NCN@Purdue Summer School, July 20-24, 2009

Low-Bias Transport in Graphene: *an introduction*

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acknowledgments

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A set of notes to accompany this lecture is available. The notes provide derivations for all of the equations presented in this lecture, as well as additional discussions for T_L > 0K*, the role the the graphene quantum capacitance, and derivations of scattering rates. See:*

D. Berdebes, T. Low, and M.S. Lundstrom, "Lecture notes on Low bias transport in graphene, July 2009."

outline

1) Introduction and Objectives

2) Theory

3) Experimental approach

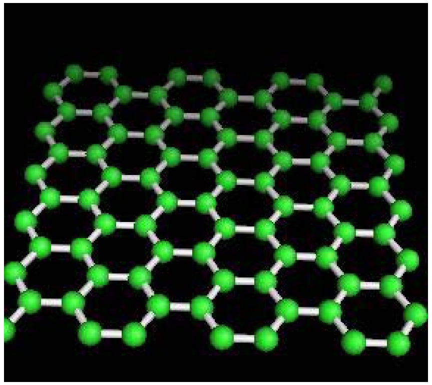
4) Results

5) Discussion

6) Summary

graphene

Graphene is a one-atom-thick planar carbon sheet with a honeycomb lattice.



source: CNTBands 2.0 on nanoHUB.org

Graphene has an unusual bandstructure that leads to interesting effects and potentially useful electronic devices.

objectives

- Describe the experimental techniques commonly-used to characterize low-bias conductance of graphene.
- Show some typical results.
- Analyze the results and discuss the general features of low-bias transport in graphene and how they are related to carrier scattering.

outline

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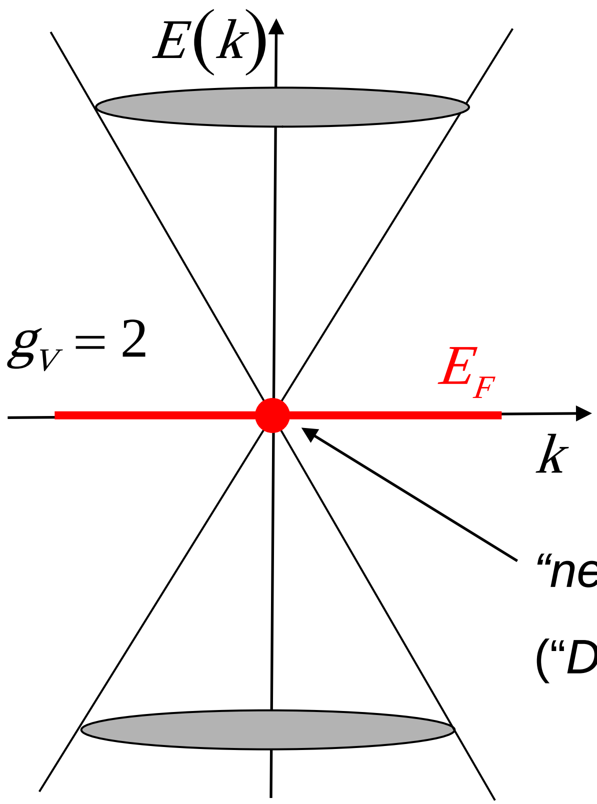
simplified graphene bandstructure

We will use a very simple description of the graphene bandstructure, which is a good approximation near the Fermi level.



"neutral point"

("*Dirac point*")

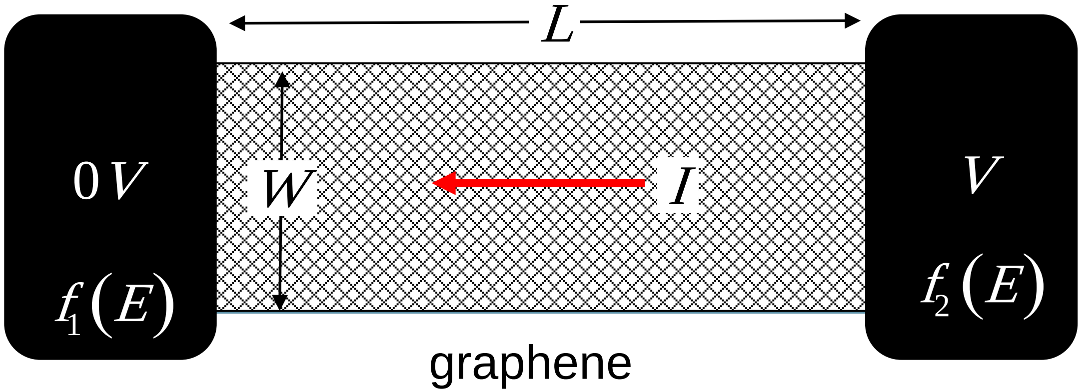
$$E(k) = \pm\hbar\upsilon_F k = \pm\hbar\upsilon_F\sqrt{k_x^2 + k_y^2}$$

$$\upsilon(k) = \upsilon_F \approx 1\times10^8\,\mathrm{cm/s}$$

$$D(E) = 2|E|/\pi\hbar^2\upsilon_F^2$$

We will refer to the $E_F > 0$ case, as "n-type graphene" and to the $E_F < 0$ case as "p-type graphene."

low-bias transport theory



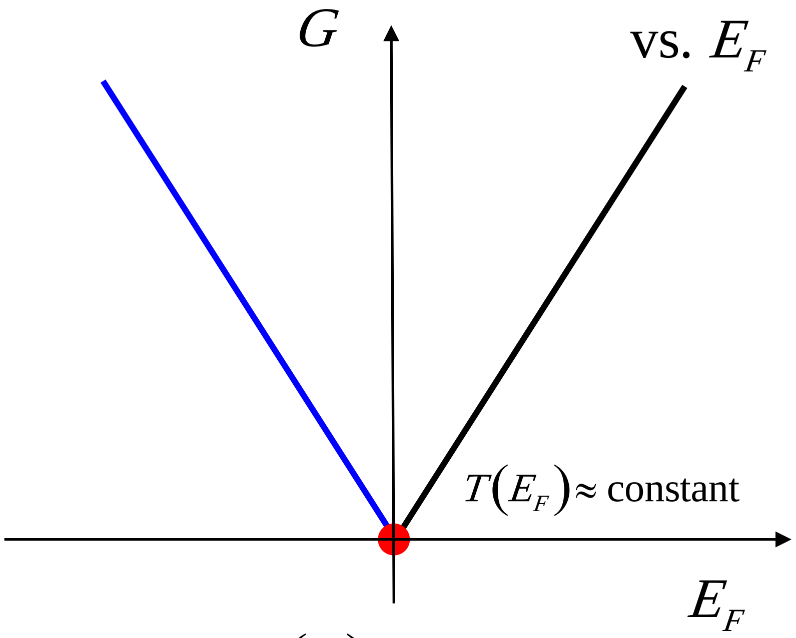
E_{F1} $\qquad$ $E_{F2} = E_{F1} - qV$

$$I = \frac{2q}{h}\int_{-\infty}^{+\infty} T(E)M(E)\left(f_1 - f_2\right)dE$$

$$G = \frac{I}{V} = \frac{2q^2}{h}\int_{-\infty}^{+\infty} T(E)M(E)\left(-\partial f_0/\partial E\right)dE$$

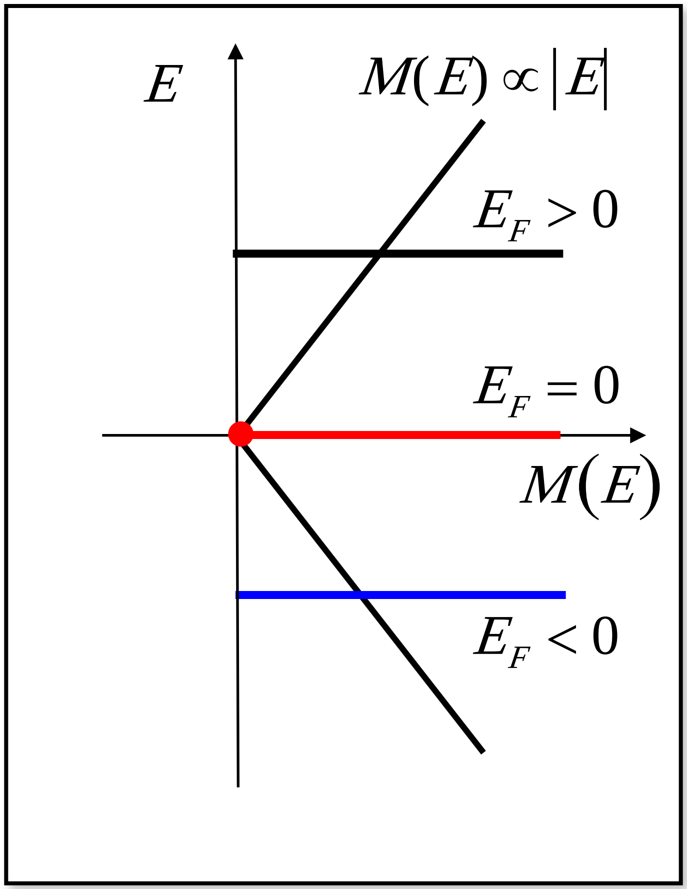
$$\left\{\begin{array}{l} f_0(E) = 1\Big/\left(1+e^{(E-E_F)/k_BT}\right) \\ T(E) \equiv \lambda(E)\big/\left(\lambda(E)+L\right) \\ M(E) = W\,2|E|/\pi\hbar\upsilon_F \end{array}\right.$$

expected results: *G* vs. E_F at T_L = 0K

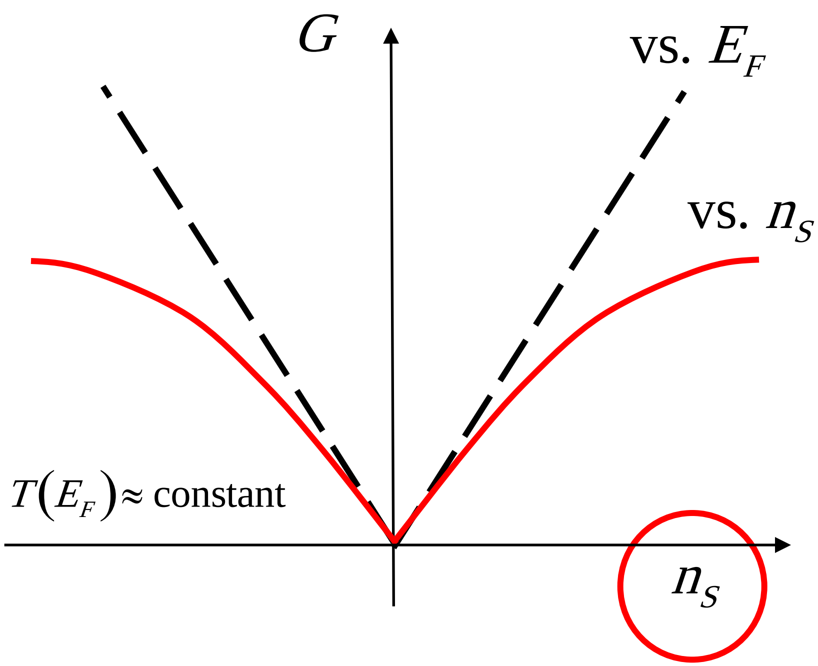


$$-\frac{\partial f_0(E)}{\partial E} = \delta(E - E_F)$$

$$\boxed{G(0\text{K}) = \frac{2q^2}{h} T(E_F) M(E_F)}$$



expected results: G vs. n_S at T_L = 0K



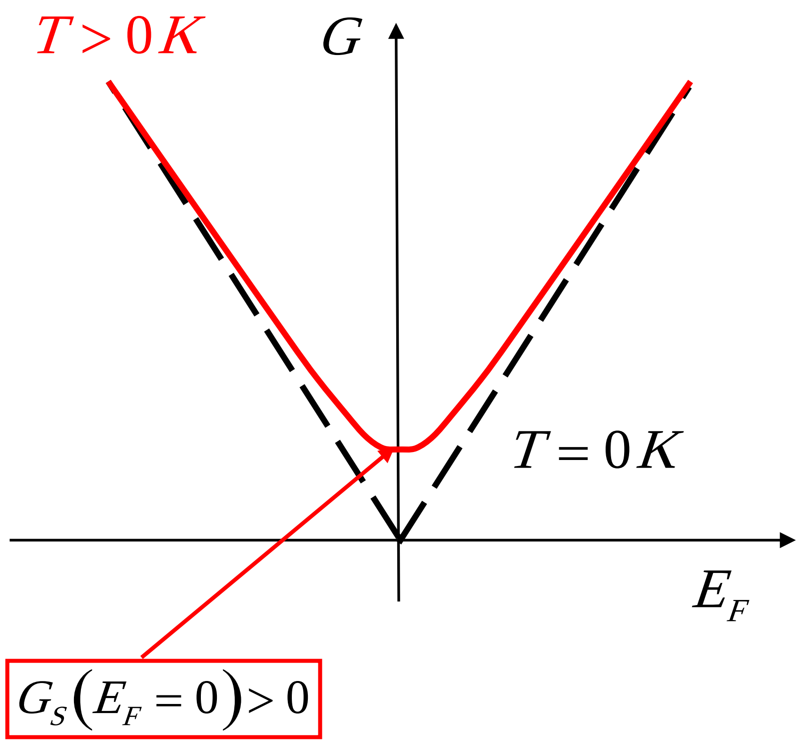
$$G = \frac{2q^2}{h} T(E_F) M(E_F)$$

$$n_S(E_F) = \frac{1}{\pi}\left(\frac{E_F}{\hbar \upsilon_F}\right)^2 \propto E_F^2$$

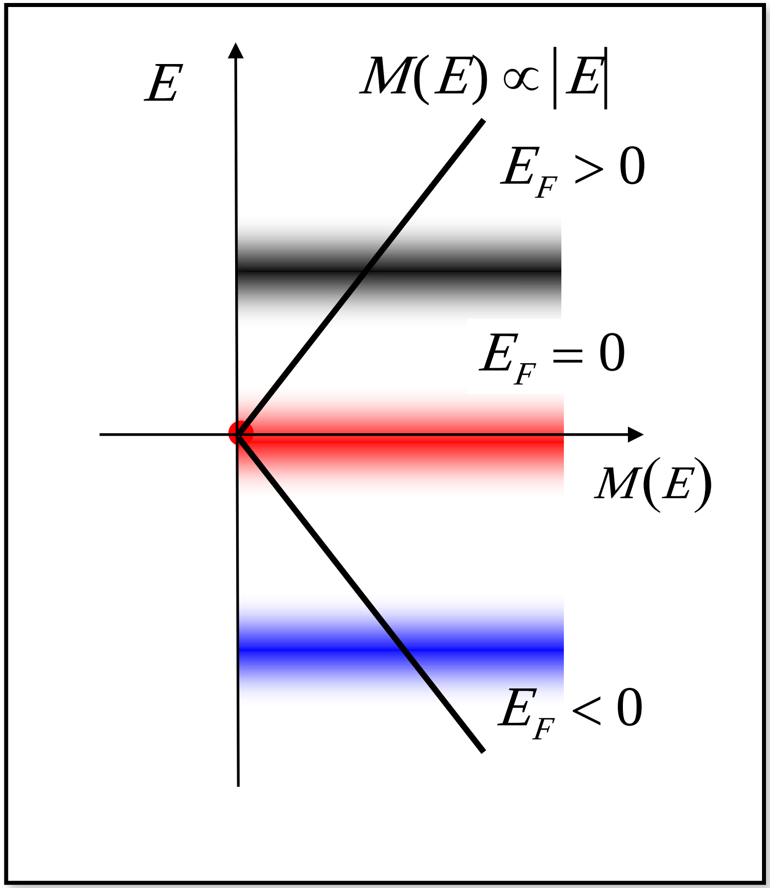
$$M(E_F) \propto E_F \propto \sqrt{n_S}$$

$$G \propto \sqrt{n_S}$$

expected results: $T_L > 0K$



$$G(T_L > 0K) = \frac{2q^2}{h}\langle T(E_F)M(E_F)\rangle$$



some key equations (T = 0K)

$$G(0K) = \frac{2q^2}{h} T(E_F) M(E_F)$$

Describes the conductance of the conduction (E > 0) or valence (E < 0) bands.

$$M(E_F) = W\, 2E_F / \pi\hbar\upsilon_F$$

(For T > 0, the total conductance is the sum of the two.)

$$T(E_F) = \lambda(E_F) \big/ \big(\lambda(E_F) + L\big)$$

$$G(0K) = \frac{2q^2}{h} \frac{\lambda(E_F)}{\lambda(E_F) + L} W \frac{2E_F}{\pi\hbar\upsilon_F}$$

$$G = G_S \frac{W}{L}$$

G_s is the "sheet conductance" or conductivity, σ

$$G_S(0\mathrm{K}) = \frac{2q^2}{h} \lambda_{app} \left(\frac{2E_F}{\pi\hbar\upsilon_F} \right)$$

$$\frac{1}{\lambda_{app}} = \frac{1}{\lambda(E_F)} + \frac{1}{L}$$

an aside

When E_F > 0, graphene is strongly degenerate and:

$$G_S(E_F) = \left(\frac{2q^2 k_B T_L}{\pi^2 \hbar^2 \upsilon_F}\right) \langle \lambda_{app} \rangle \mathcal{F}_0\left(E_F / k_B T_L\right) \approx \frac{2q^2}{h} \lambda_{app}\left(E_F\right)\left(\frac{2E_F}{\pi \hbar \upsilon_F}\right)$$

T_L > 0K result ≈ T_L = 0K result

$$n_S = \left(\frac{2}{\pi}\right)\left(\frac{k_B T_L}{\hbar \upsilon_F}\right)^2 \mathcal{F}_1\left(E_F / k_B T_L\right) \approx \frac{1}{\pi}\left(\frac{E_F}{\hbar \upsilon_F}\right)^2$$

questions

- How is G vs. E_F (or G vs. n_S) measured experimentally?
- How do the results compare to theory?
- What do the results us about scattering in graphene?

For a comprehensive review of theoretical aspects of graphene, see: A. H. Castro Neto, F. Guinea, N. M. R. Peres, Braga, Portugal, K. S. Novoselov, and A. K. Geim. "The electronic properties of graphene," *Reviews of Modern Physics*, Vol. 81, pp. 109-162, 2009.

outline

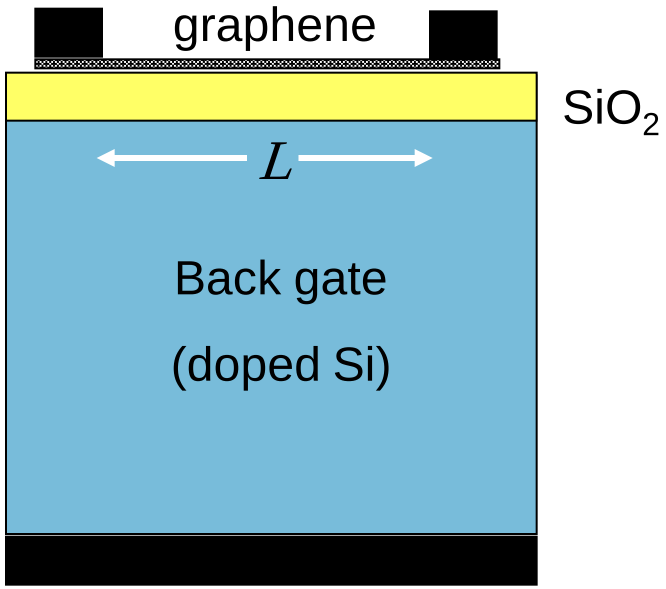
gate-modulated conductance in graphene

1) The location of the Fermi level (or equivalently the carrier density) is experimentally controlled by a "gate."

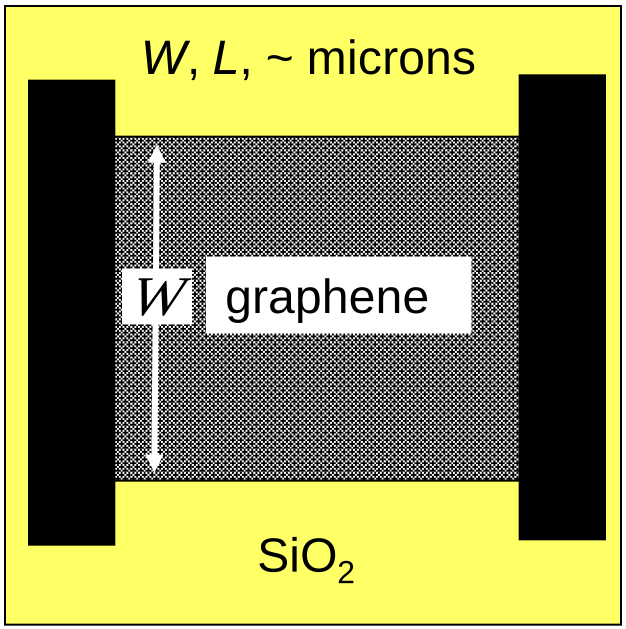
2) In a typical experiments, a layer of graphene is place on a layer of SiO_2, which is on a doped silicon substrate. By changing the potential of the Si substrate (the "back gate"), the potential in the graphene can be modulated to vary E_F and, therefore, n_S.

experimental structure (2-probe)

(4-probe is used to eliminate series resistance and for Hall effect measurements.



Side view

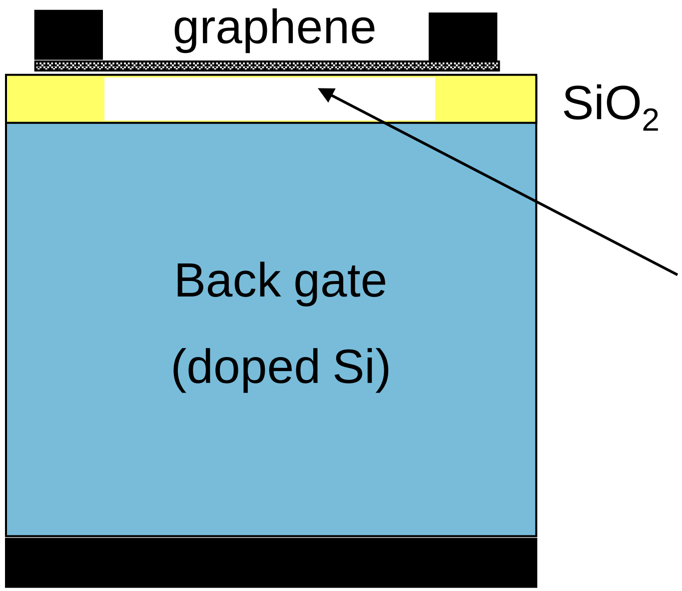


Top view

Typically, Cr/Au or Ti/Au are used for the metal contacts.

The thickness of SiO_2 is typically 300nm or 90nm, which makes it possible to see a single layer of graphene.

suspended graphene



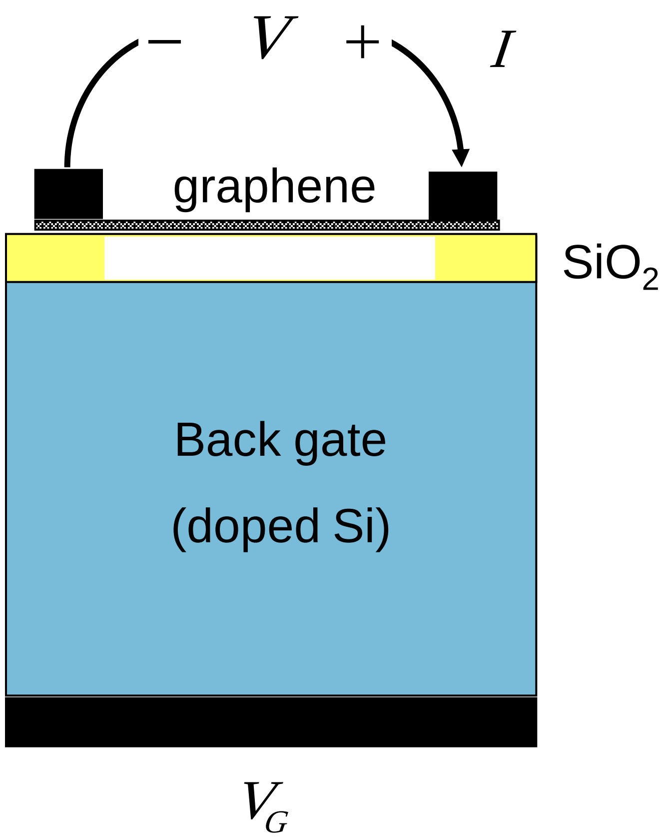
Side view

SiO_2 removed by etching.
This eliminates charges in the SiO_2 and after annealing produces higher quality graphene.

“Temperature-Dependent Transport in Suspended Graphene”

K. Bolotin, et al., *Phys. Rev. Lett*. **101**, 096802 (2008)

measurements



$$G = I/V \qquad R = V/I$$

At a fixed temperature:

$$G(V_G) \text{ or } R(V_G)$$

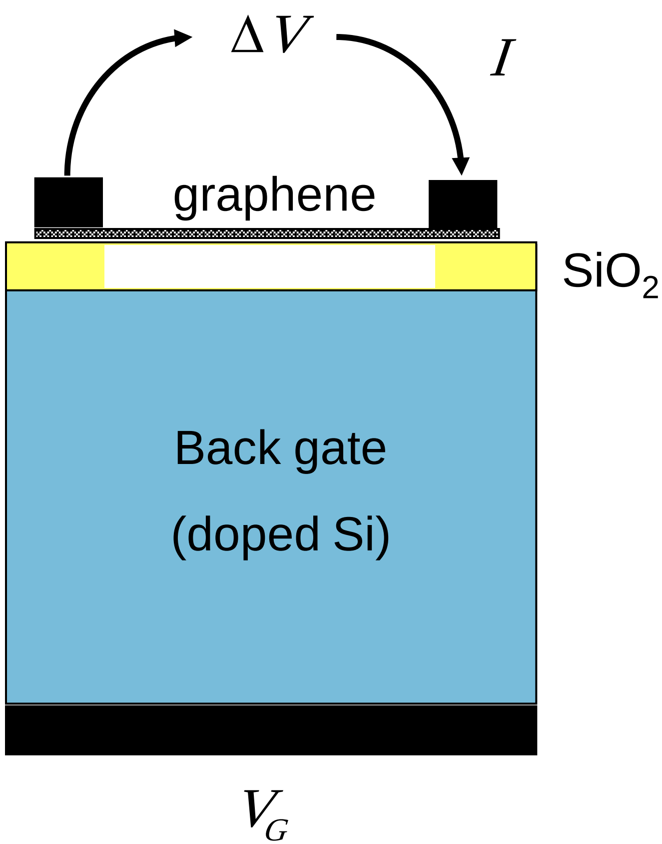
At a fixed gate voltage:

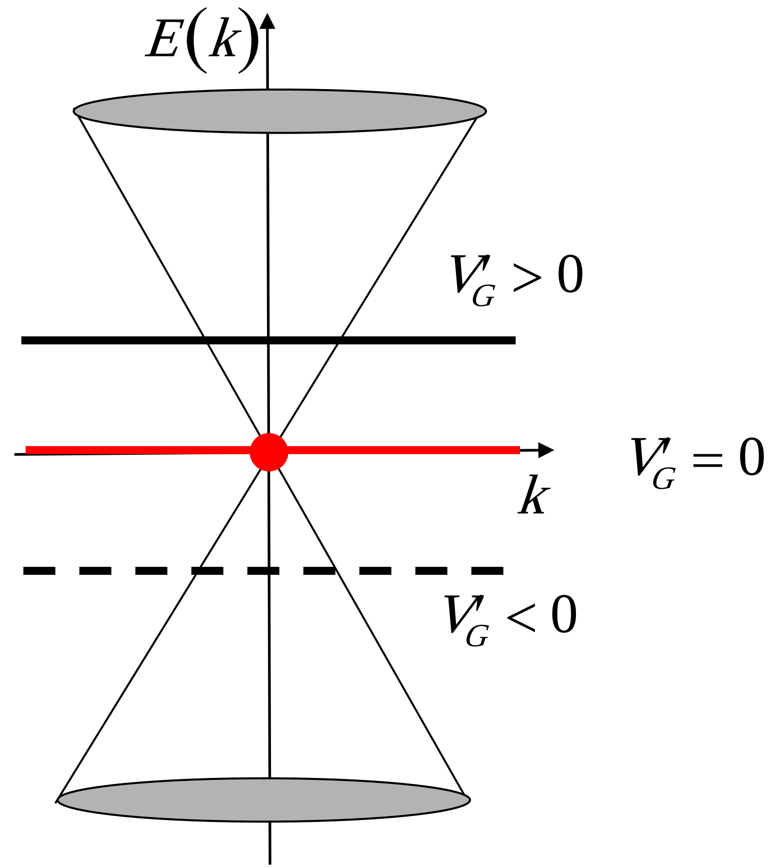
$$G(T_L) \text{ or } R(T_L)$$

Frequently the sheet conductance or sheet resistance is reported (and this is usually referred to as the 'conductivity' or the 'resistivity.')

$$G = G_S(W/L) \qquad R = R_S(L/W)$$

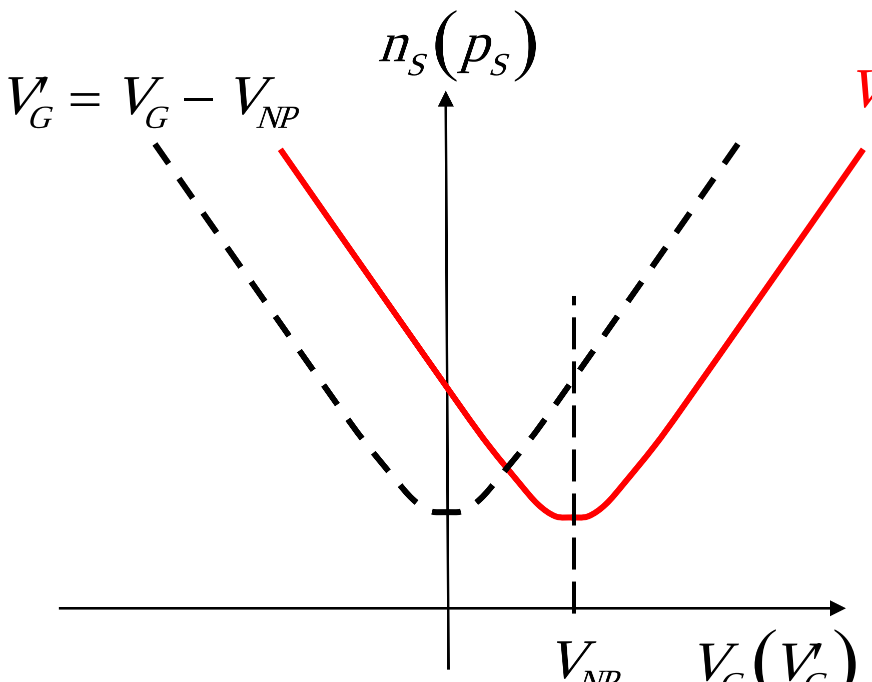
using a gate voltage to change the Dirac point (or E_F)





$$V_G' = V_G - V_{NP}$$

gate voltage - carrier density relation



If the oxide is not too thin (so that the quantum capacitance of the graphene is not important), then:

$$qn_S = C_{ins} V'_G$$

$$C_{ins} = \frac{\varepsilon_{ins}}{t_{ins}}$$

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sheet conductance vs. V_G

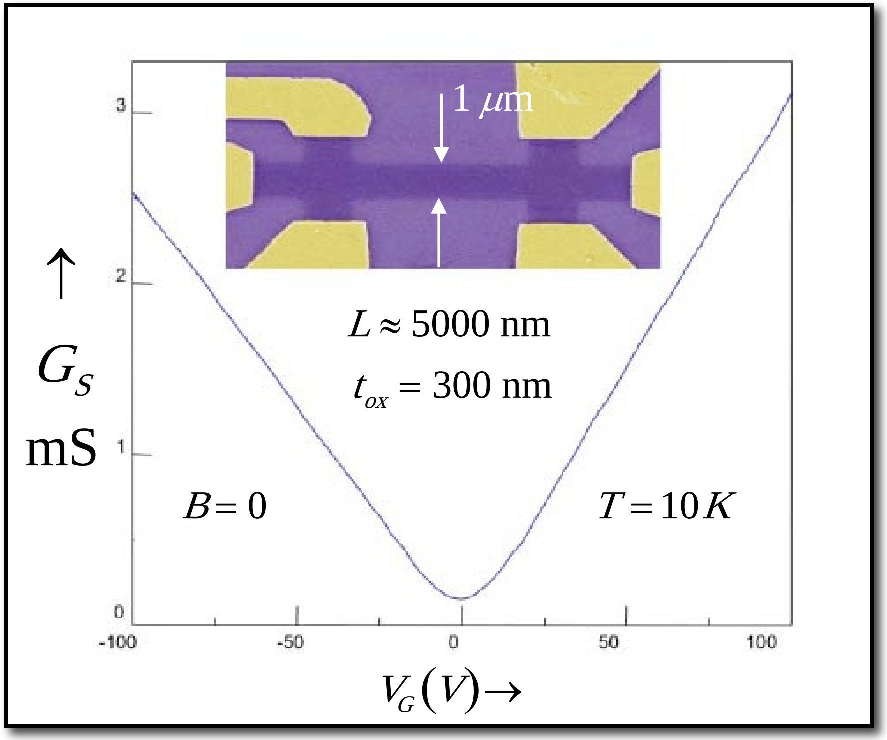


Fig. 30 in A. H. Castro, et al.,"The electronic properties of graphene," *Rev. of Mod. Phys.*, **81**, 109, 2009.

$$G = G_S W/L$$

$$G_S(E_F) \approx \frac{2q^2}{h} \lambda_{app}(E_F) \left(\frac{2E_F}{\pi \hbar \upsilon_F} \right)$$

$$n_S = C_{ox} V_G \approx \frac{1}{\pi} \left(\frac{E_F}{\hbar \upsilon_F} \right)^2$$

$$\lambda_{app}(E_F) = \frac{G_S / \left(2q^2/h \right)}{2\sqrt{n_S/\pi}}$$

$$\left(T_L = 0 \text{ K} \right)$$

mean-free-path ($V_G = 100$V)

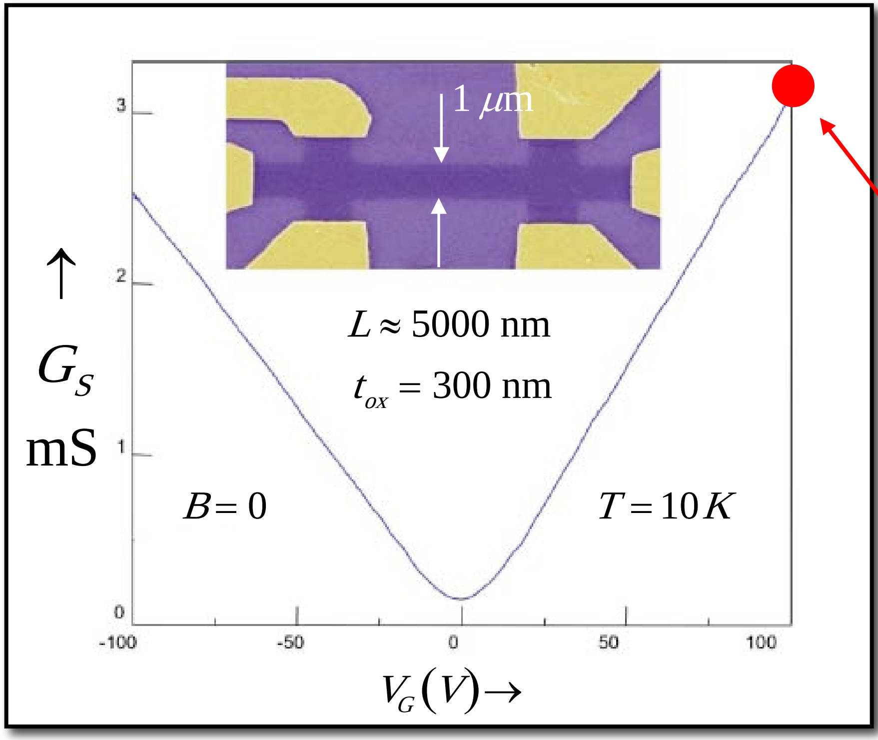


Fig. 30 in A. H. Castro, et al.,"The electronic properties of graphene," *Rev. of Mod. Phys.*, **81**, 109, 2009.

$$G_S \approx 3.0 \text{ mS}$$

$$n_S \approx 7.1 \times 10^{12} \text{ cm}^{-2}$$

$$E_F \approx 0.3 \, \text{eV}$$

$$\lambda_{app}\left(0.3 \text{ eV}\right) \approx 130 \text{ nm}$$

$$\lambda\left(0.3 \, \text{eV}\right) << L$$

mean-free-path ($V_G = 50V$)

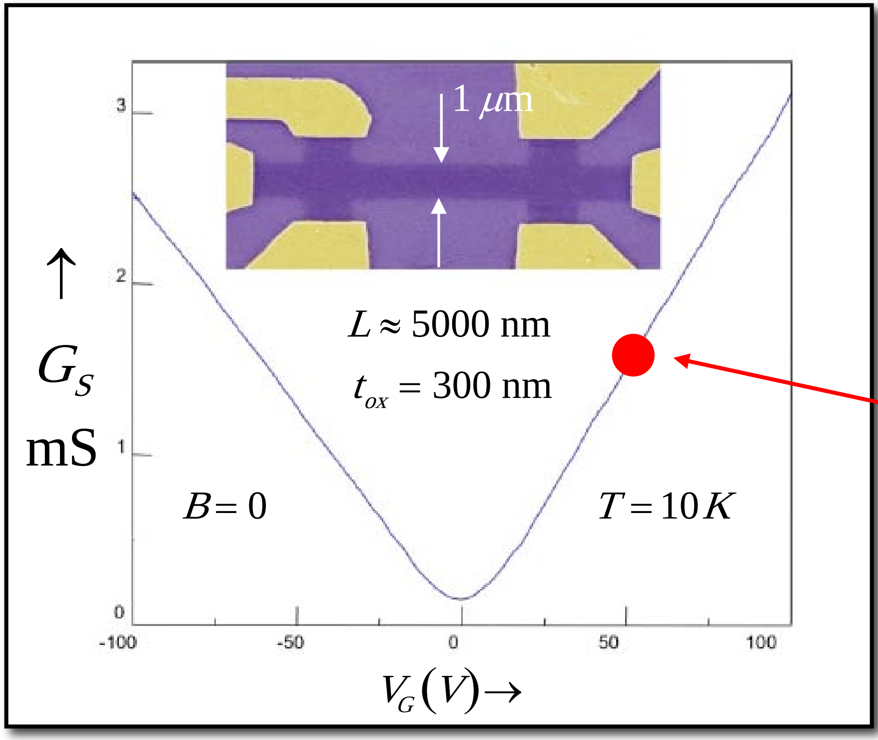


Fig. 30 in A. H. Castro, et al.,"The electronic properties of graphene," *Rev. of Mod. Phys.*, **81**, 109, 2009.

$$G_S \approx 1.5\ \text{mS}$$

$$n_S \approx 3.6 \times 10^{12}\ \text{cm}^{-2}$$

$$E_F \approx 0.2\ \text{eV}$$

$$\lambda_{app}\left(0.2\ \text{eV}\right) \approx 90\ \text{nm}$$

$$\frac{\lambda\left(0.2\ \text{eV}\right)}{\lambda\left(0.3\ \text{eV}\right)} \approx 0.69$$

$$\frac{0.2\ \text{eV}}{0.3\ \text{eV}} \approx 0.67$$

$$\lambda\left(E_F\right) \propto E_F$$

mobility

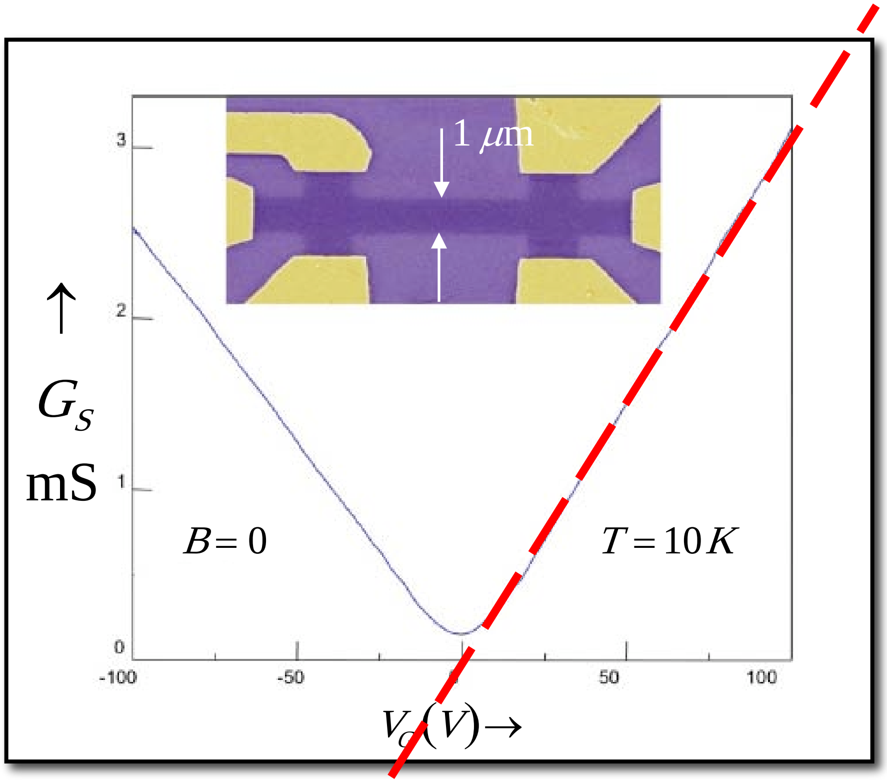


Fig. 30 in A. H. Castro, et al.,"The electronic properties of graphene," *Rev. of Mod. Phys.*, **81**, 109, 2009.

Since, $G_S \sim n_S$, we can write:

$$G_S \equiv n_S q \mu_n$$

and deduce a mobility:

$$\mu_n \approx 12{,}500 \text{ cm}^2/\text{V-sec}$$

Mobility is constant, but mean-free-path depends on the Fermi energy (or n_S).

$V_G = 0$

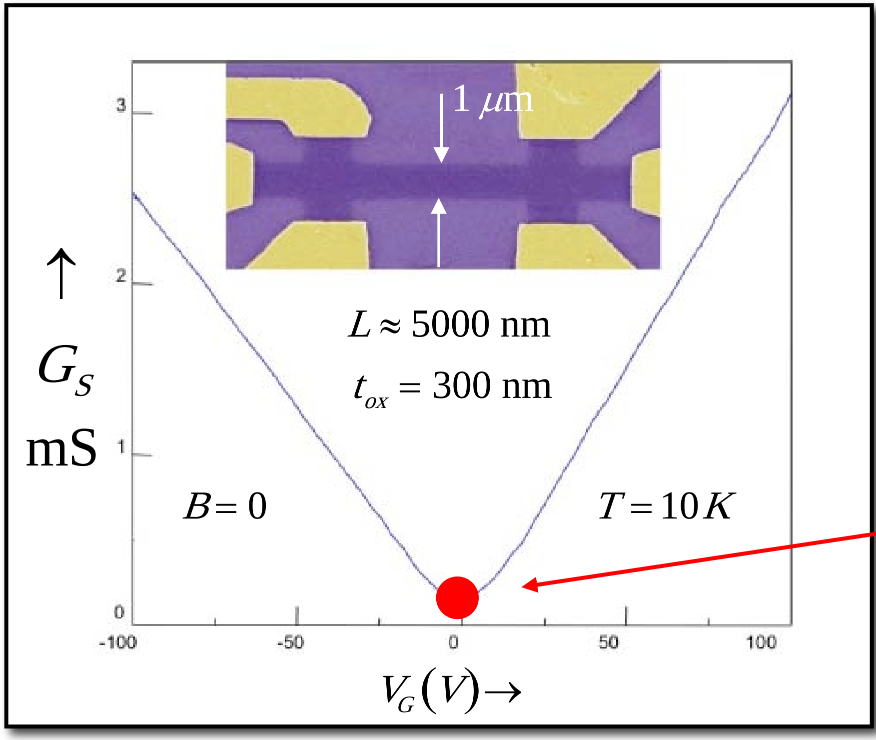


Fig. 30 in A. H. Castro, et al.,"The electronic properties of graphene," *Rev. of Mod. Phys.*, **81**, 109, 2009.

$$G_S \approx 0.16 \text{ mS}$$

$$n_S = C_{ox} V_G \approx 0\ ?$$

$$\lambda_{app} = \frac{G_S}{\left(2q^2/h\right) 2\sqrt{n_S/\pi}}$$

$$\lambda_{app} \rightarrow \infty\ ?$$

$$\left(T_L = 0 \text{ K}\right)$$

electron-hole puddles

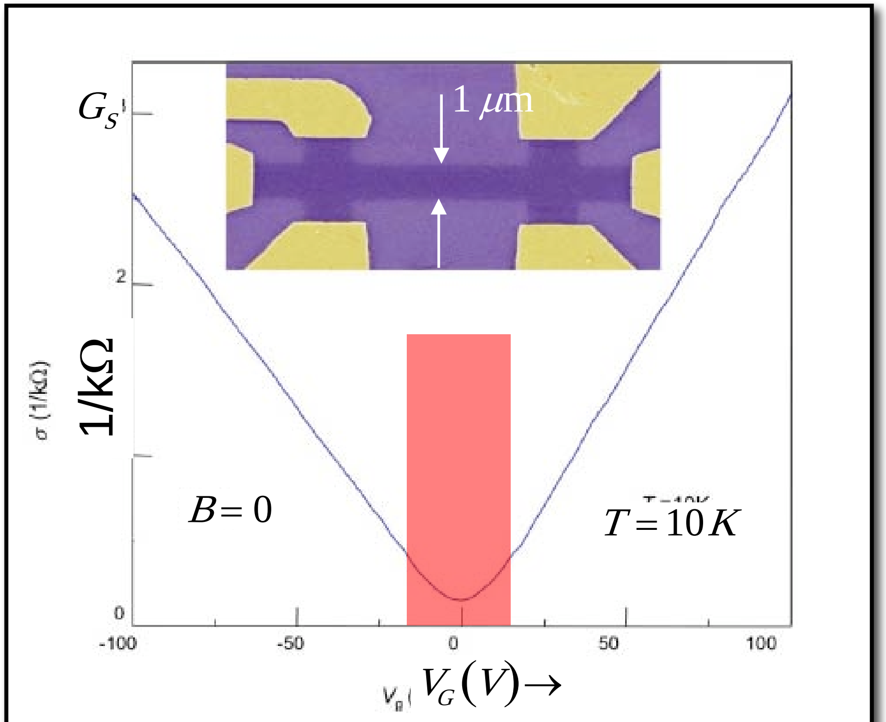
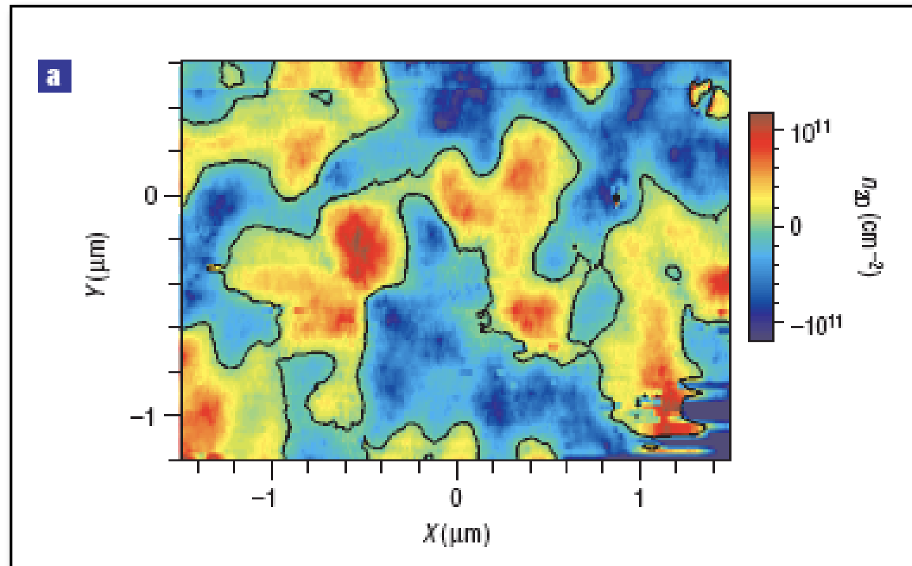
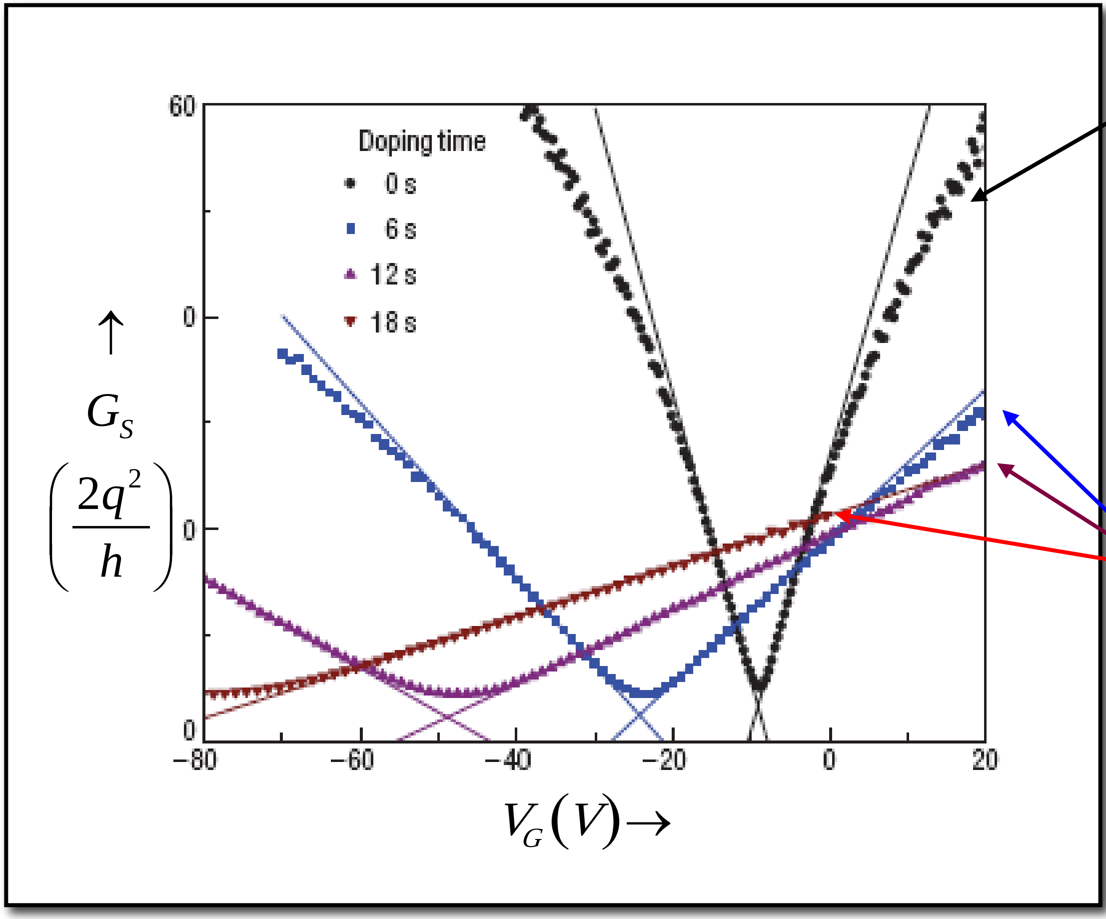


Fig. 30 in A. H. Castro, et al.,"The electronic properties of graphene," *Rev. of Mod. Phys.*, **81**, 109, 2009.



J. Martin, et al, "Observation of electron–hole puddles in graphene using a scanning single-electron transistor," *Nature Phys.*, **4**, 144, 2008

effect of potassium doping

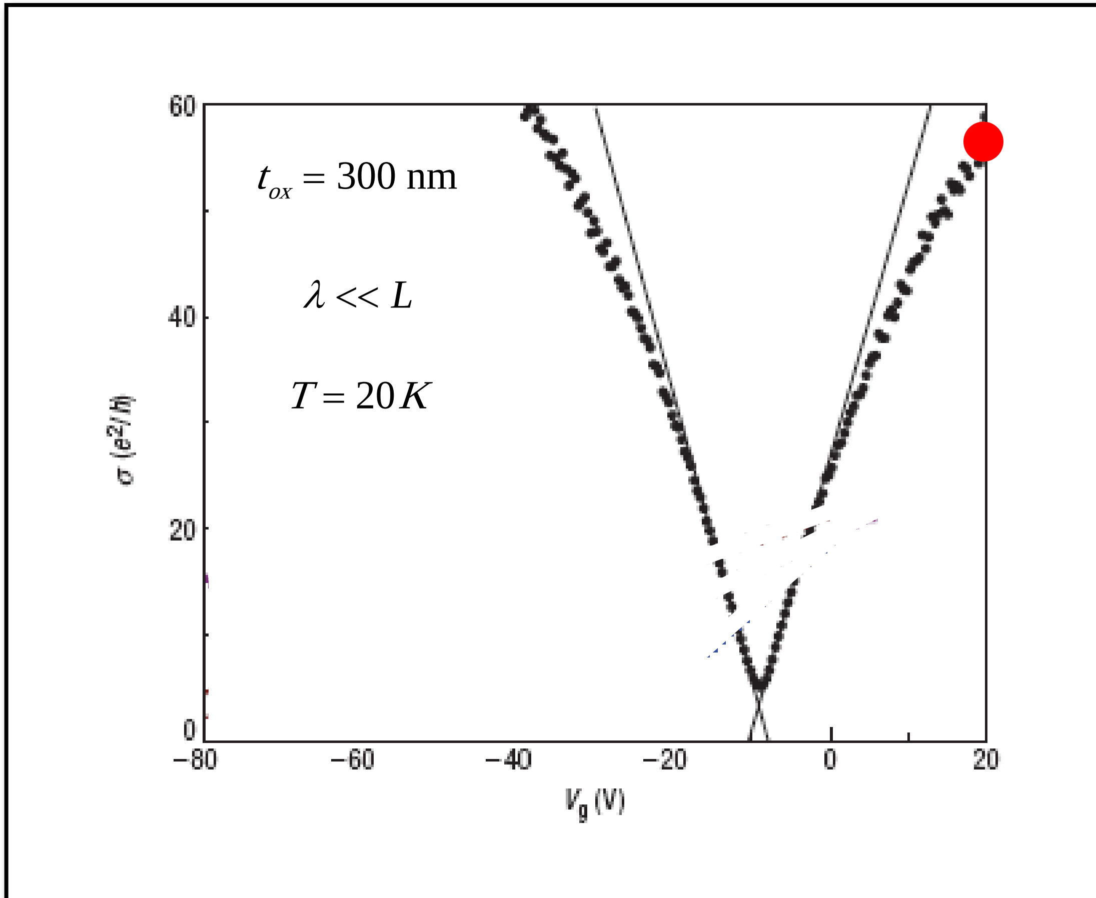


For nominally undoped samples, G_S vs. n_S is non-linear.

As doping increases, G_S vs. n_S becomes more linear, mobility decreases, and the NP shifts to the left.

J.-H. Chen, C. Jang, S. Adam, M. S. Fuhrer, E. D. Williams,and M. Ishigami, “Charged-impurity scattering in graphene,” *Nature Phys*., **4**, 377-381, 2008.

nominally undoped sample



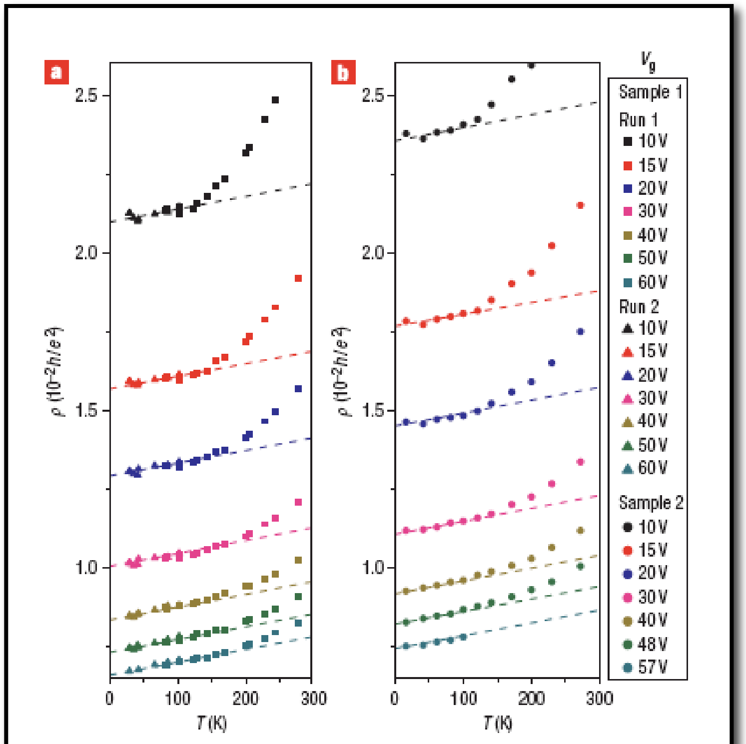
$$\lambda_{app} = \frac{G_S / \left(2q^2/h\right)}{2\sqrt{n_S/\pi}} \approx 164 \text{ nm}$$

$$\lambda << L$$

J.-H. Chen, C. Jang, S. Adam, M. S. Fuhrer, E. D. Williams,and M. Ishigami, "Charged-impurity scattering in graphene," *Nature Phys*., **4**, 377-381, 2008.

temperature dependence

Away from the conductance minimum, the conductance decreases as T_L increases (or resistivity increases as temperature increases).



$$T_L < 100\,K: \quad R_S \propto T_L$$

(acoustic phonon scattering - intrinsic)

$$T_L > 100\,K: \quad R_S \propto e^{\hbar\omega_0/k_B T_L}$$

(optical phonons in graphene or surface phonons at SiO_2 substrate)

J.-H. Chen, J. Chuan, X. Shudong, M. Ishigami, and M.S. Fuhrer, “Intrinsic and extrinsic performance limits of graphene devices on SiO_2,” *Nature Nanotechnology*, **3**, pp. 206-209, 2008.

phonons and temperature dependence

$$R_S = \frac{1}{G_S} \propto \frac{1}{\lambda} \propto N_\beta$$

$$N_\beta = \frac{1}{e^{\hbar\omega(\beta)/k_B T_L} - 1}$$

acoustic phonons:

$$\hbar\omega < k_B T_L$$

$$N_\beta \approx \frac{k_B T_L}{\hbar\omega}$$

$$R_S \propto T_L$$

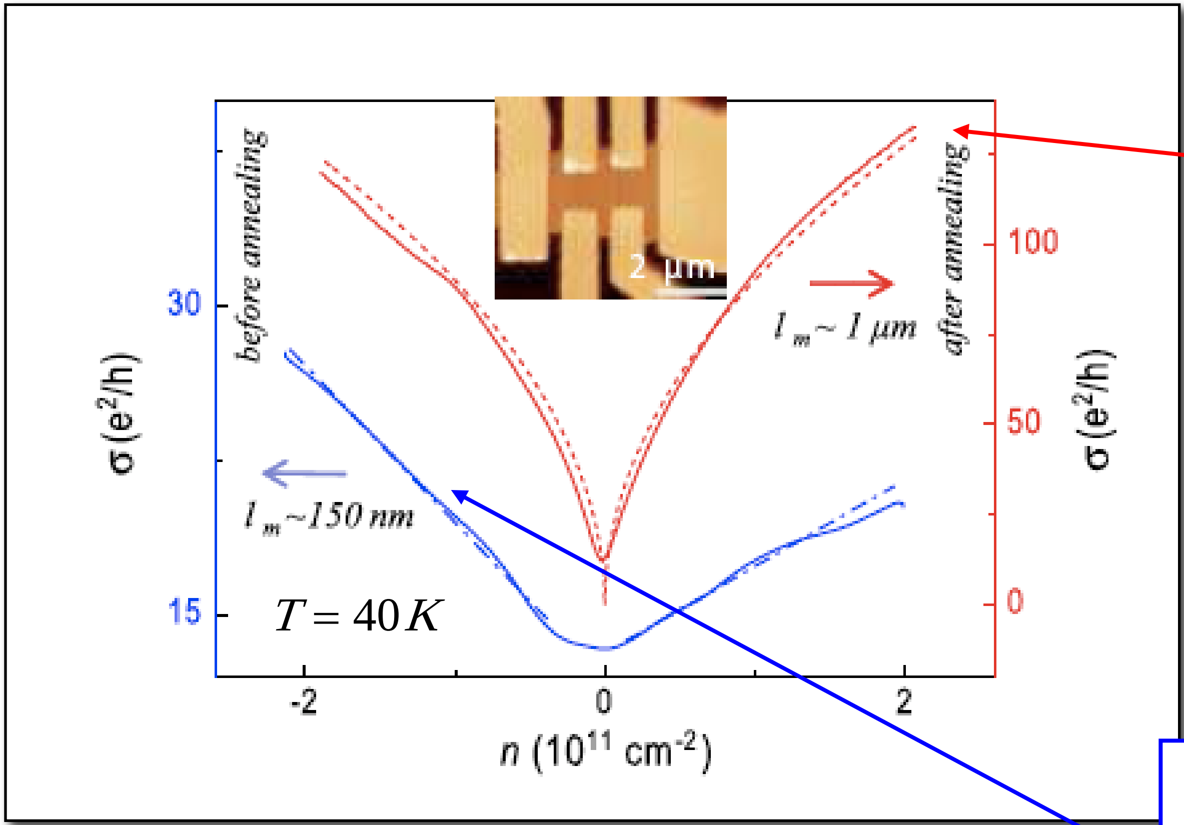
optical phonons:

$$\hbar\omega_0 \approx k_B T_L$$

$$N_\beta = \frac{1}{e^{\hbar\omega_0/k_B T_L} - 1}$$

$$R_S \propto \frac{1}{e^{\hbar\omega_0/k_B T_L} - 1}$$

unannealed vs. annealed suspended graphene



$$G_S \propto \sqrt{n_S}$$

$$\lambda_{app} \approx 1300 \text{ nm}$$

expected from ballistic theory

$$G_S = n_S q \mu_n + G_{res}$$

K. I. Bolotin, K. J. Sikes, J. Hone, H. L. Stormer, and P. Kim, “Temperature dependent transport in suspended graphene,” 2008

about mobility

$$G_S(E_F) \approx \frac{2q^2}{h} \lambda_{app}(E_F) \left(\frac{2E_F}{\pi \hbar \upsilon_F} \right)$$

$$G_S(E_F) \propto \lambda_{app}(E_F) \sqrt{n_S}$$

$$G_S \equiv n_S q \mu_n$$

$$\mu_n \propto \frac{\lambda_{app}(E_F)}{\sqrt{n_S}}$$

Case 1):

$$\lambda_{app} \propto E_F \propto \sqrt{n_S}$$

$$G_S \propto n_S$$

μ_n constant

Case 2):

λ_{app} constant

$$G_S \propto \sqrt{n_S}$$

$$\mu_n \propto 1 / \sqrt{n_S}$$

experimental summary: graphene on SiO_2

1) Low conductance samples often show $G_S \sim n_S$ (away from the minimum)

2) Higher conductance samples are frequently non-linear (G_S rolls off at higher n_S)

3) $G_S(T)$ decreases with temperature ("metallic") for large n_S

4) $R_S \sim T_L$ for $T_L < 100$K and superlinear for $T_L > 100$K

5) Best mobilities for graphene on SiO_2 are ~30,000 cm^2/V-s at $T_L = 5$K

6) Asymmetries between $+V_G$ and $-V_G$ are often seen.

experimental summary: suspended graphene

1) Before annealing $G_S \sim n_S$ (away from the minimum)

2) After annealing, G_S increases and G_S vs. n_S becomes non-linear

3) After annealing, G_S is close to the ballistic limit

4) Best mean-free-paths are ~ 1µm at T_L = 5K

5) G_S decreases with T_L for large n_S but increases with T_L near the Dirac point.

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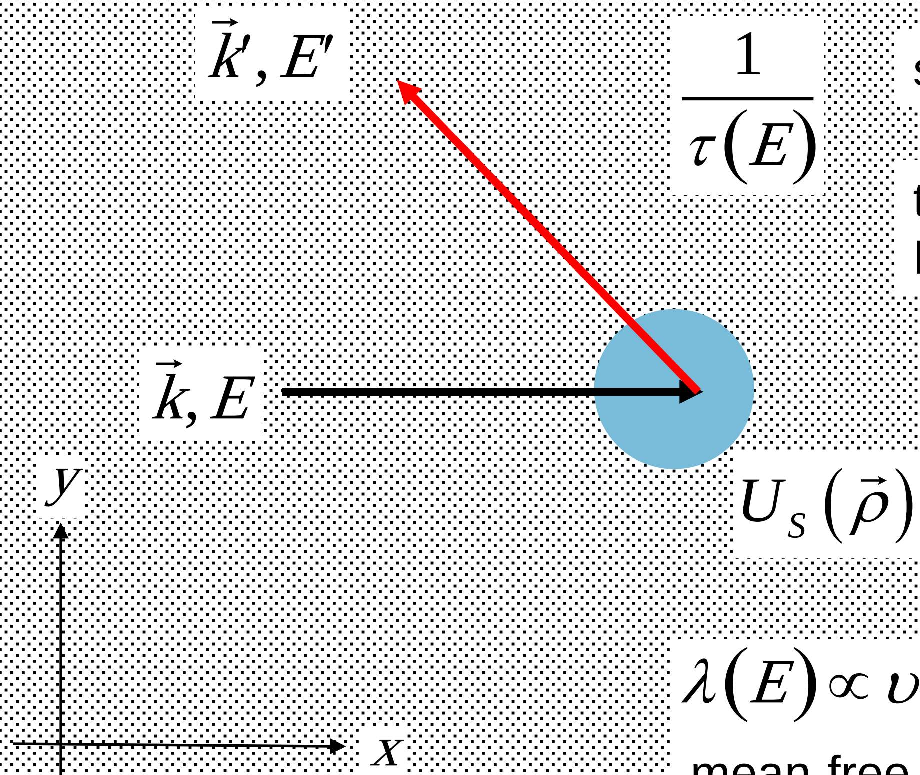
conductance and scattering

$$G(0K) = \frac{2q^2}{h} \frac{\lambda(E_F)}{\lambda(E_F) + L} W \frac{2E_F}{\pi\hbar\upsilon_F}$$

λ(*E*) is the mean-free-path (technically, the mfp for “backscattering”), which is determined by the dominant scattering processes.

scattering



$$\frac{1}{\tau(E)}$$

scattering rate per sec

typically computed from FGR

$$\lambda(E) \propto \upsilon_F \tau(E)$$

mean-free-path *for backscattering*

scattering

$$\lambda(E) = \frac{\pi}{2} \upsilon_F \tau_m(E)$$ (elastic or isotropic scattering)

For many scattering mechanisms (e.g. acoustic phonon, point defect), the scattering rate is proportional to the density of final states:

$$\frac{1}{\tau(E)} \propto D(E) \propto E \qquad \tau(E) \propto E^{-1}$$

The energy-dependent mean-free-path is:

$$\lambda(E) \propto 1/E$$

What does this type of scattering do to the conductance?

effect of short range / ADP scattering

Assume T_L = 0 K and diffusive transport (just to keep the math simple)

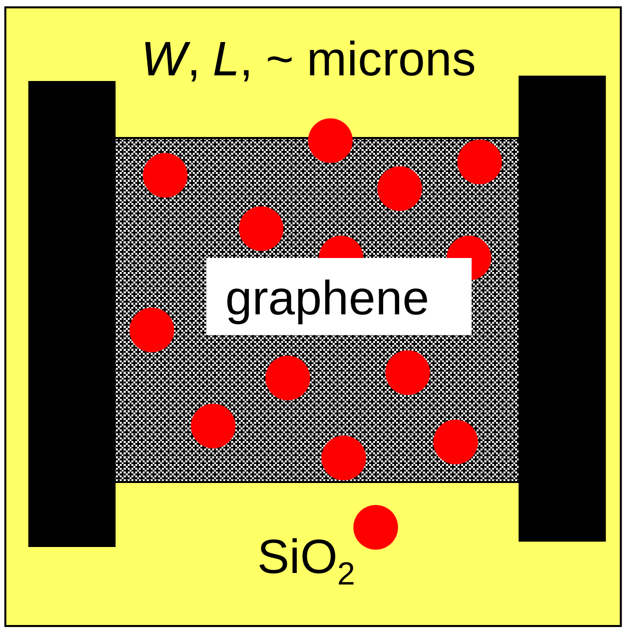
$$G_S = \frac{2q^2}{h}\lambda\left(E_F\right)\left(\frac{2E_F}{\pi\hbar\upsilon_F}\right) \qquad\qquad \lambda\left(E_F\right) \propto 1/E_F$$

$$G_S = \text{constant!}$$

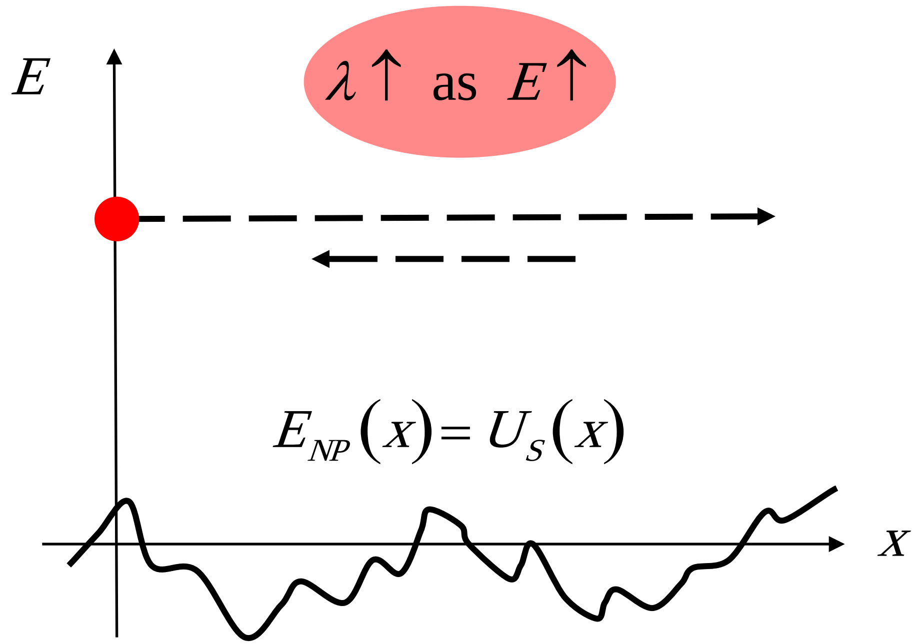
For short range or ADP scattering, G_S is constant.

N.H. Shon and T. Ando, *J. Phys. Soc. Japan,* **67**, 2421, 1998.

long range (charged impurity) scattering



Top view



For screened or unscreened charged impurity scattering, the mfp is **proportional** *to energy.*

Random charges introduce random fluctuations in $E(k)$, which act a scattering centers.

High energy electrons don't "see" these fluctuations and are not scattered as strongly.

effect of charged impurity scattering

Assume $T_L = 0$ K and diffusive transport (just to keep the math simple)

$$G_S = \frac{2q^2}{h}\lambda\left(E_F\right)\left(\frac{2E_F}{\pi\hbar\upsilon_F}\right) \qquad\qquad \lambda\left(E_F\right) \propto E_F$$

$$G_S \propto n_S \qquad \left(\mu_n \quad \text{constant}\right)$$

For charged impurity scattering, G_S vs. n_S is linear.

T. Ando, J. Phys. Soc. Japan, 75, 074716, 2006

N.M.R. Peres, J.M.B. Lopes dos Santos, and T. Stauber, *Phys. Rev. B,* **76**, 073412, 2007.

comment on linear G vs. n_S

The observation of a linear $G(n_S)$ characteristic is frequently taken as experimental evidence of charged impurity scattering, but…

Theoretical work shows that strong, neutral defect scatter can lead to a linear G *vs.* n_S characteristics…

T. Stauber, N.M.R. Peres, and F. Guinea, "Electronic transport in graphene: A semiclassical approach including midgap states," *Phys. Rev. B*, **76**, 205423, 2007.

Even more recent experimental work on intentionally damaged graphene bears this out…

TJ.-H. Chen, W.G. Callen, C. Jang, M.S. Fuhrer, and E.D. Williams, "Defect Scattering in graphene," *Phys. Rev. Lett.*, **102**, 236805, 2009.

the energy-dependent mfp

Mobility is not always the best way to characterize the quality of a graphene film, but mean-free-path is always a well-defined quantity.

We can extract the mean-free path vs. energy from measured data.

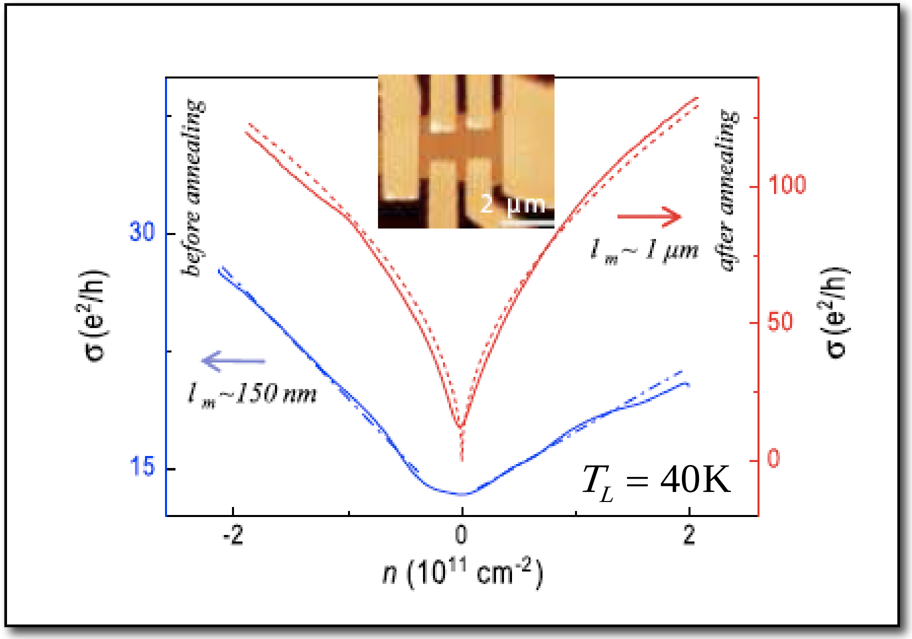
$$\left.\begin{aligned} G_S\left(0\mathrm{K}\right) &= \frac{2q^2}{h}\lambda_{app}\left(E_F\right)\left(\frac{2E_F}{\pi\hbar\upsilon_F}\right) \\ n_S\left(0\mathrm{K}\right) &= \frac{1}{\pi}\left(\frac{E_F}{\hbar\upsilon_F}\right)^2 \end{aligned}\right\}$$

$$\lambda_{app}\left(E_F\right) = \frac{G_S\left(V_g\right)/\left(2q^2/h\right)}{2\sqrt{n_S\left(V_g\right)/\pi}}$$

$$\frac{1}{\lambda_{app}\left(E_F\right)} = \frac{1}{\lambda\left(E_F\right)} + \frac{1}{L}$$

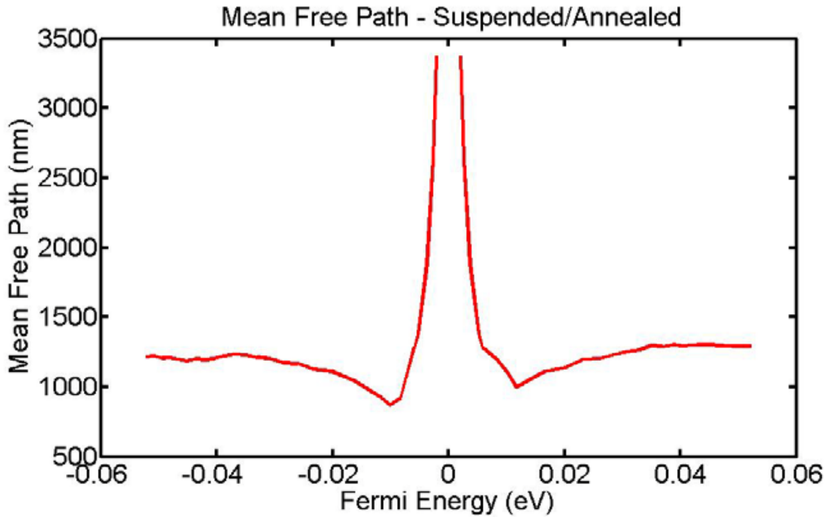
*The apparent mfp is the **shorter** of the actual mfp and the sample length.*

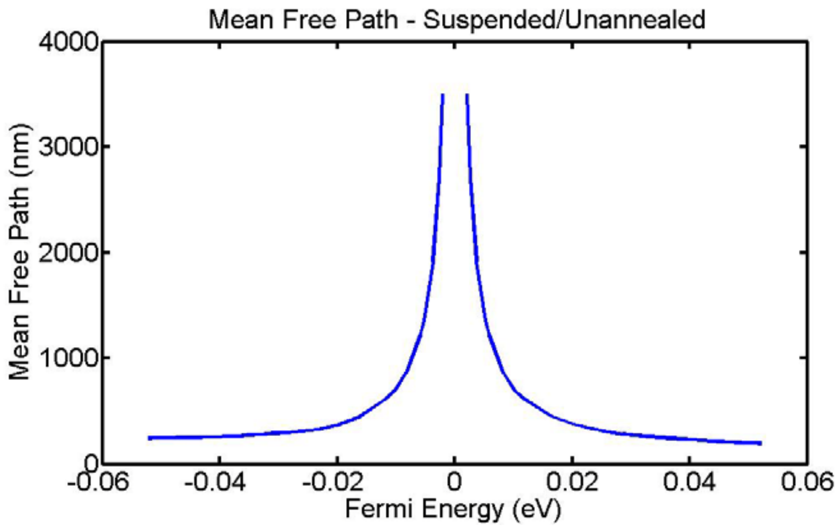
example



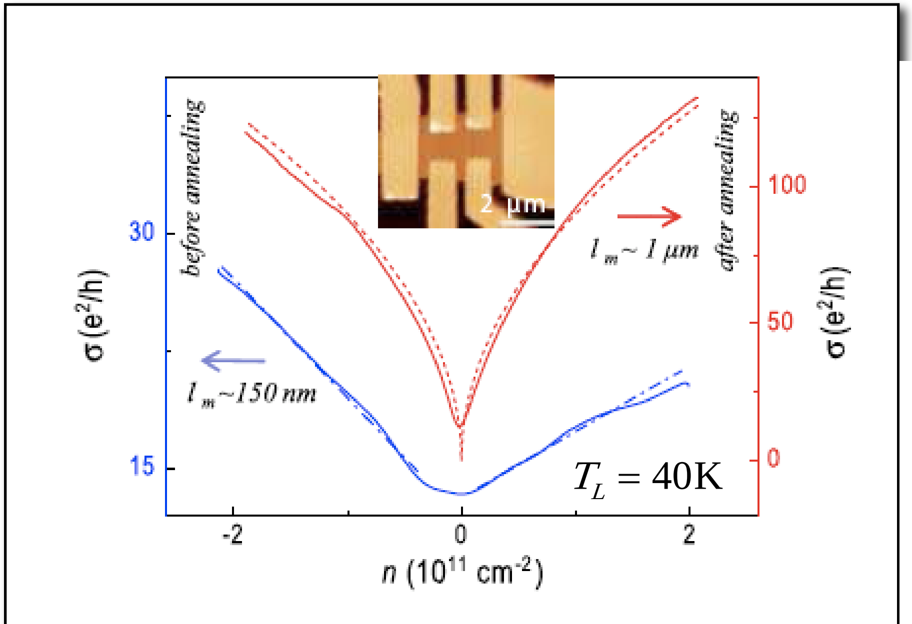
"Temperature-Dependent Transport in Suspended Graphene"

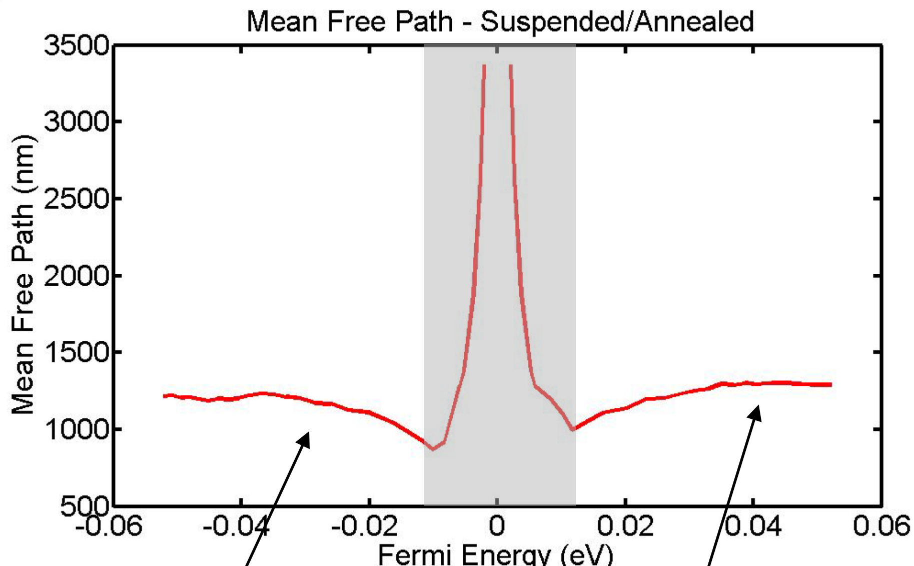
K. Bolotin, et al., *Phys. Rev. Lett*. **101**, 096802 (2008)





suspended, annealed





"Temperature-Dependent Transport in Suspended Graphene"

K. Bolotin, et al., *Phys. Rev. Lett*. **101**, 096802 (2008)

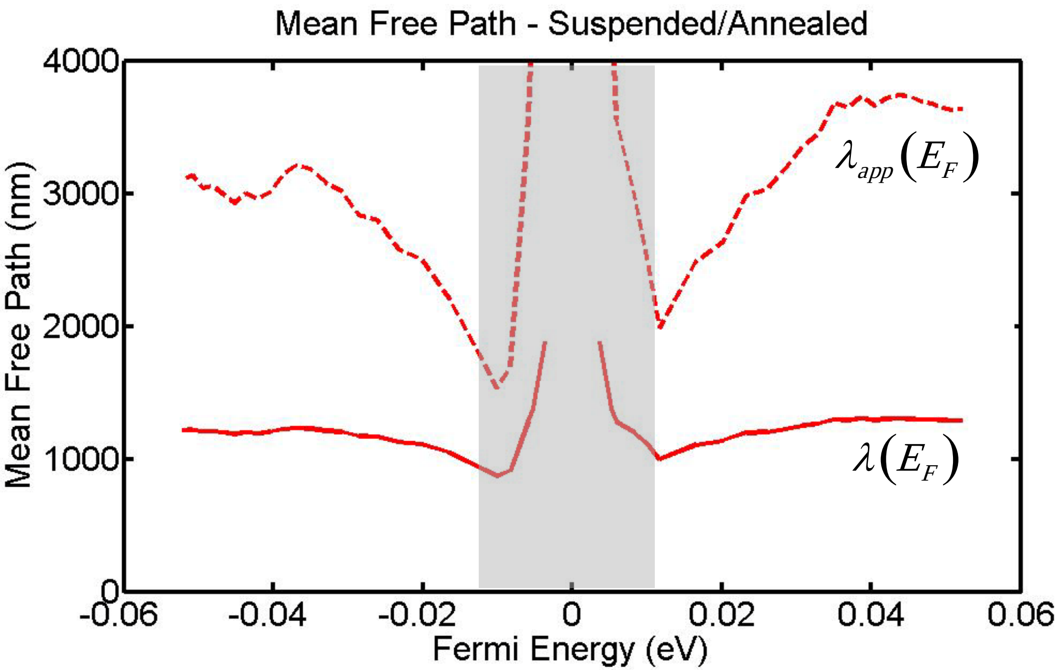
apparent mfp increases with energy

apparent mfp independent of energy - approximately L

suspended, annealed

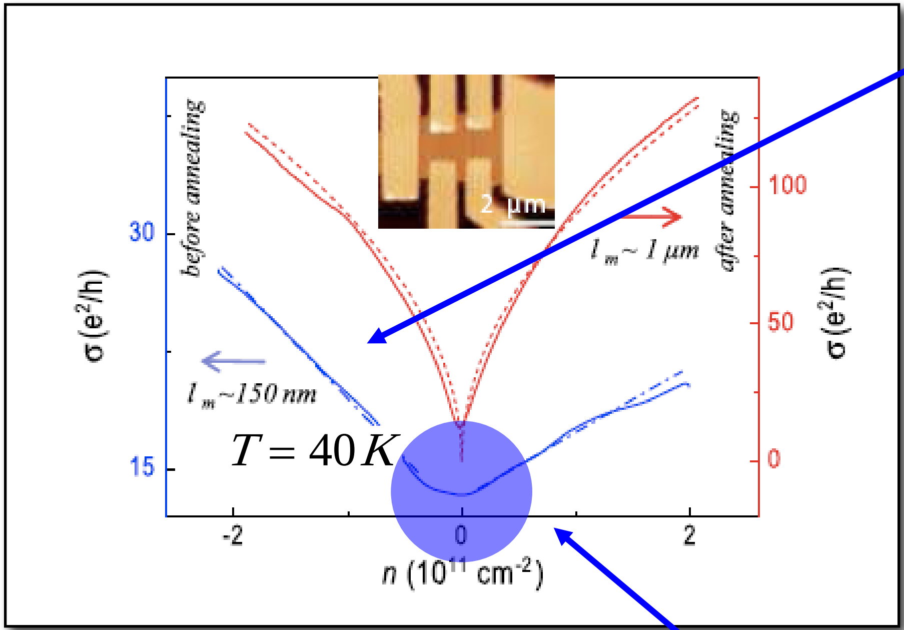
$$\lambda_{app}(E_F) = \frac{G_S(V_g)/(2q^2/h)}{2\sqrt{n_S(V_g)/\pi}}$$

$$\frac{1}{\lambda_{app}(E_F)} = \frac{1}{\lambda(E_F)} + \frac{1}{L}$$



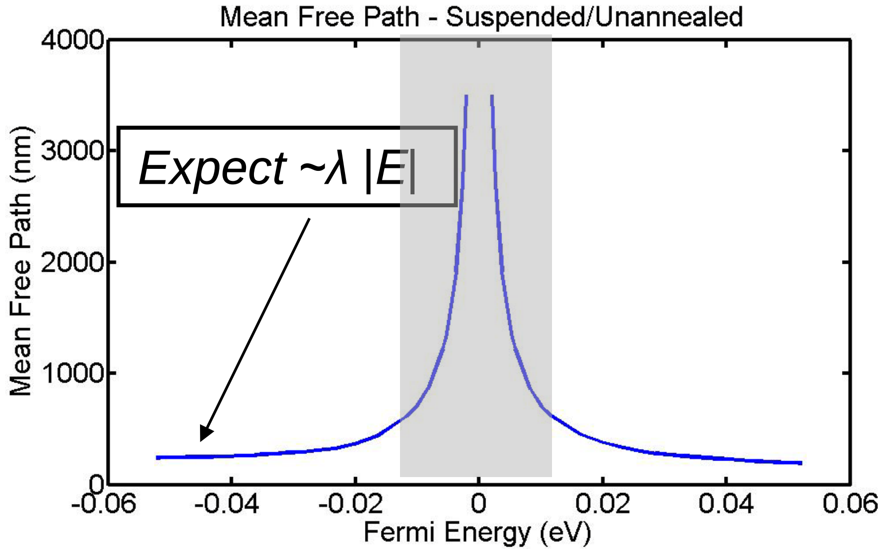
(data from: K. Bolotin, et al., *Phys. Rev. Lett*. **101**, 096802, 2008)

suspended, unannealed



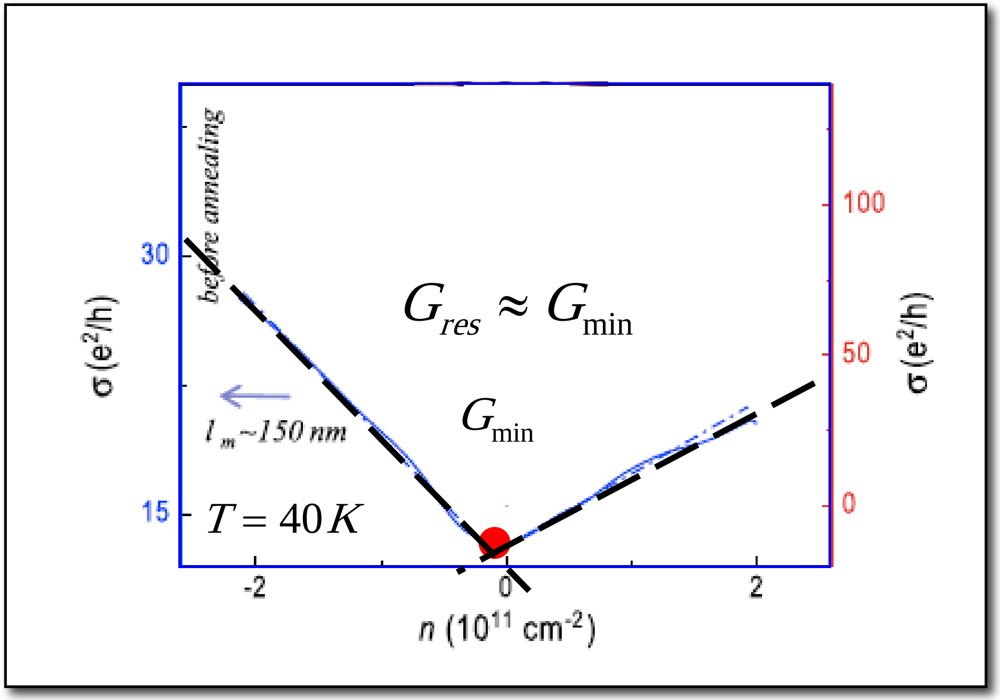
K. Bolotin, et al., *PRL* **101**, 096802 (2008)

linear G_S vs. n suggests charged impurity scattering.



analysis complicated by large residual resistance.

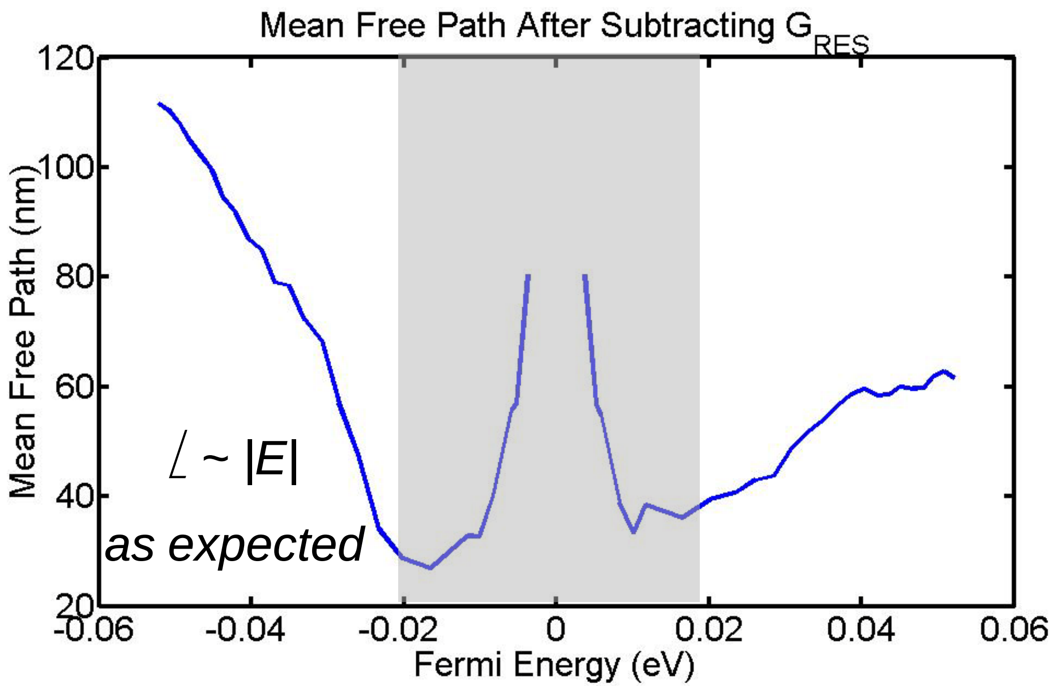
minimum and residual conductance



K. Bolotin, et al., *PRL* **101**, 096802 (2008)

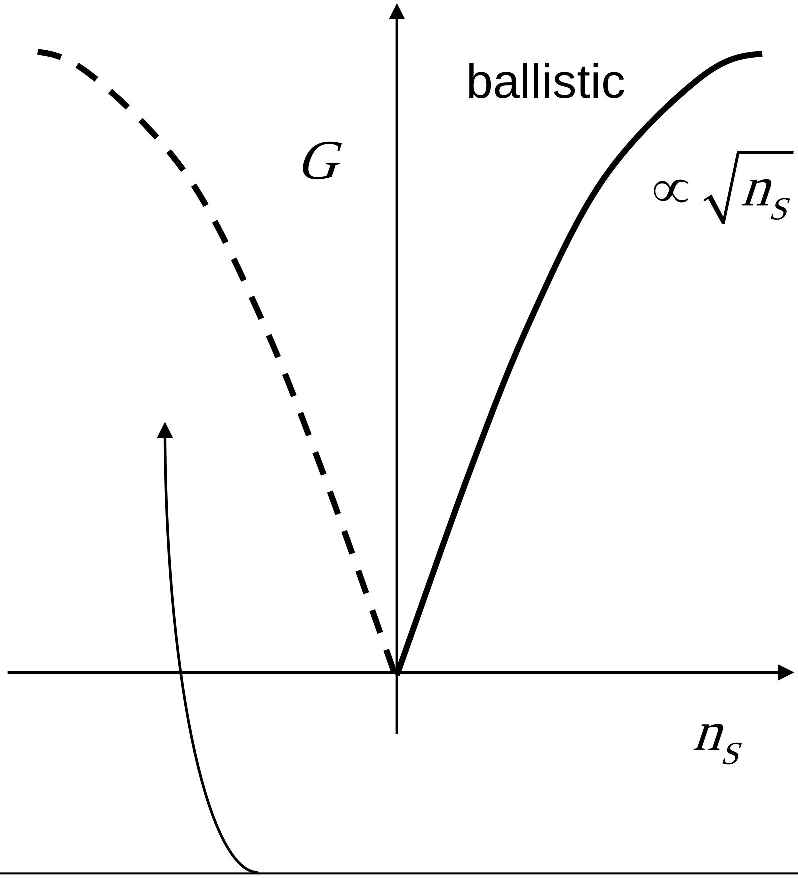
$$G_{res} \approx 14\,\frac{q^2}{h} \qquad G(n_S) = G_{res} + (q\mu_1)\,n_S$$

suspended, unannealed



$$\lambda_{app}\left(E_F\right)=\frac{\left[G_S\left(V_g\right)-G_{res}\right]\Big/\left(2q^2/h\right)}{2\sqrt{n_S\left(V_g\right)\big/\pi}}$$

general picture of G_S vs. n_S (ballistic)

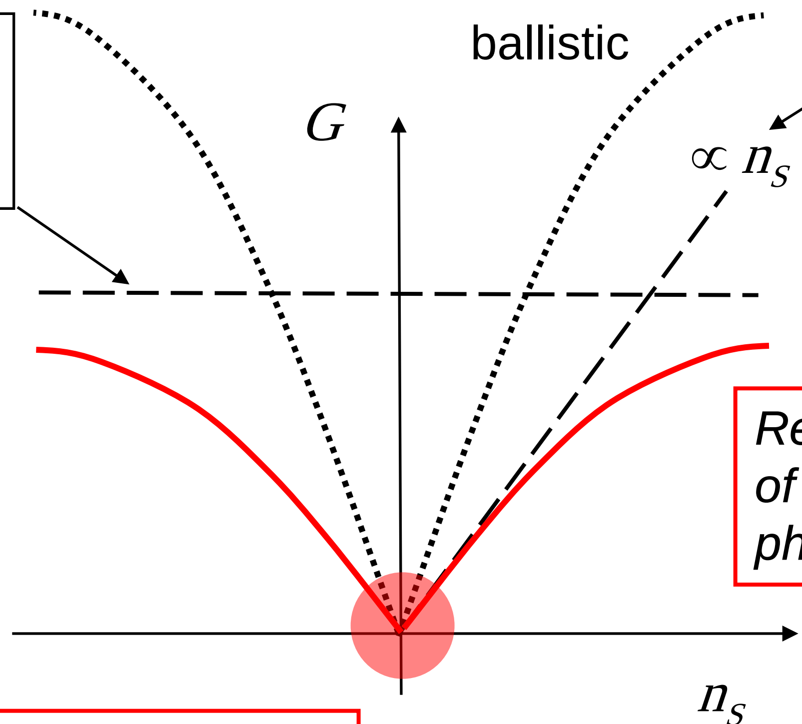


If the mfp is small and constant, then G is also proportional to sqrt(n_S*), but the magnitude is less than the ballistic limit.*

We have discussed V_g (n_S) > 0, but by symmetry, the same thing *should* occur for p-type graphene (E_F < 0).

general picture of G_S vs. n_S (diffusive)

Short range or acoustic phonon scattering.



Charged-impurity (long-range) scattering.

Constant mobility.

Result is a combination of charged impurity and phonon scattering.

Non-zero residual resistance commonly observed.

outline

1) Introduction and Objectives

2) Theory

3) Experimental approach

4) Results

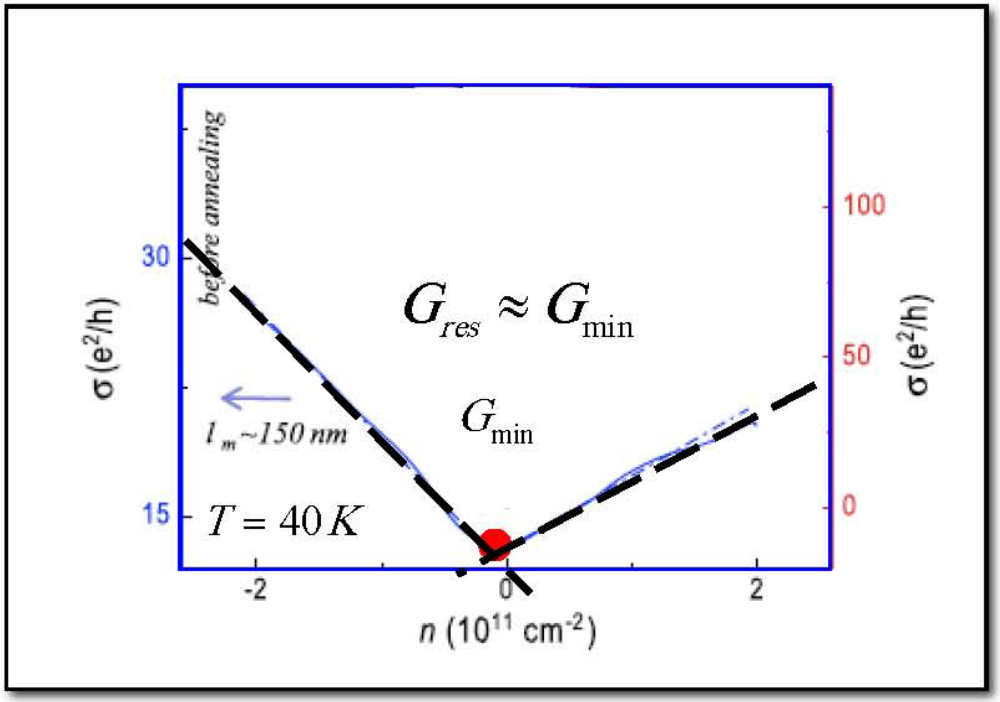
5) Discussion

6) Summary

summary

- The general features of the graphene conductance vs. gate voltage are readily understood (but still being discussed).
- Data can be analyzed by extracting the mean-free-path for backscattering and relating it to the underlying scattering mechanisms.
- More sophisticated theoretical treatments include screening, remote, polar phonons, etc.
- Actual experiments are frequently non-ideal (e.g. not symmetrical about V_{NP}, non monotonic behavior, variations due to sample state, uncertainties in W and L, etc.
- But the material presented here gives a general framework and starting point for analyzing experimental data.

minimum and residual conductance



K. Bolotin, et al., *PRL* **101**, 096802 (2008)

$$G_{res} \approx 14\frac{q^2}{h} \qquad G(n_S) = G_{res} + (q\mu_1)n_S$$

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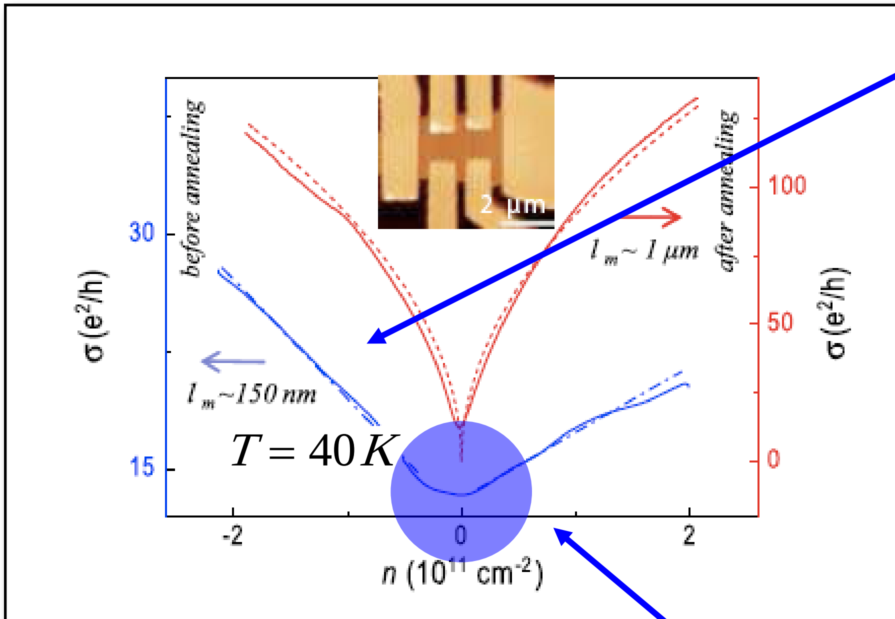

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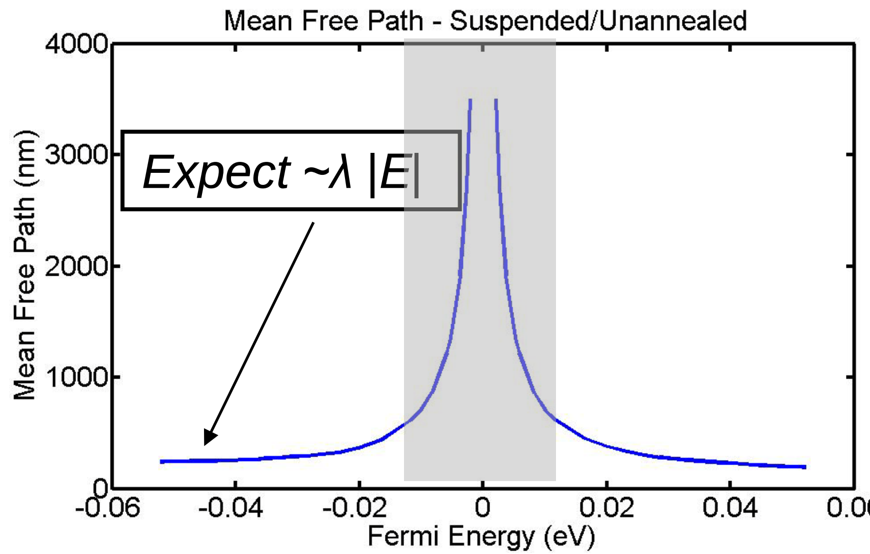

analysis complicated by large residual resistance.