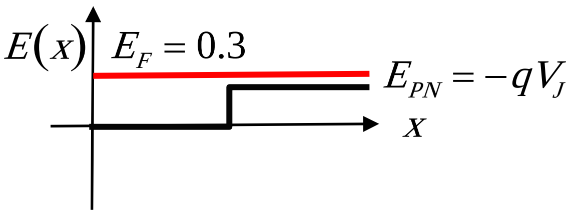
B. Huard, J. A. Sulpizio, N. Stander, K. Todd, B. Yang, and D. Goldhaber-Gordon, "Transport Measurements Across a Tunable Potential Barrier in Graphene, *Phys. Rev. Lett*., **98**, 236803, 2007.

NEGF simulation of abrupt junctions

$$G_{PN} < G_{NN}$$







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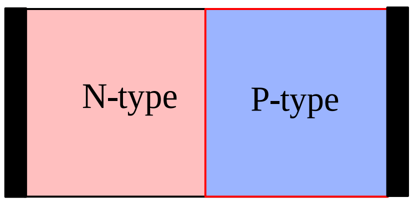
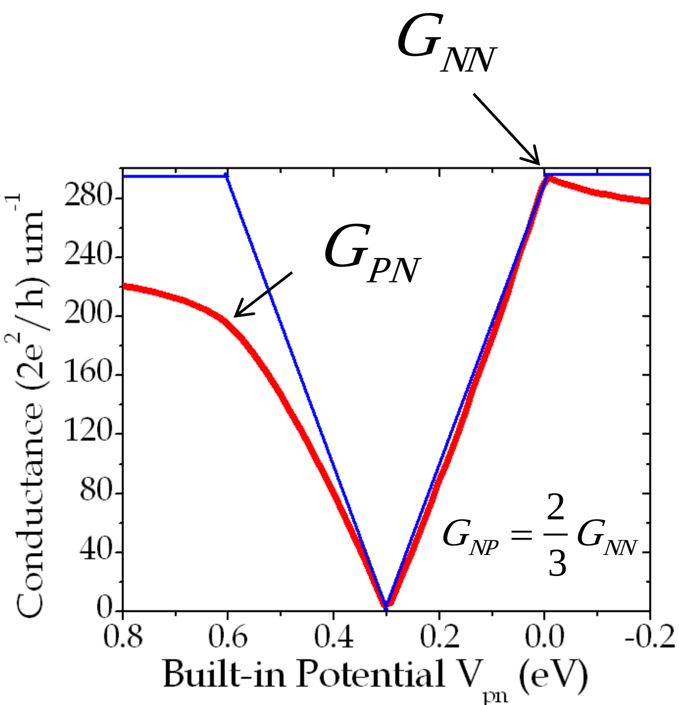
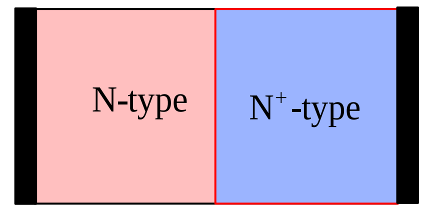
conductance of abrupt graphene junctions

$E_F = 0.3$





$E_F = 0.3$



$$G_{NP} = \frac{2q^2}{h} \sum_{k_y} T\left(k_y\right)$$

$$G_{NN}/W = \frac{2q^2}{h} M\left(E_F\right)$$

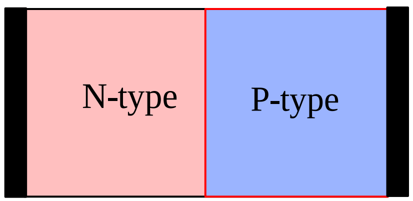
$$T(\theta) = \cos^2\theta = \left(\frac{k_x}{k_F}\right)^2 = 1 - \left(\frac{k_y}{k_F}\right)^2$$

$$G_{NP}/W = \frac{2}{3}\left(\frac{2q^2}{h} M\left(E_F\right)\right)$$

$$G_{NP} < G_{NN}$$

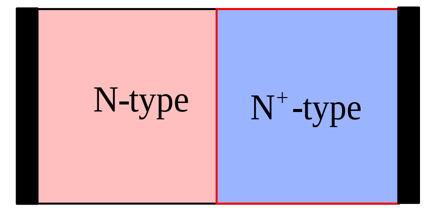
conductance of graded graphene junctions

$E_F = 0.3$



$$G_{NP} << G_{NN}$$

$E_F = 0.3$



$$G_{NP} = \frac{2q^2}{h} \sum_{k_y} T(k_y)$$

$$G_{NN}/W = \frac{2q^2}{h} M(E_F)$$

$$T(\theta) = e^{-\pi k_F d \sin^2 \theta} = e^{-\pi k_F d (k_y/k_F)^2}$$

$$G_{NP} = G_{NN} \big/ \sqrt{k_F d}$$

$$k_F d >> 1 \rightarrow \lambda_F << d$$

T. Low, et al., *IEEE TED*, **56**, 1292, 2009

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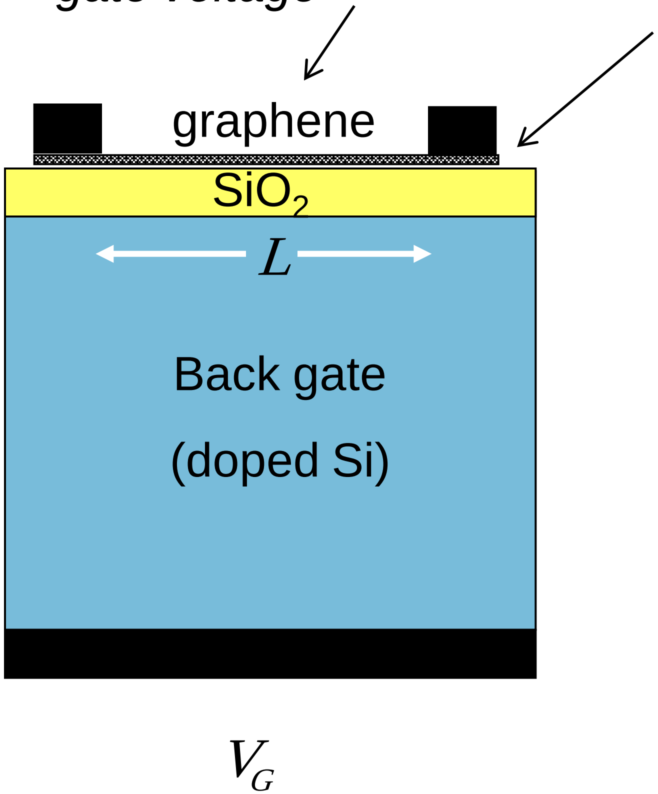
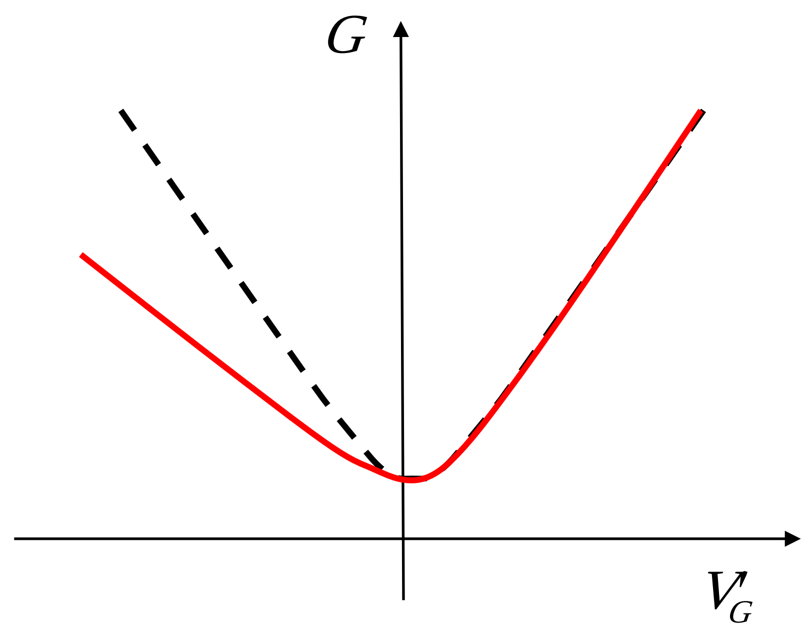
conductance vs. gate voltage measurements

either N-type or P-type depending on the back gate voltage

either N-type or P-type depending on the metal workfunction





B. Huard, N. Stander, J.A. Sulpizo, and D. Goldhaber-Gordon, “Evidence of the role of contacts on the observed electron-hole asymmetry in graphene,” *Pbys. Rev. B*., **78**, 121402(R), 2008.

outline

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conclusions

1) For abrupt graphene PN junctions transmission is reduced due to wavefunction mismatch.
2) For graded junctions, tunneling reduces transmission and sharply focuses it.
3) Normal incident rays transmit perfectly
4) The conductance of a graphene PN junction can be considerably less than that of an NN junction.
5) Graphene PN junctions may affect measurements and may be useful for focusing and guiding electrons.

questions