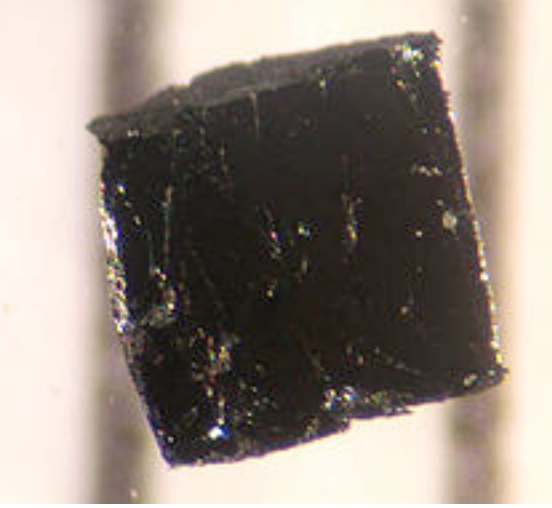
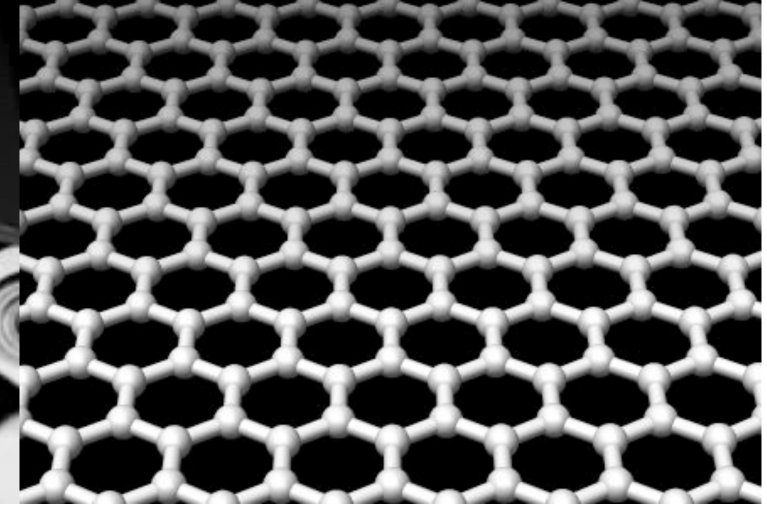
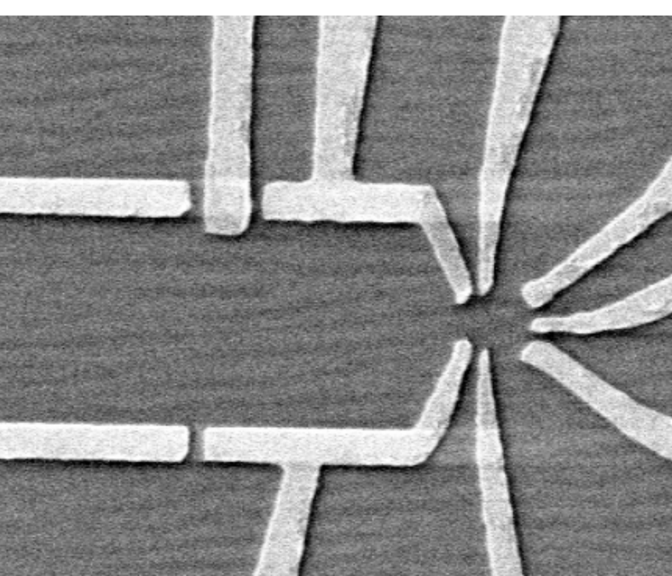




Modeling in Physics

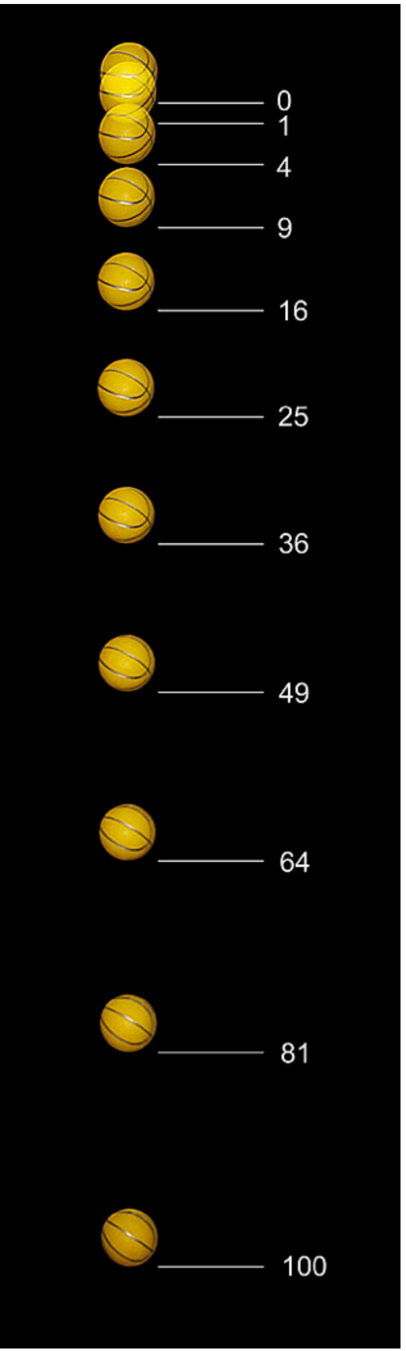
Bruno Uchoa
Department of Physics and Astronomy
University of Oklahoma

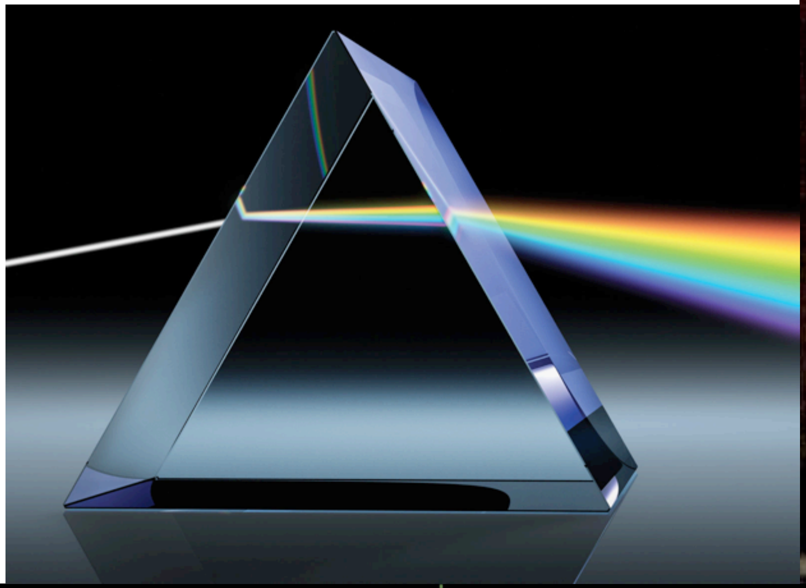


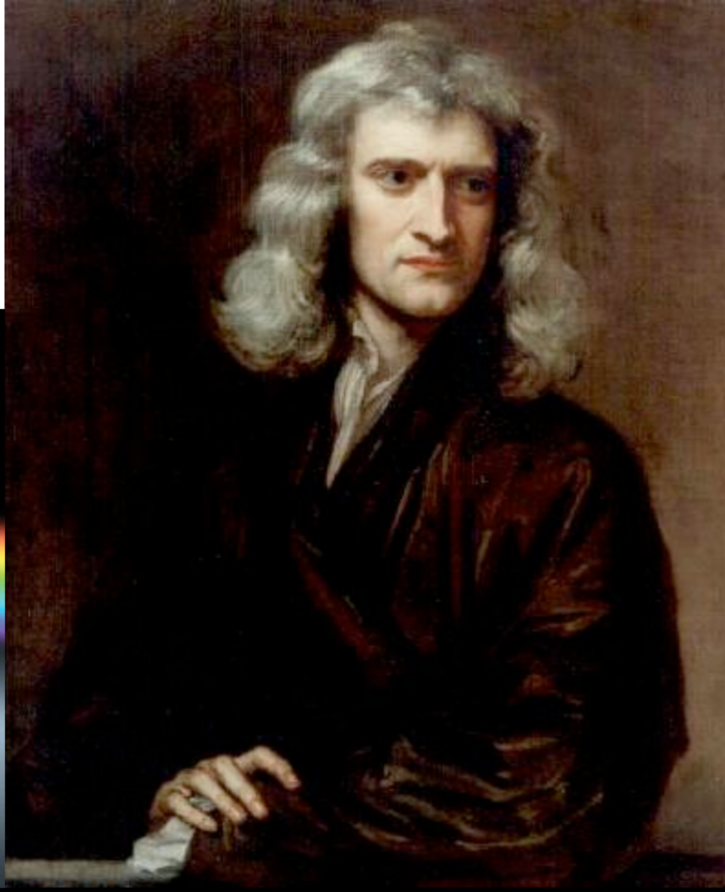

Lecture 2

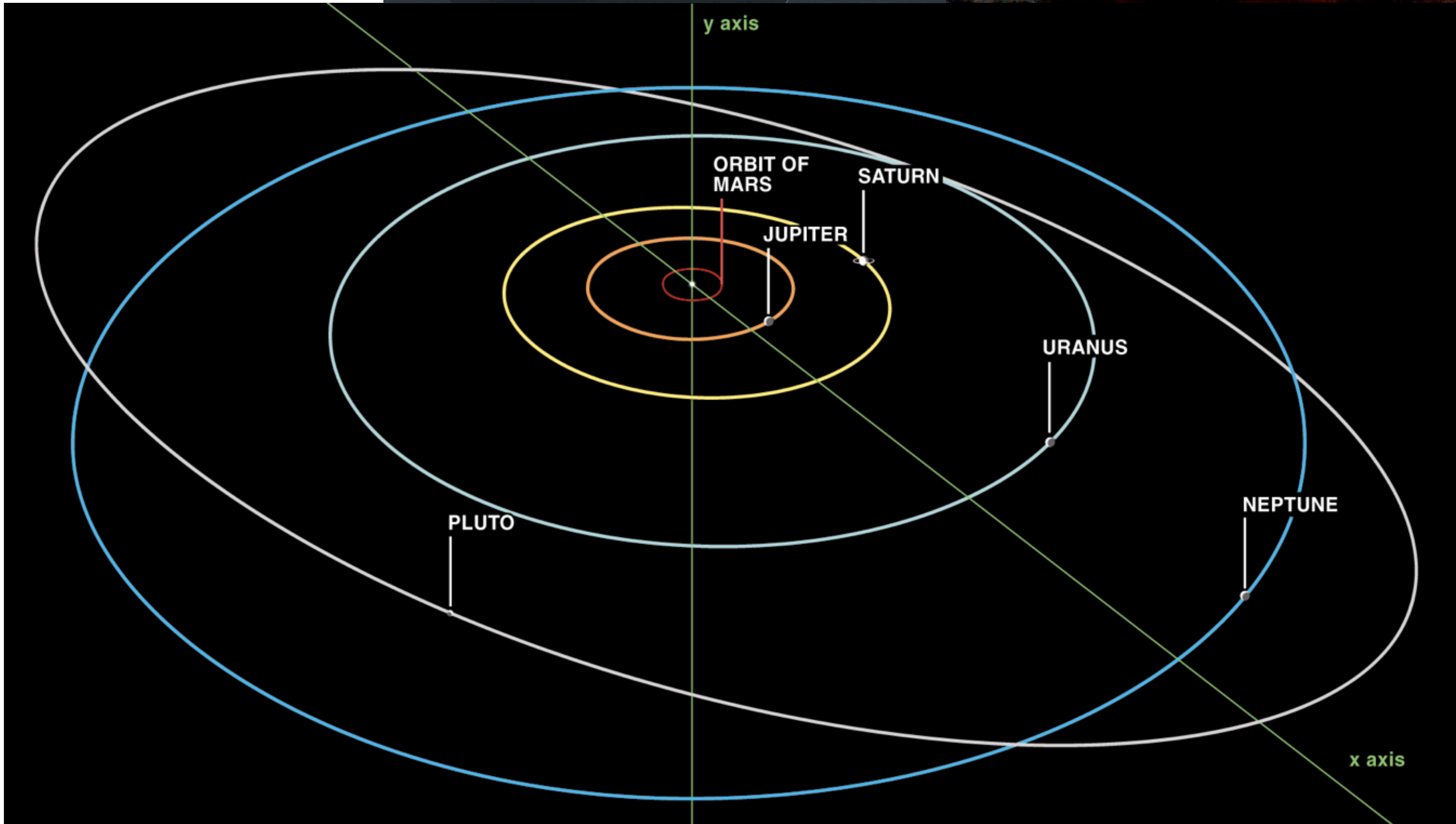
The interpretation of quantum mechanics

For centuries the world was believed to be fully explained by Classical laws



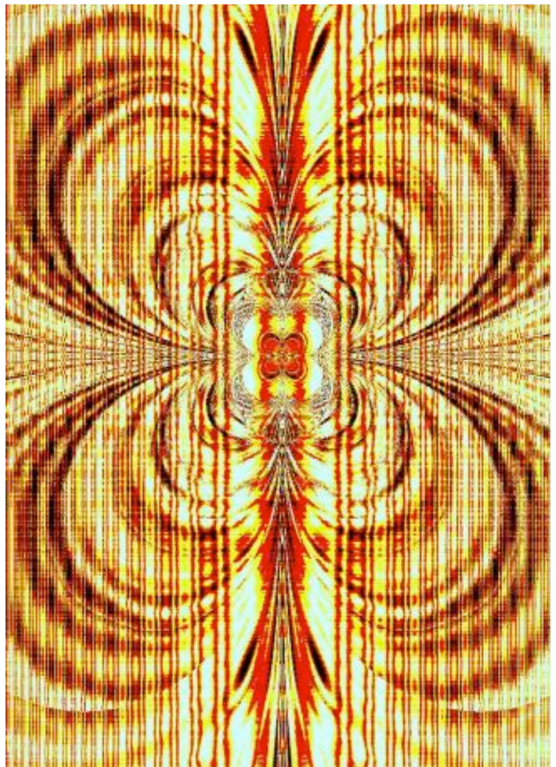






Electromagnetism

(J. C. Maxwell)

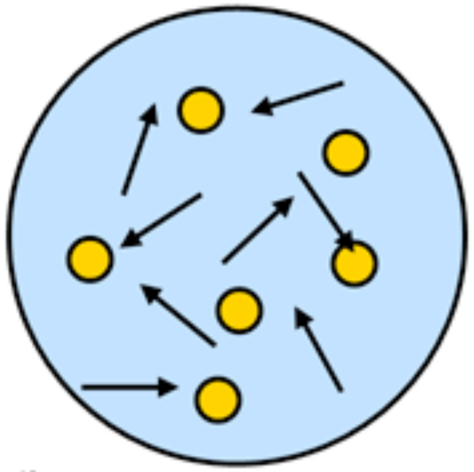


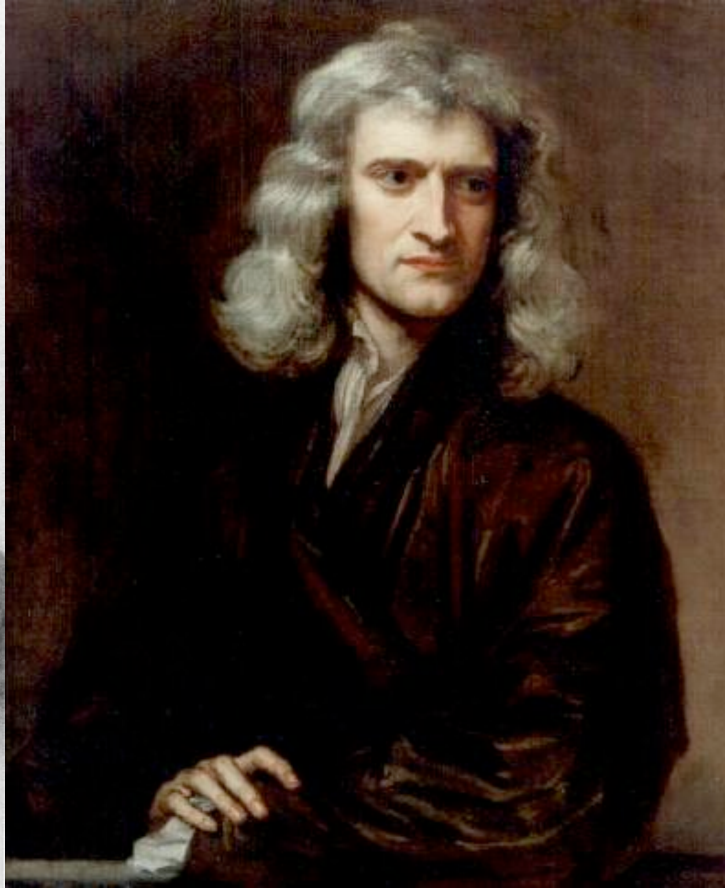
Undulatory physics

(C. Huygens)

Thermodynamics

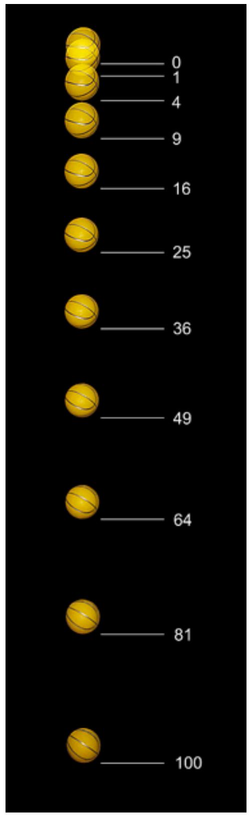
(L. Boltzmann)





Gravitation

(I. Newton)



Electromagnetism

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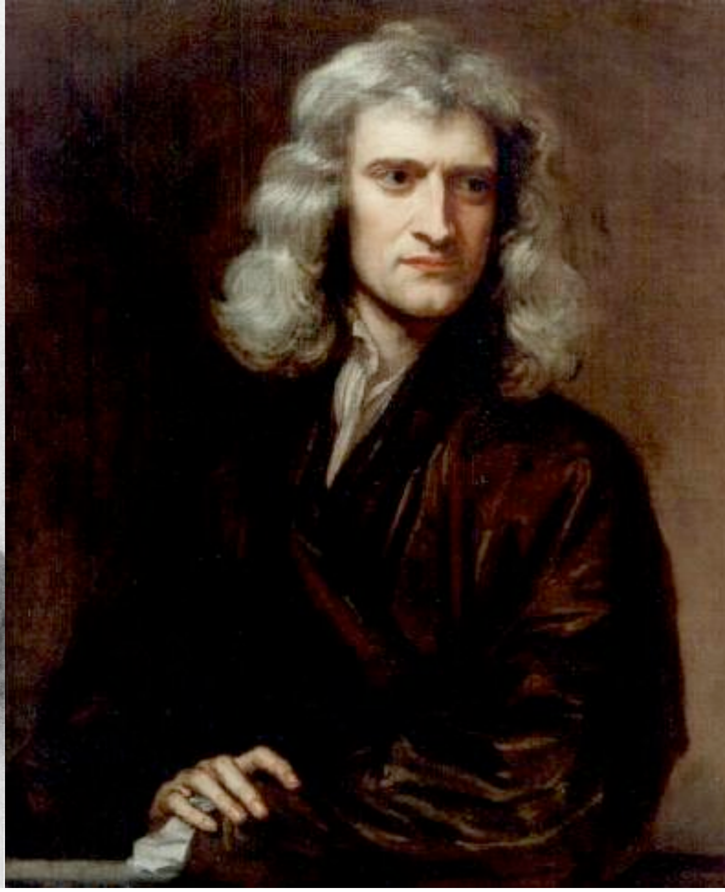

Gravitation

(I. Newton)

By the end of the nineteenth century, all the main fields of physics seemed completely understood.

In 1874, Max Plank decided to get a PhD in physics.
His advisor told him that he should pick a different field because there was not much left to explain.

He replied that he did not wish to discover new things, but only to understand the known fundamentals of the field.

In 1874, Max Plank decided to get a PhD in physics.
His advisor told him that he should pick a different field because there was not much left to explain.

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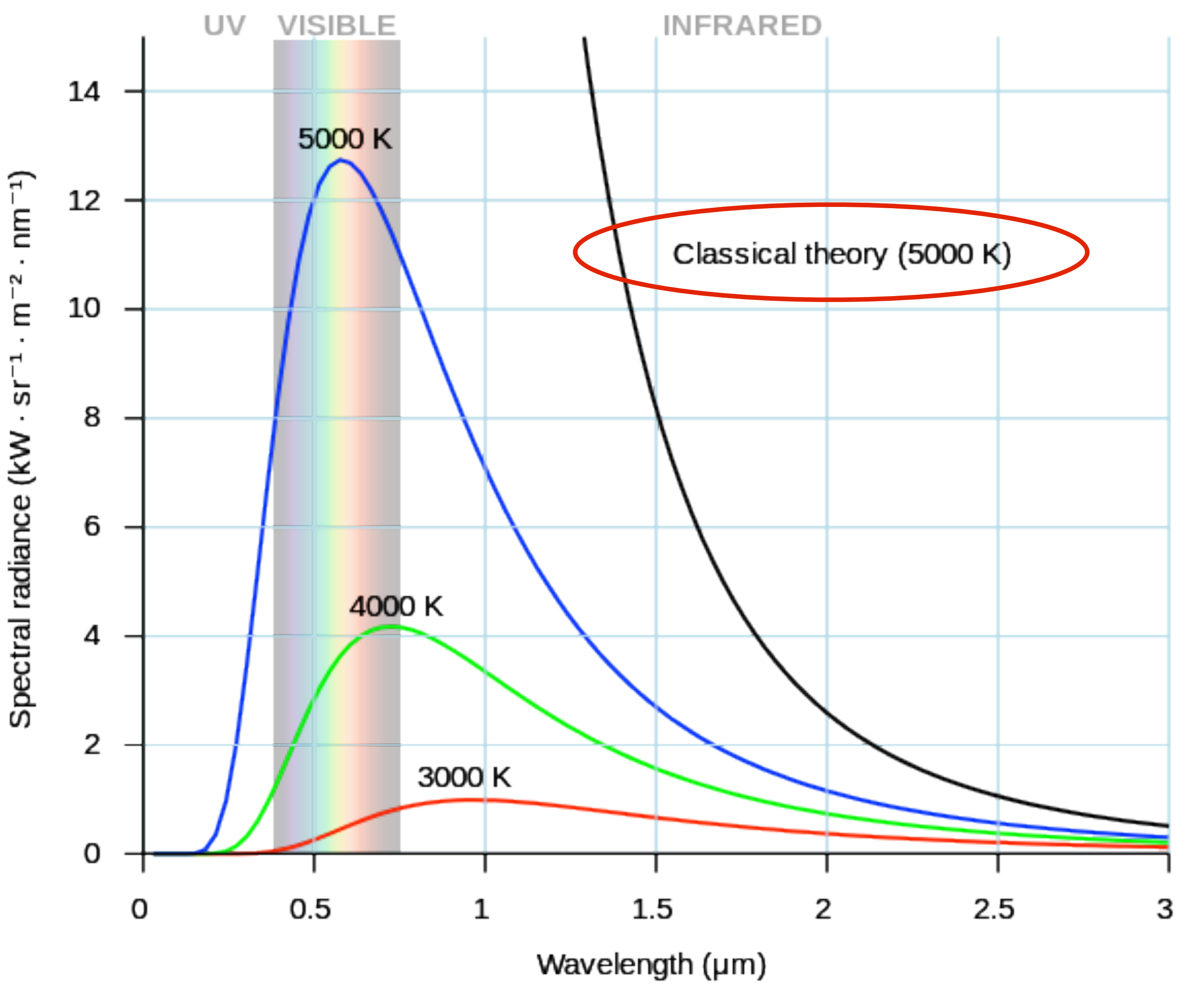
The Nobel Prize in Physics 1918
Max Karl Ernst Ludwig Planck

The Nobel Prize in Physics 1918 was awarded to Max Planck *"in recognition of the services he rendered to the advancement of Physics by his discovery of energy quanta"*.

Ultraviolet catastrophe



Hypothesis: quantization of the photon energy

$$E = \hbar\nu$$

Planck's constant

The energy is quantized in small unit packages called quanta. The energy of each package is proportional to the frequency of the radiation

People like Lorentz and others did not believe it, and set Planck's constant to zero as in classical theory.

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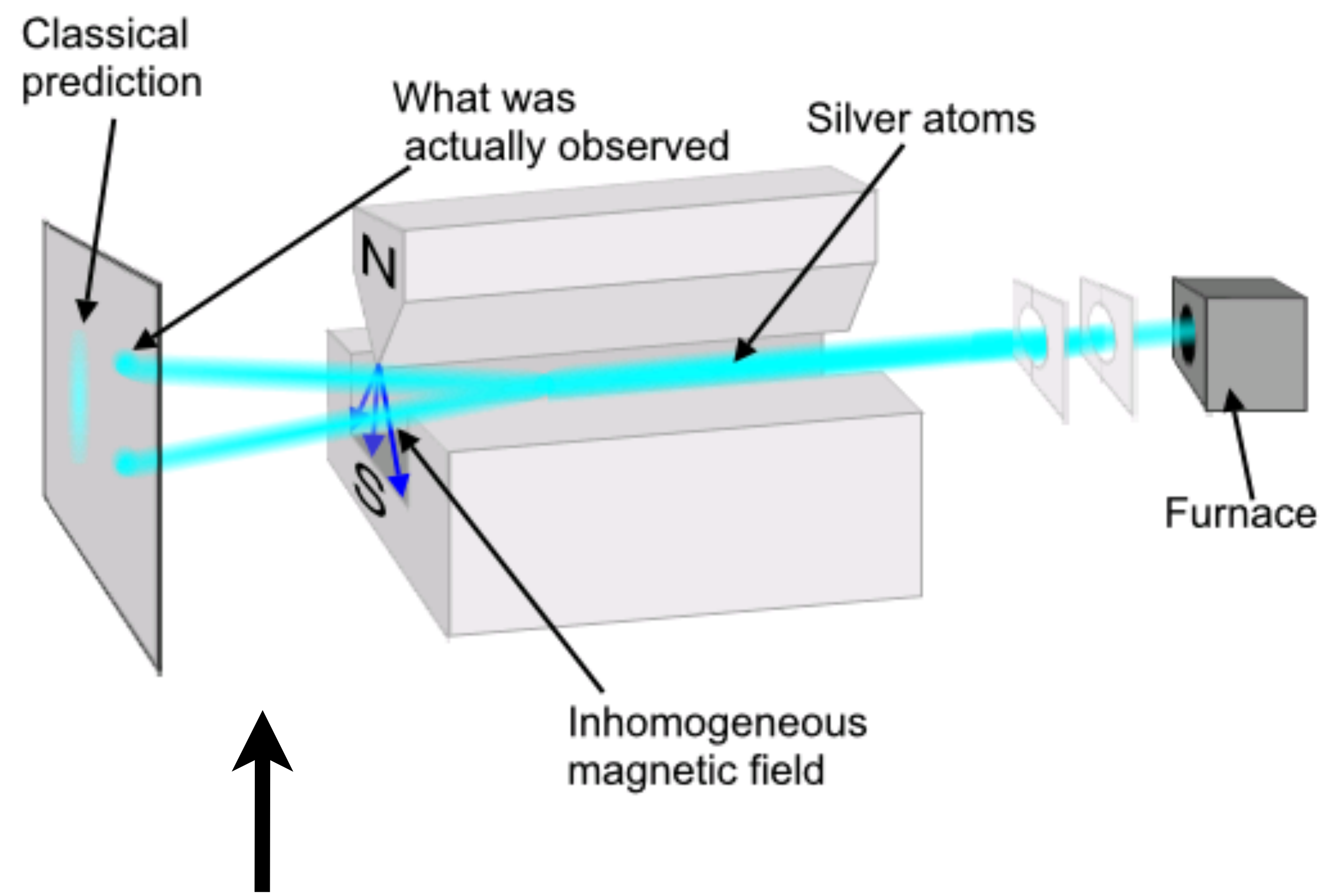
But Einstein understood that Planck's hypothesis explained the photoelectric effect.

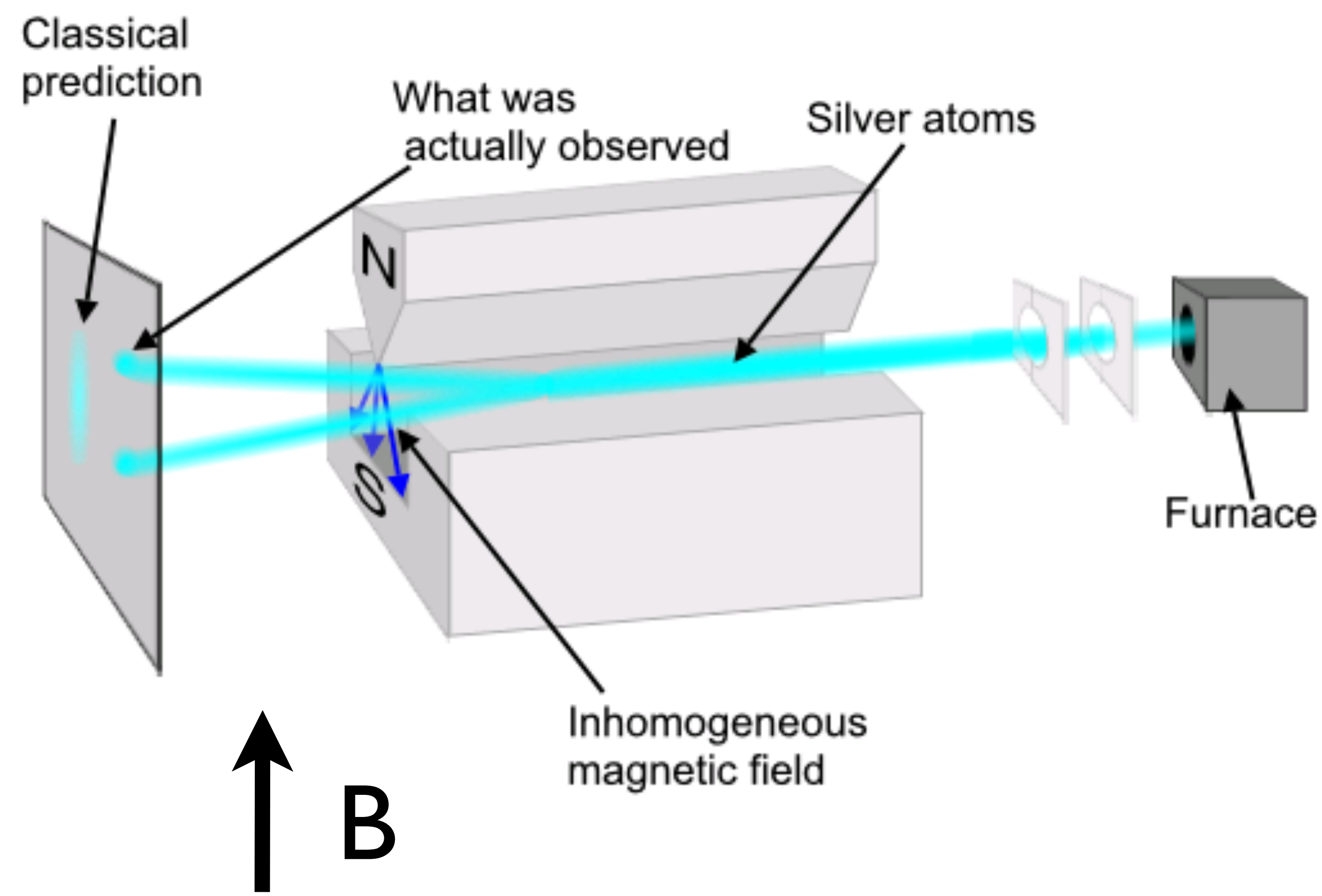
The Nobel Prize in Physics 1921
Albert Einstein

The Nobel Prize in Physics 1921 was awarded to Albert Einstein *"for his services to Theoretical Physics, and especially for his discovery of the law of the photoelectric effect".*

Stern Gerlatch experiment (1922)



In 1925, R. Kronig suggested that this two component degree of freedom could be the spin of the electron!



Magnetic moment

$$\boxed{\vec{\mu}_e \propto \mathbf{S}}$$

Force:

$$\mathbf{F} \propto \frac{\partial}{\partial z}\left(\vec{\mu}_e \cdot \mathbf{B}\right) \approx \vec{\mu}_e \cdot \frac{\partial \mathbf{B}}{\partial z}$$

In 1925, R. Kronig suggested that this two component degree of freedom could be the spin of the electron!

W. Pauli strongly criticized the idea:
the velocity at the surface of a sphere with the size of an electron would be larger than the speed of light, what violates relativity.

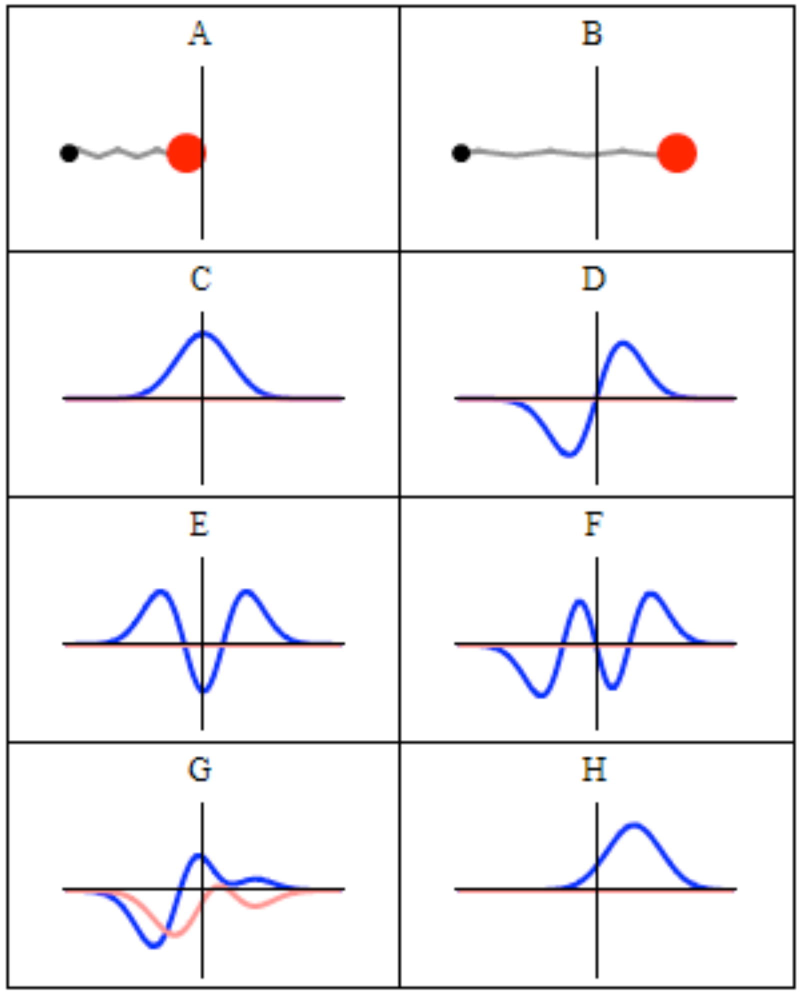
Kronig changed his mind and did not publish the paper.

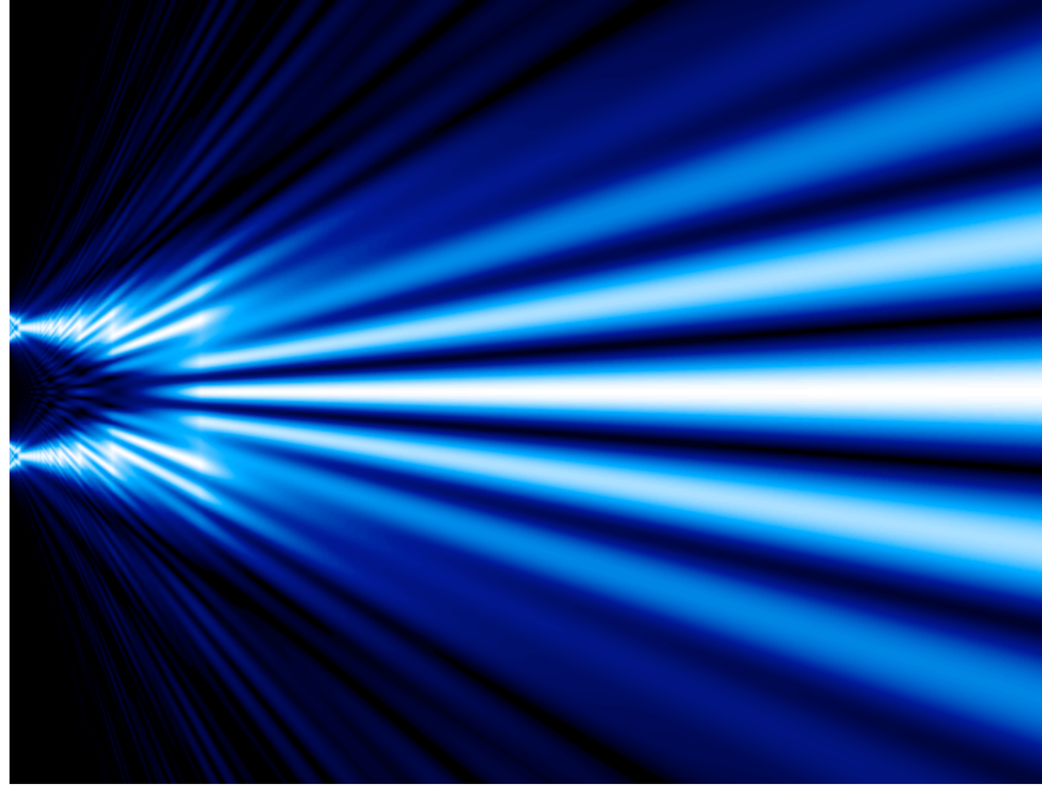
Lesson 1: In science, never trust common wisdom.
If you believe you have a good idea, publish it!

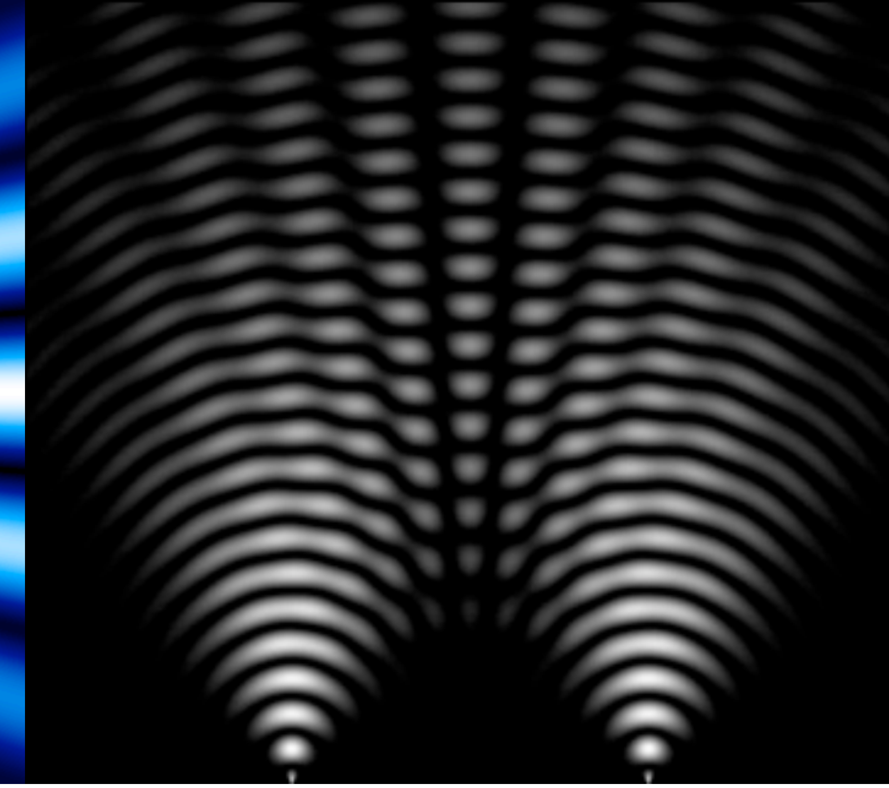
Schrodinger equation

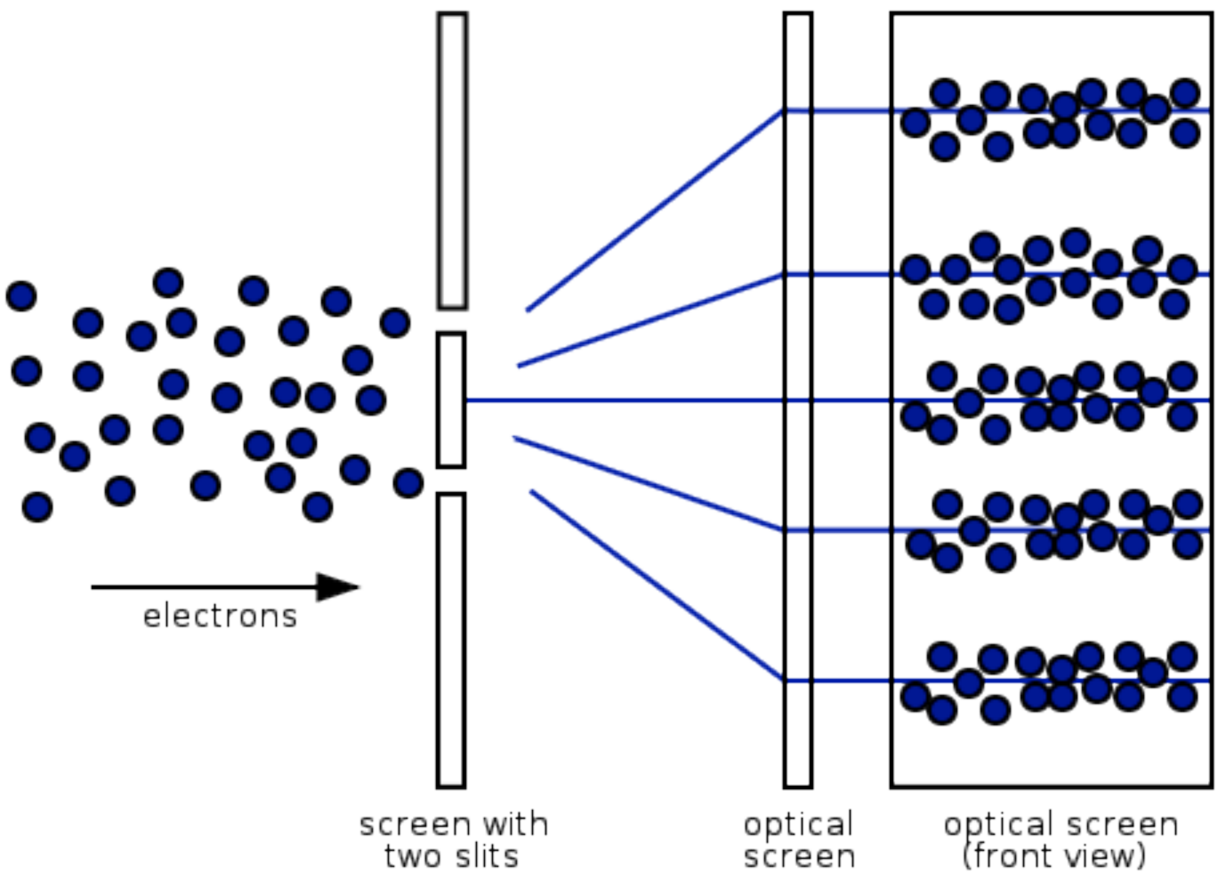
$$i\hbar\frac{\partial}{\partial t}\psi(\vec{r},t) = \left[-\frac{\hbar^2}{2m}\nabla^2 + U(\vec{r},t)\right]\psi(\vec{r},t)$$

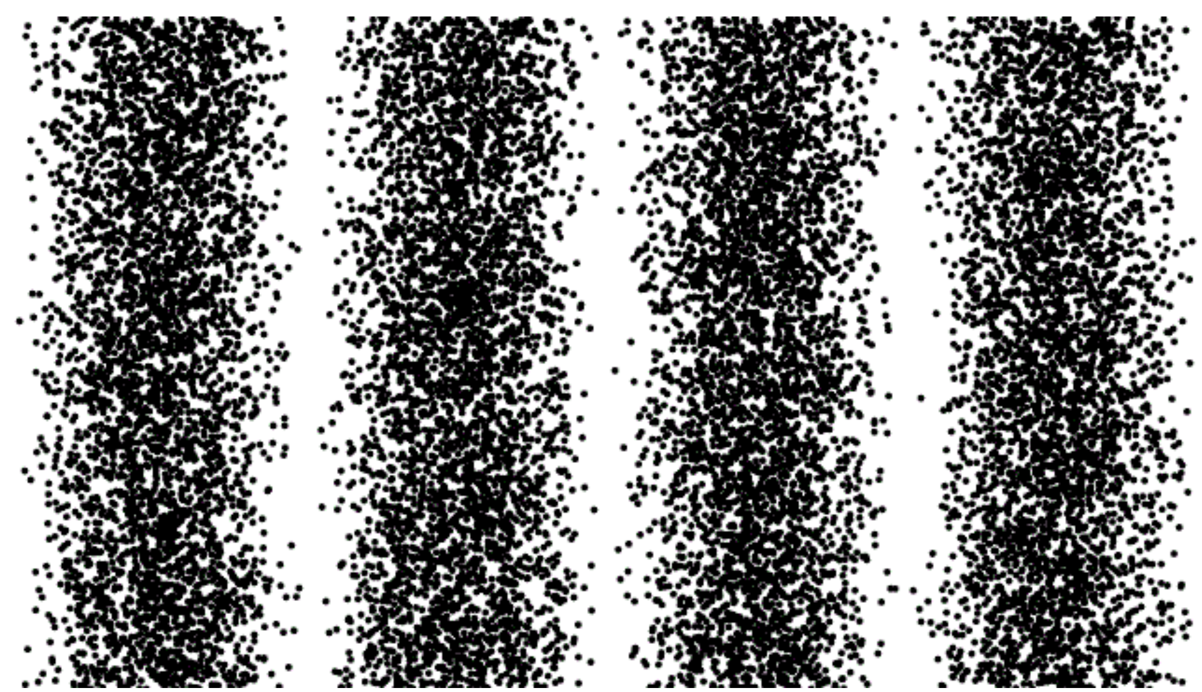
Pauli did not know about the Schrodinger Equation, which was only proposed at that time...



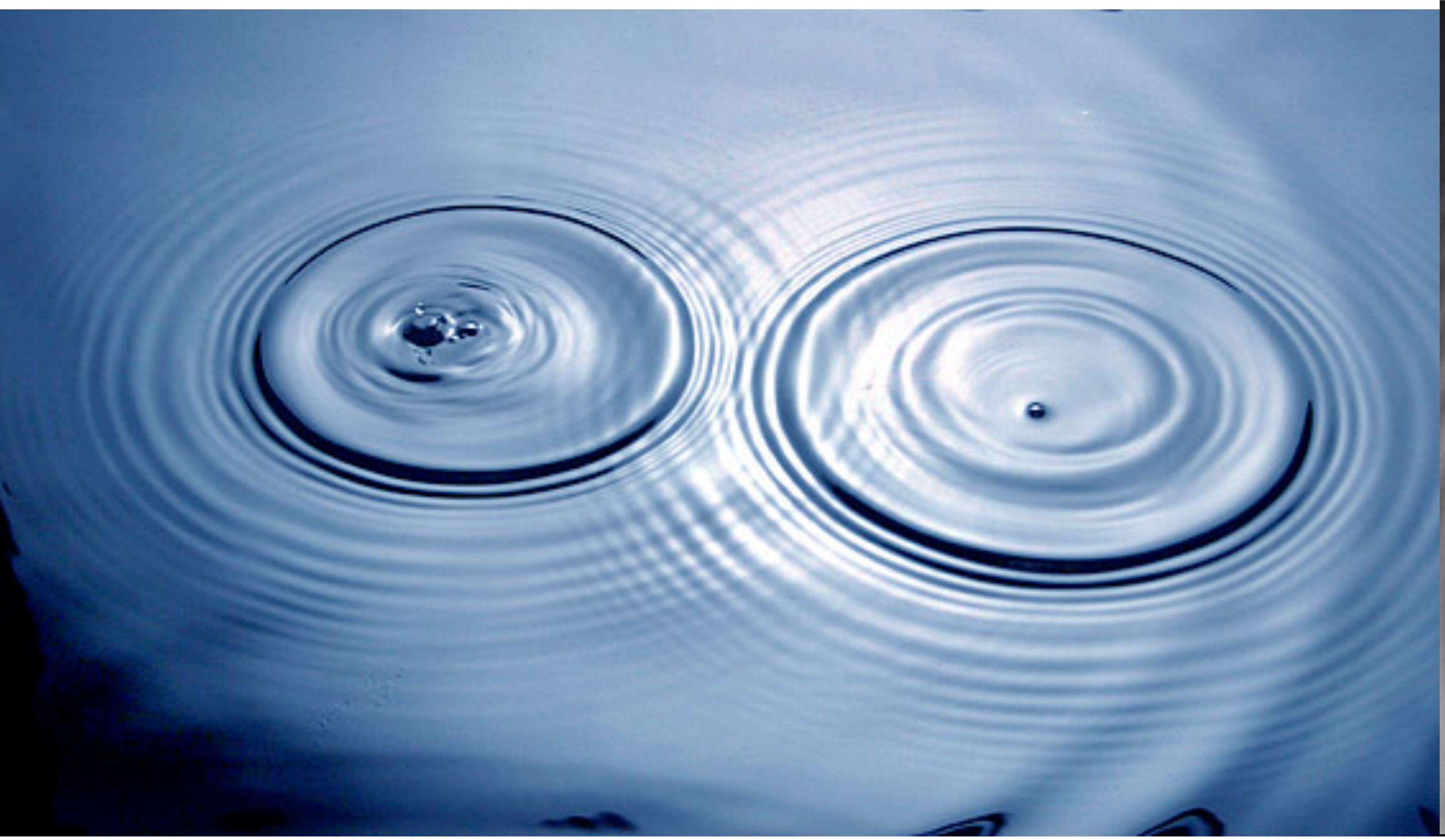


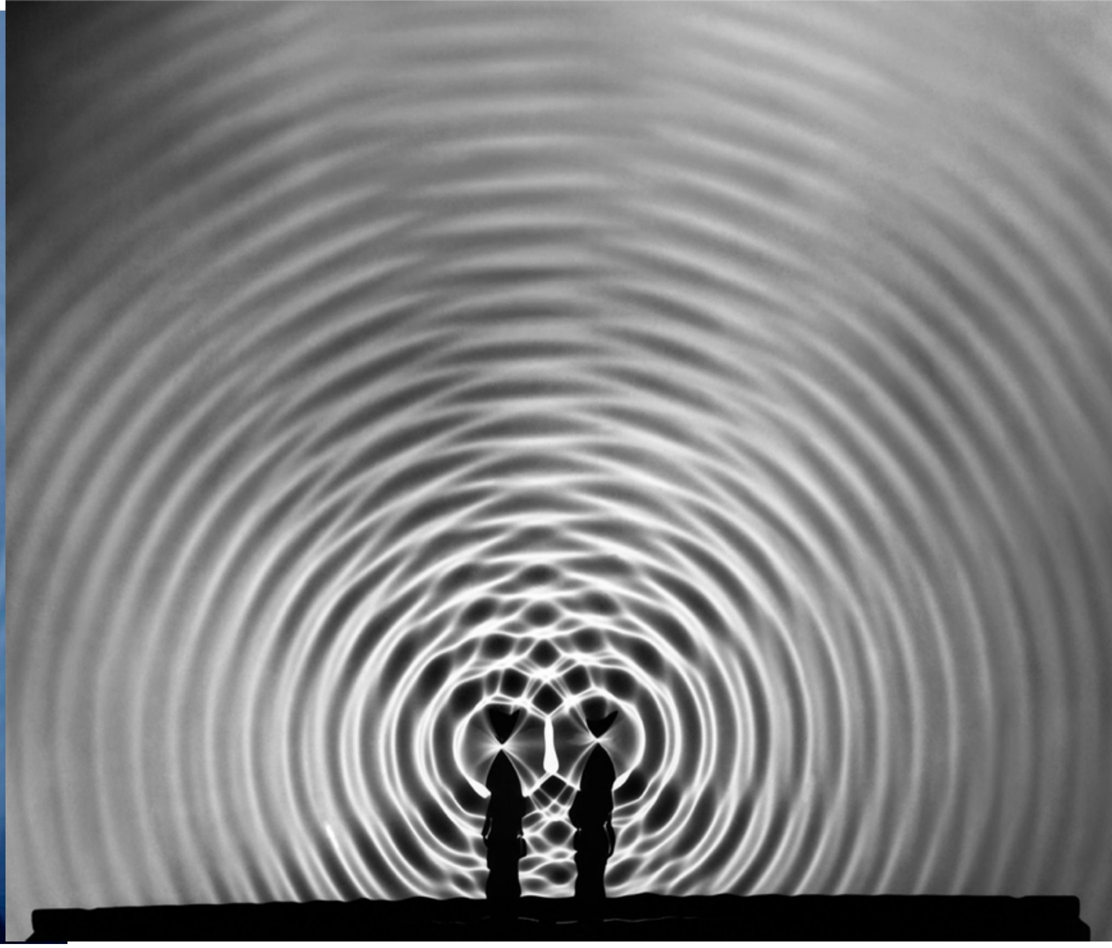


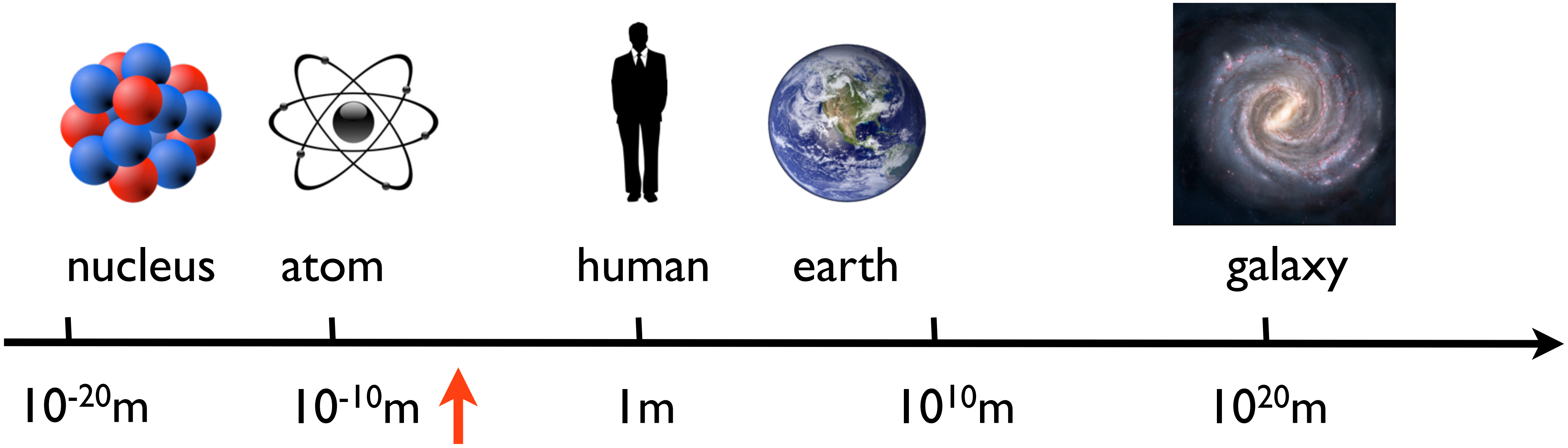




Electrons also behave as waves!







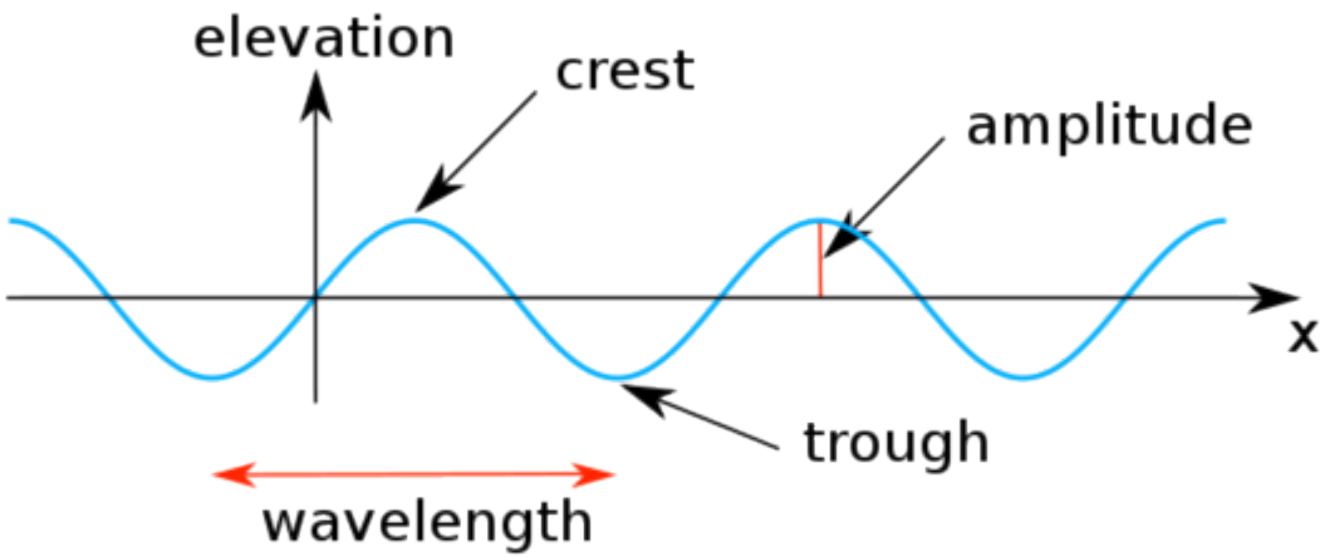
There are short length scales below which the world becomes quantum mechanical!

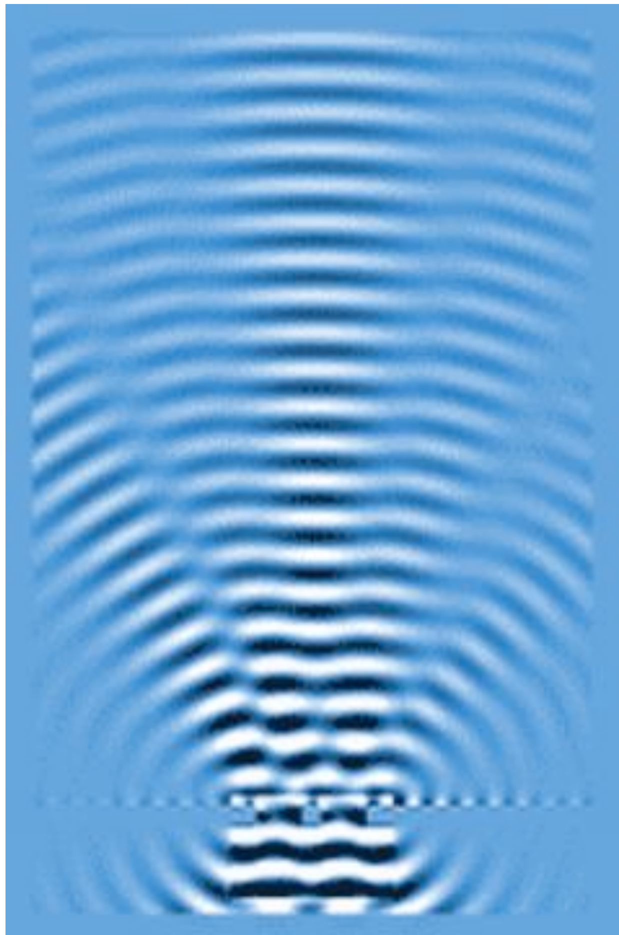
Particle-wave duality

$$\lambda = \frac{h}{p}$$

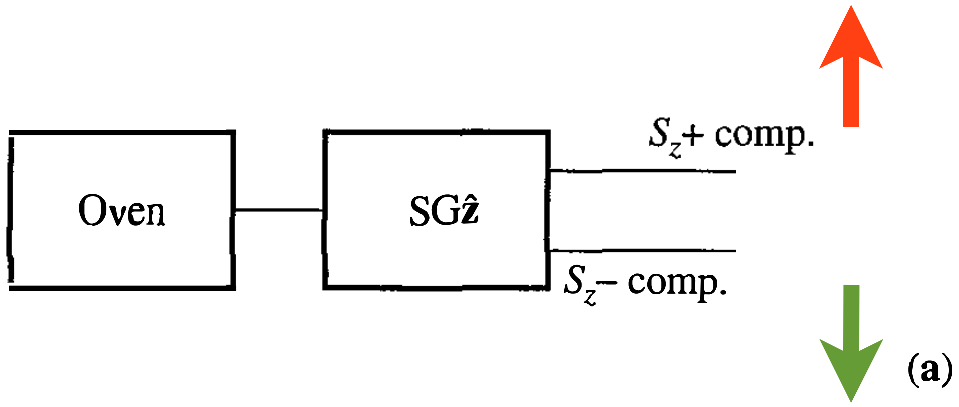
momentum

wavelength

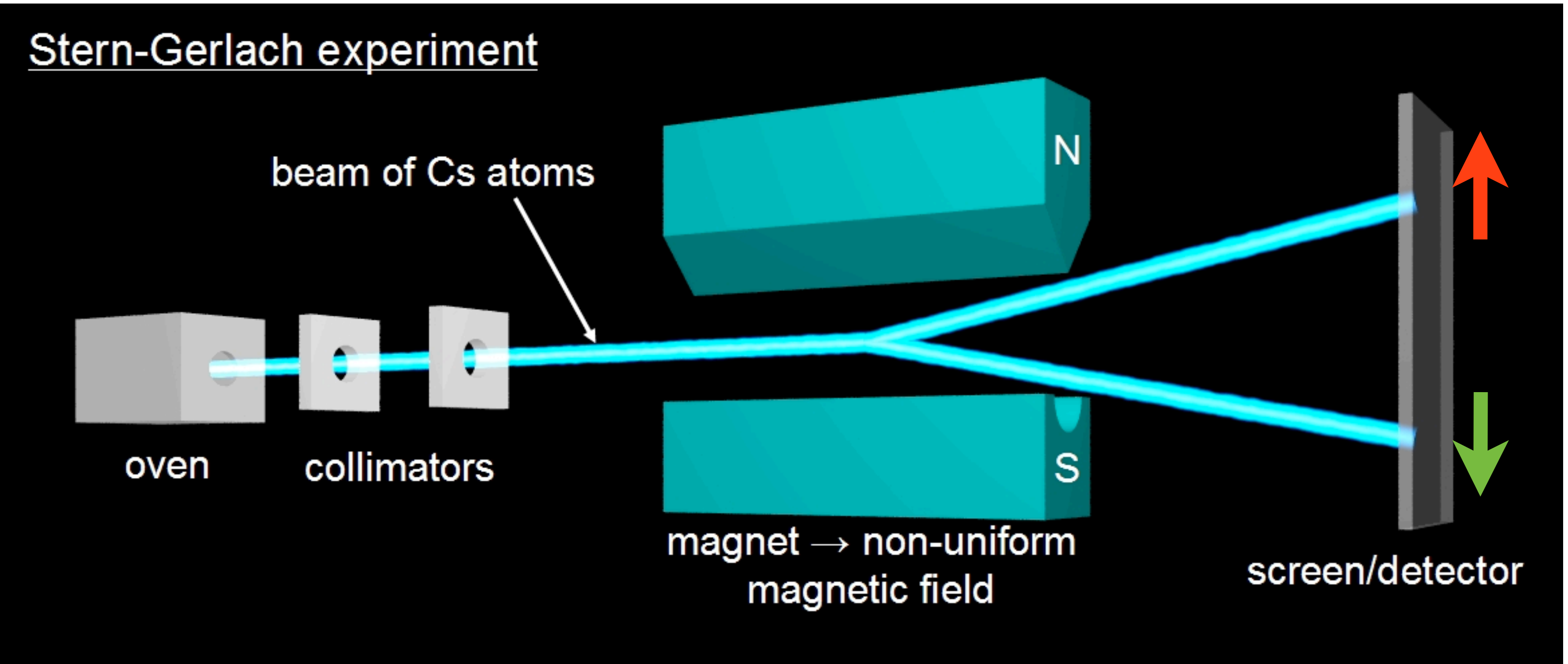




Experiment

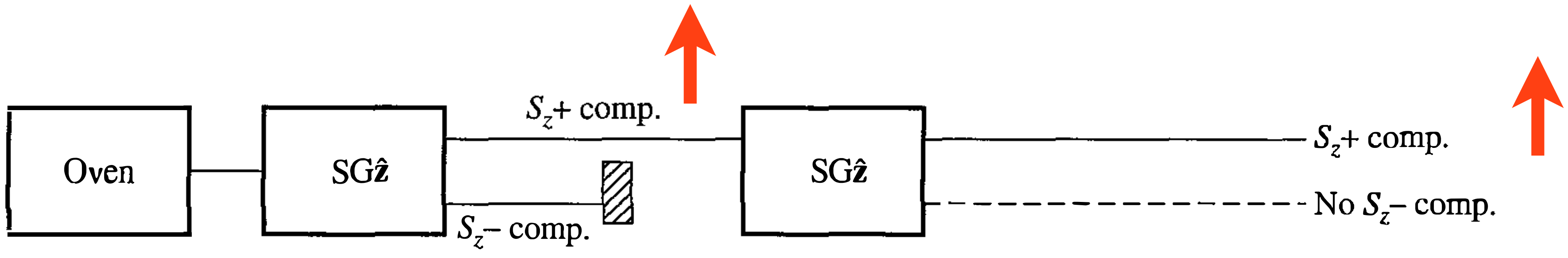


Oven
SGẑ
S_z+ comp.
S_z– comp.
(a)

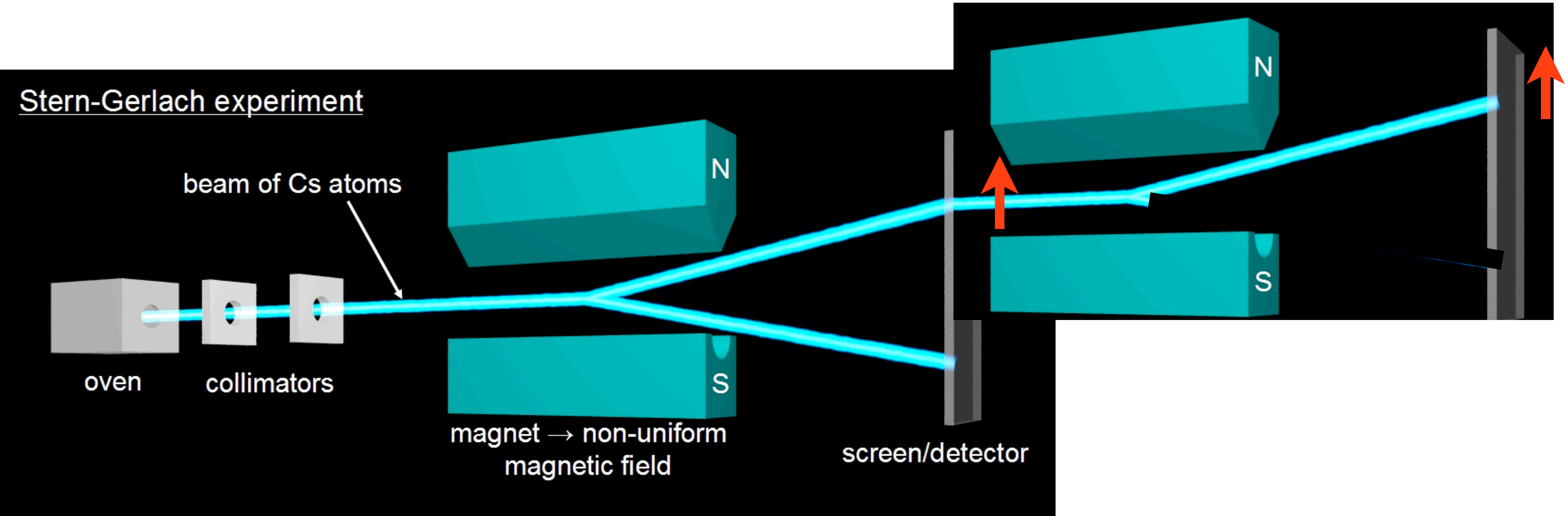


Stern-Gerlach experiment
beam of Cs atoms
N
S
oven
collimators
magnet → non-uniform magnetic field
screen/detector

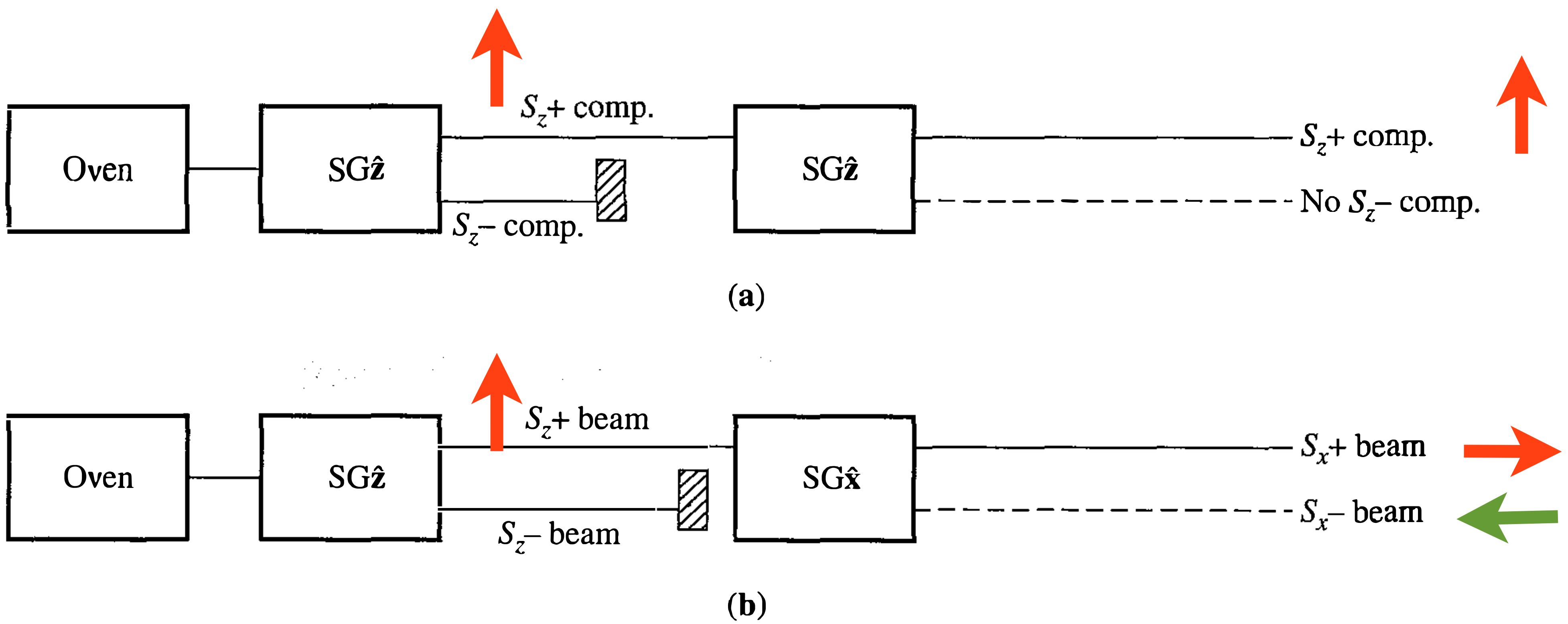
Experiment

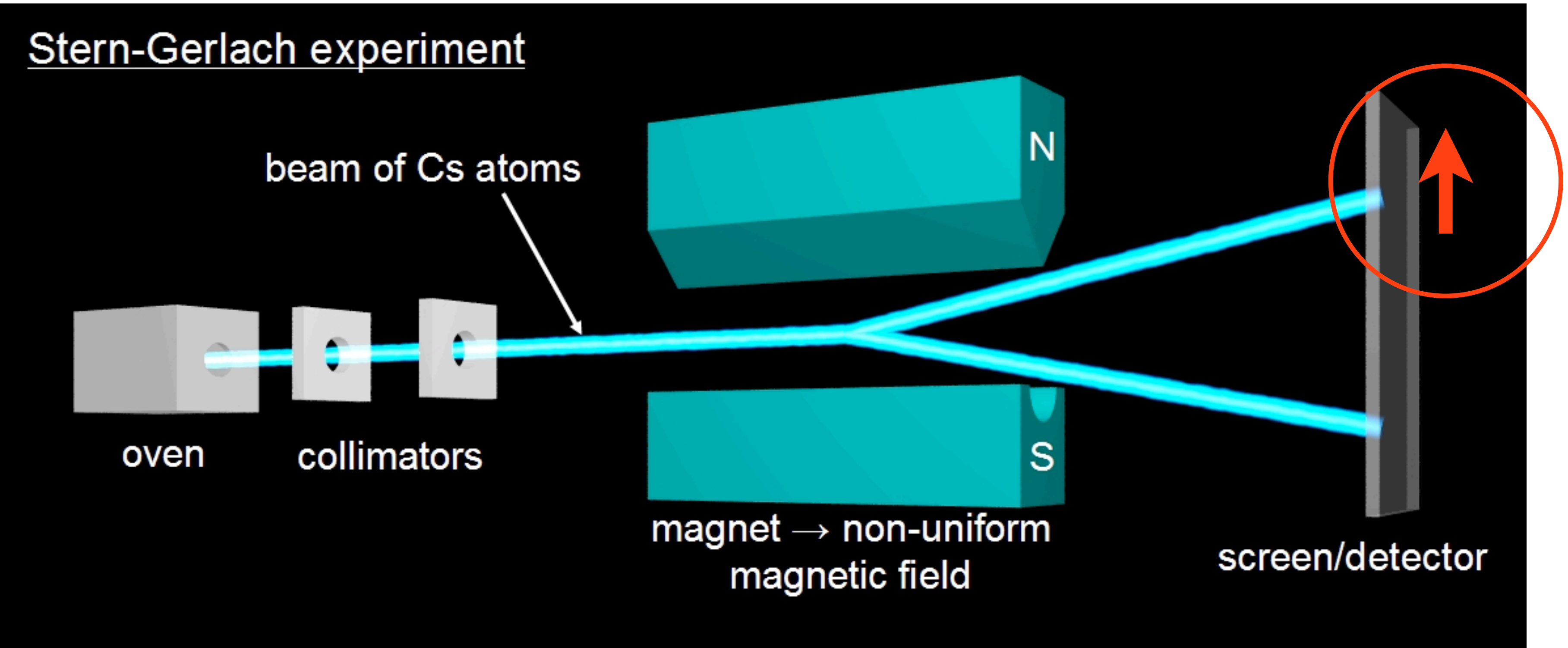


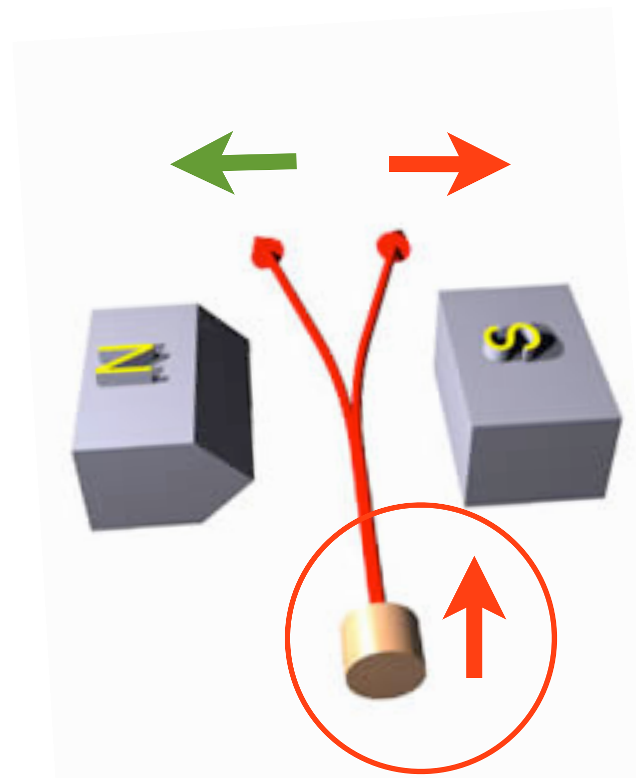
(a)



Experiment

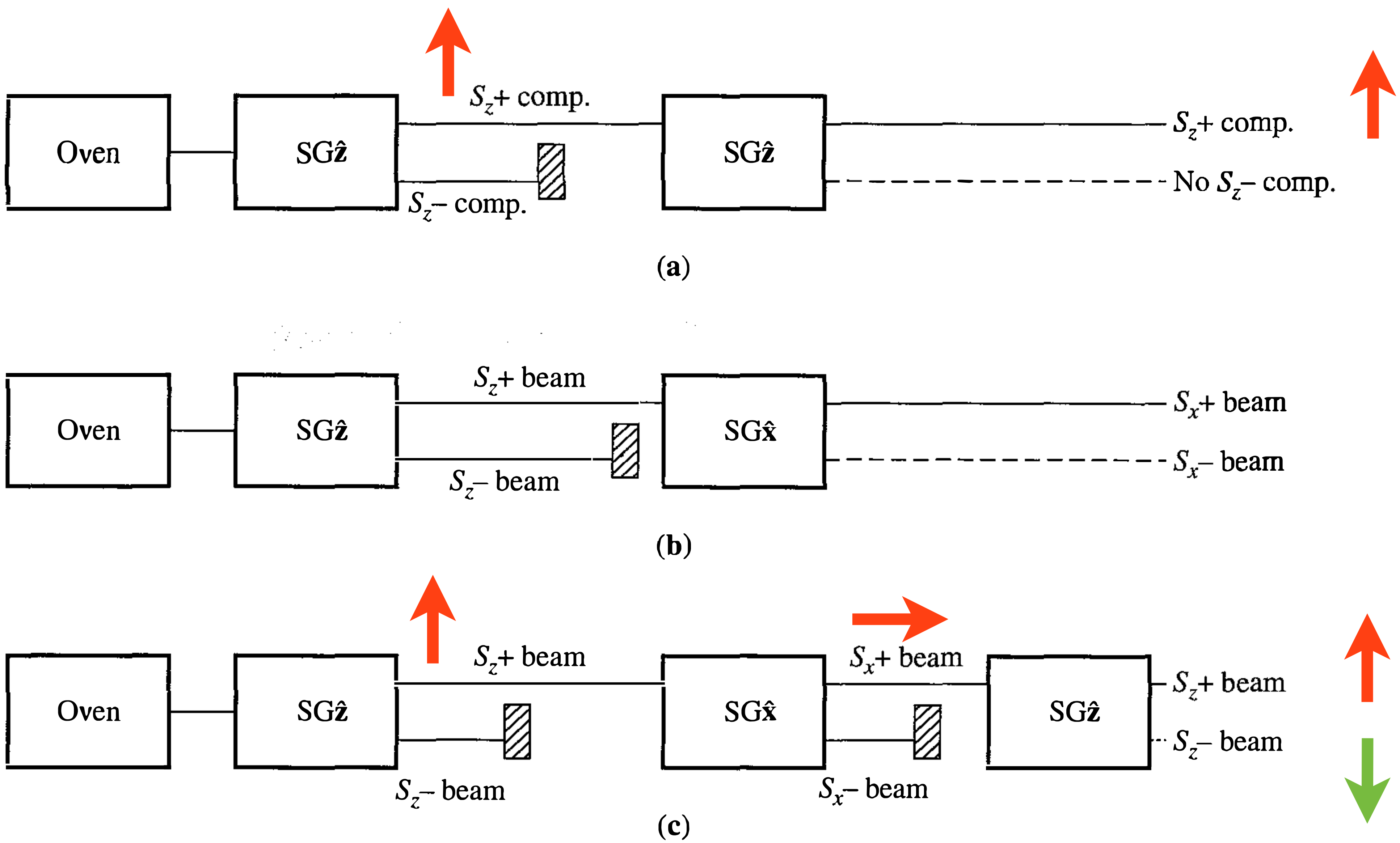






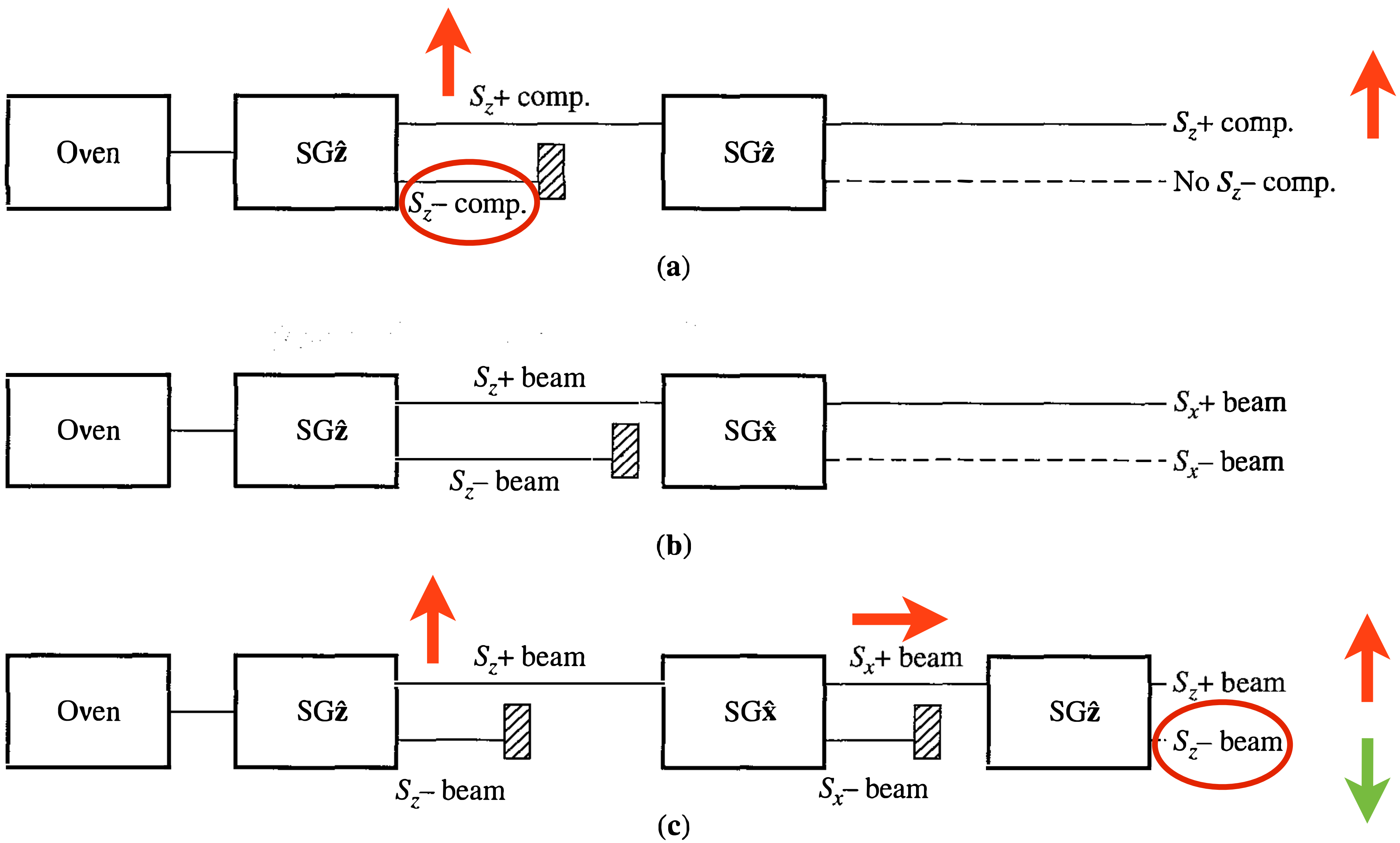
Horizontal apparatus

Experiment



The down component is recovered back!

Experiment



The down component is recovered back!

Pauli's interpretation: The spin...

Spin operators

$$S_z = \frac{\hbar}{2}\left(|+\rangle\langle+| - |-\rangle\langle-|\right)$$

$$S_y = \frac{\hbar}{2} i\left(|-\rangle\langle+| - |+\rangle\langle-|\right)$$

$$S_x = \frac{\hbar}{2}\left(|+\rangle\langle-| + |-\rangle\langle+|\right)$$

Pauli matrices

$$S_x = \frac{\hbar}{2}\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \qquad S_y = \frac{\hbar}{2}\begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix} \qquad S_z = \frac{\hbar}{2}\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

Pauli's interpretation: The spin...

Spin operators

$$S_z = \frac{\hbar}{2}\left(|+\rangle\langle+| - |-\rangle\langle-|\right)$$

Eigenstates:

$|+\rangle$ Up state

$|-\rangle$ Down state

Pauli matrices

$$S_x = \frac{\hbar}{2}\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \qquad S_y = \frac{\hbar}{2}\begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix} \qquad S_z = \frac{\hbar}{2}\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

Pauli's interpretation: The spin...

The observables of a measurement along the x direction are the two eigenstates of the S_x operator!

$$|+\rangle_x = \frac{1}{\sqrt{2}}\left(|+\rangle + |-\rangle\right)$$

$$|-\rangle_x = \frac{1}{\sqrt{2}}\left(|+\rangle - |-\rangle\right)$$

Pauli matrices

$$S_x = \frac{\hbar}{2}\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \qquad S_y = \frac{\hbar}{2}\begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix} \qquad S_z = \frac{\hbar}{2}\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

Interpretation of the spin

$|+\rangle$ Up state

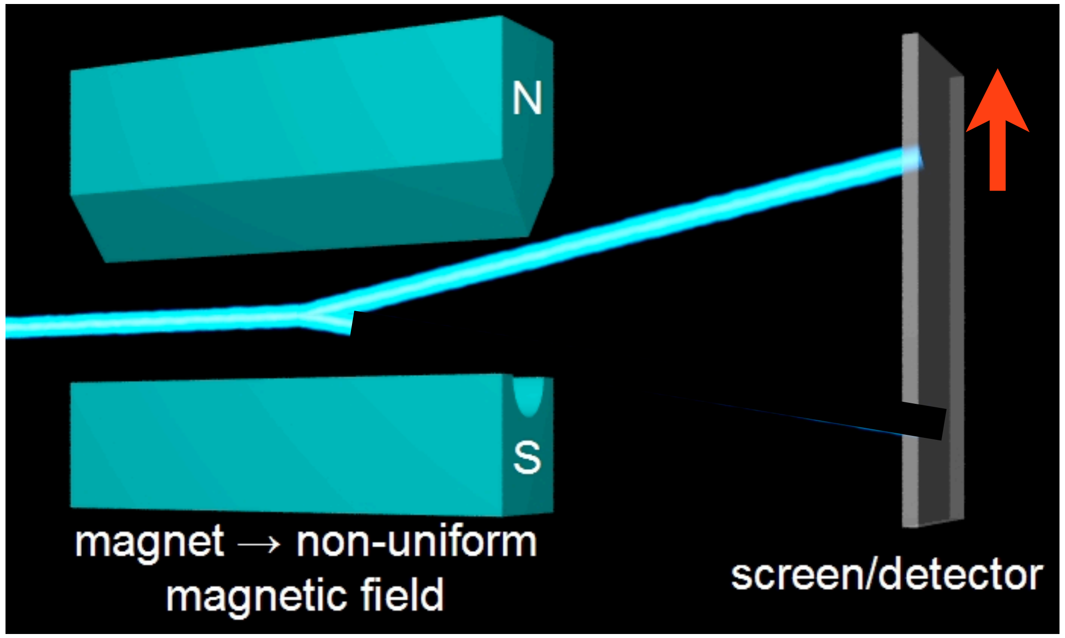
$|-\rangle$ Down state

The spin has an unusual property that the the "up" state of the spin component measured along the vertical direction is a superposition of both "up" and "down" states of the spin component for a measurement along the horizontal (x) direction

$$
\begin{aligned}
|+\rangle &= \frac{1}{\sqrt{2}}(|+\rangle_x + |-\rangle_x) \\
|-\rangle &= \frac{1}{\sqrt{2}}(|+\rangle_x - |-\rangle_x)
\end{aligned}
$$

Spin uncertainty

$|+\rangle$ ↑

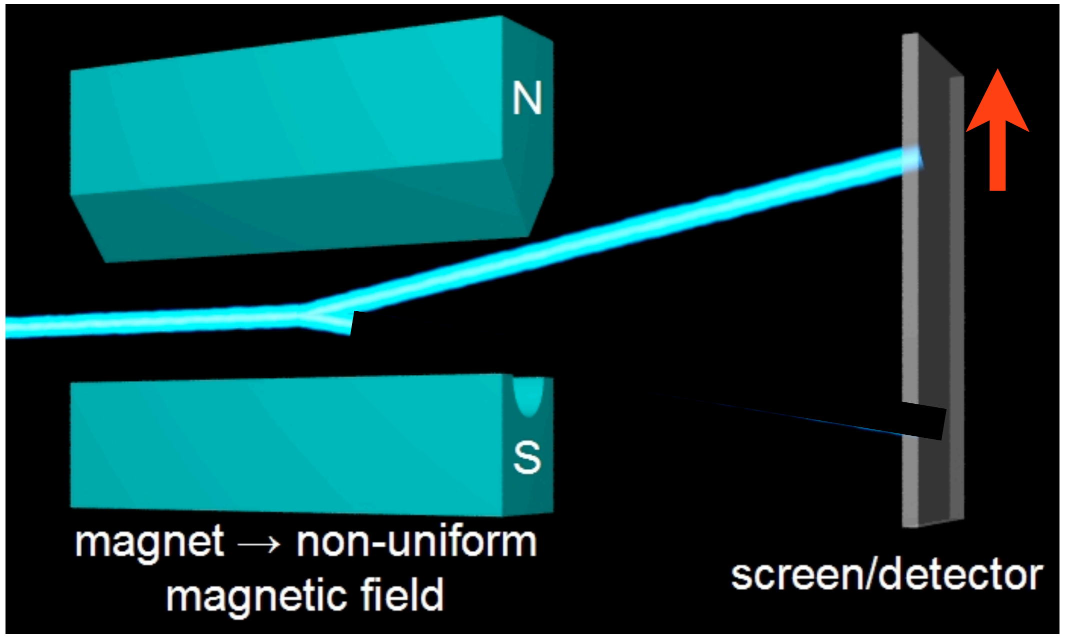


$|+\rangle$

Measurement is sharp (no uncertainty)

Spin uncertainty

$|+\rangle$

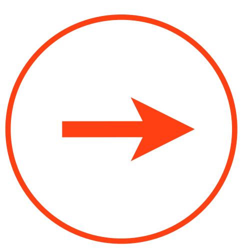


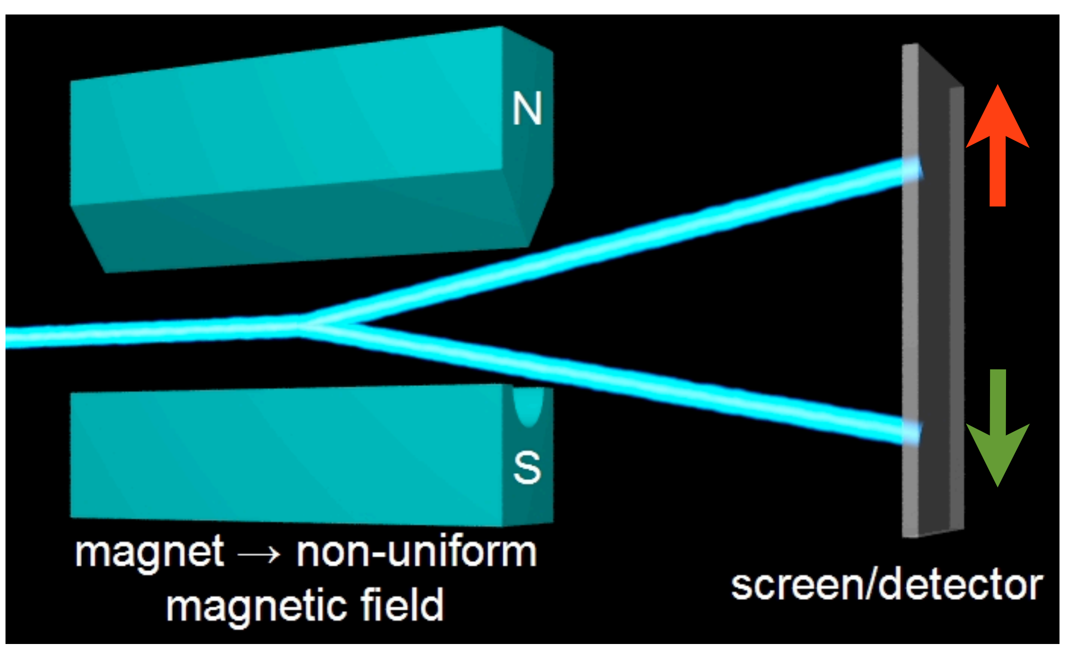
$|+\rangle$

S$_z$ measurement

$$|+\rangle_x = \frac{1}{\sqrt{2}}(|+\rangle + |-\rangle)$$

$|+\rangle_x$



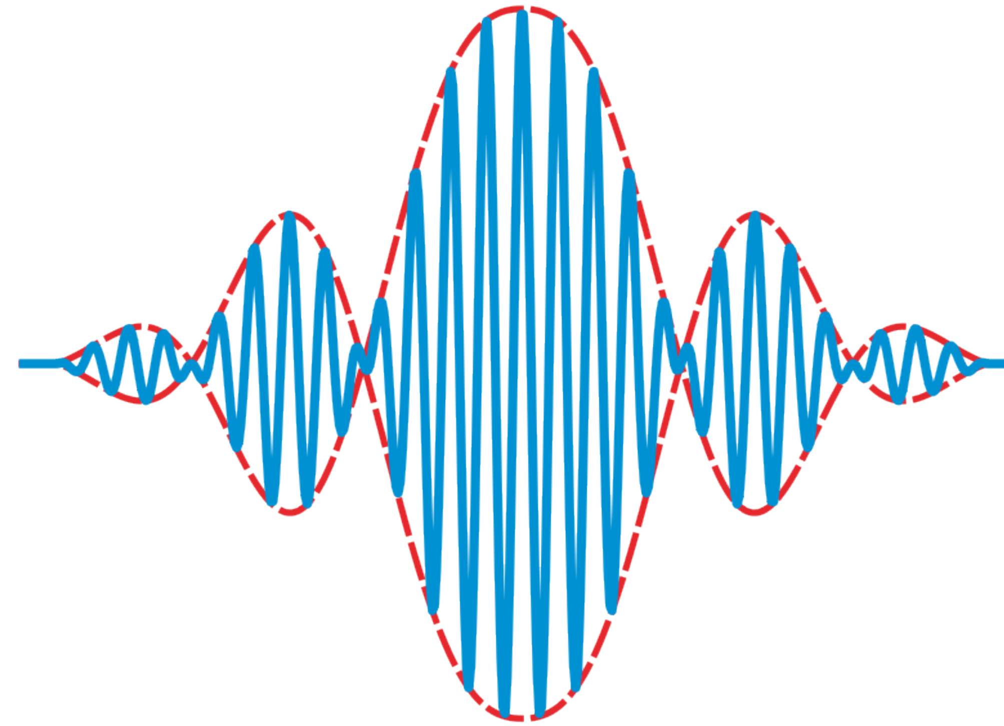


$|+\rangle$

$|-\rangle$

There is a 50% chance of measuring up or down!

Uncertainty Principle



$$\Delta p \Delta x \geq \frac{\hbar}{2}$$

$$\Delta E \Delta t \geq \frac{\hbar}{2}$$

Conjugated variables

Uncertainty of a given observable A:

$$\Delta A = \sqrt{\langle\psi|A^2|\psi\rangle - \langle\psi|A|\psi\rangle^2}$$

Spin uncertainty

For spins, if one prepares a state $|\psi\rangle = |+\rangle$ which is an eigenstate of S_z,

$$\Delta S_z = \sqrt{\langle\psi|S_z^2|\psi\rangle - \langle\psi|S_z|\psi\rangle^2} = 0$$

(sharp measurement)

But for a measurement along S_x, since $|+\rangle = \frac{1}{\sqrt{2}}\left(|+\rangle_x + |-\rangle_x\right)$,

$$\Delta S_x = \sqrt{\langle\psi|S_x^2|\psi\rangle - \langle\psi|S_x|\psi\rangle^2} = \frac{\hbar}{2}$$

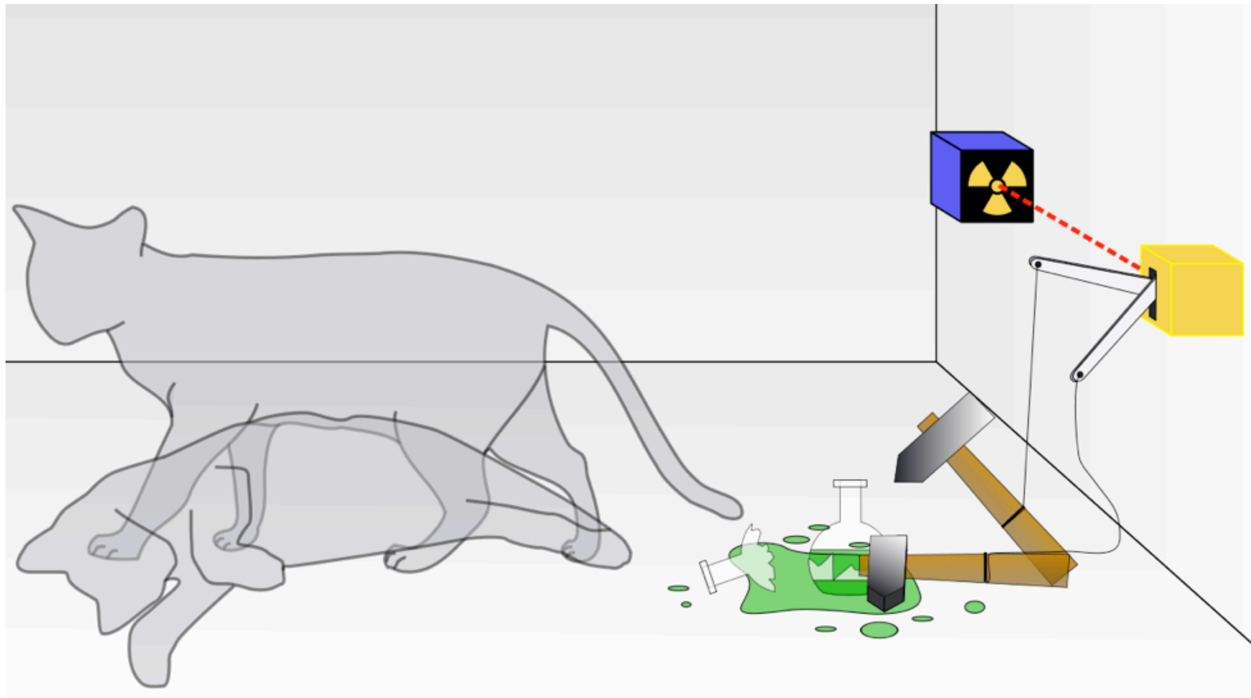
(uncertainty)

There is a 50% chance of measuring $\pm\hbar/2$!

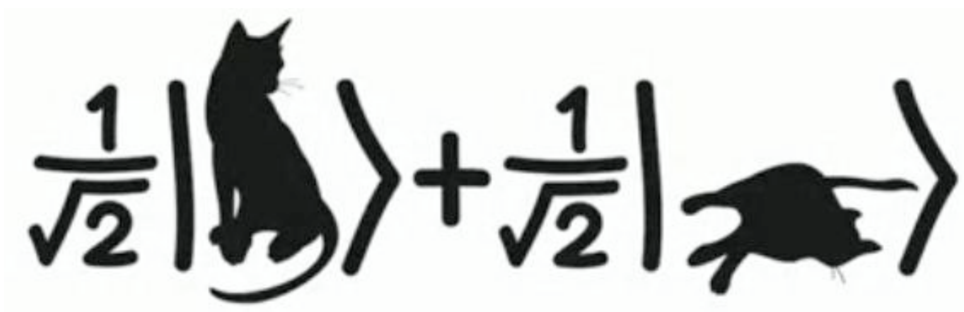
Schrodinger's cat

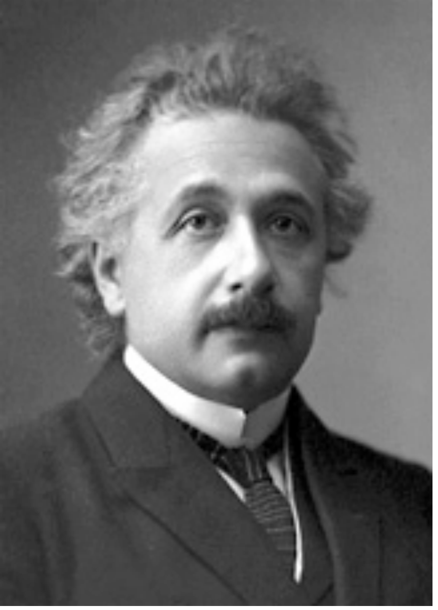
Schrodinger had problems with the probabilistic interpretation of quantum mechanics

"One can even set up quite ridiculous cases. A cat is penned up in a steel chamber, along with the following device (which must be secured against direct interference by the cat): in a Geiger counter, there is a tiny bit of radioactive substance, so small, that perhaps in the course of the hour one of the atoms decays, but also, with equal probability, perhaps none; if it happens, the counter tube discharges and through a relay releases a hammer that shatters a small flask of hydrocyanic acid. If one has left this entire system to itself for an hour, one would say that the cat still lives if meanwhile no atom has decayed. The psi-function of the entire system would express this by having in it the living and dead cat (pardon the expression) mixed or smeared out in equal parts."

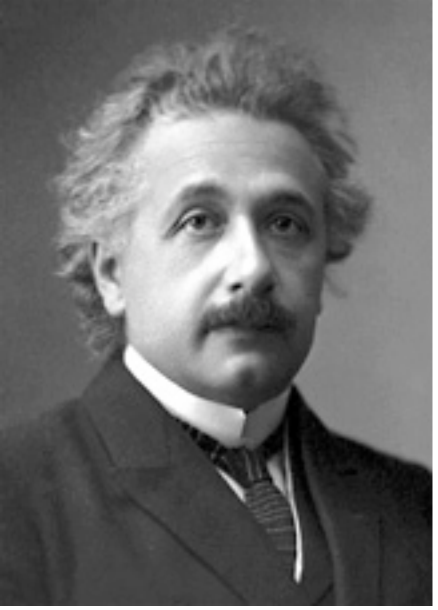


Cat quantum state

$|\Psi\rangle =$ 



Einstein did not believe that quantum mechanics could be a complete theory. There should be a set of hidden variables that would specify the true quantum state.

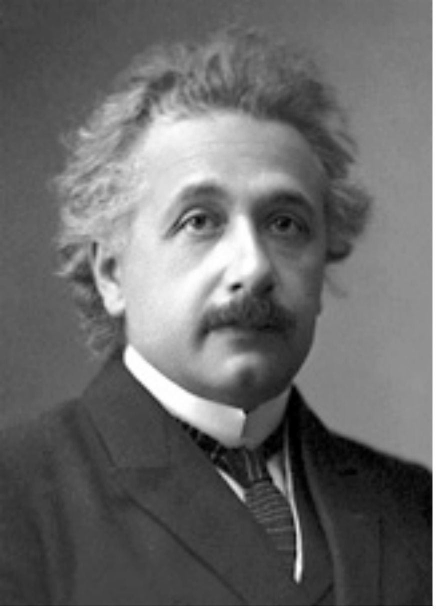


Einstein did not believe that quantum mechanics could be a complete theory. There should be a set of hidden variables that would specify the true quantum state.

Bohr and Heisenberg strongly believed that the probabilistic nature of quantum mechanics was not an artifact but the correct description of reality.

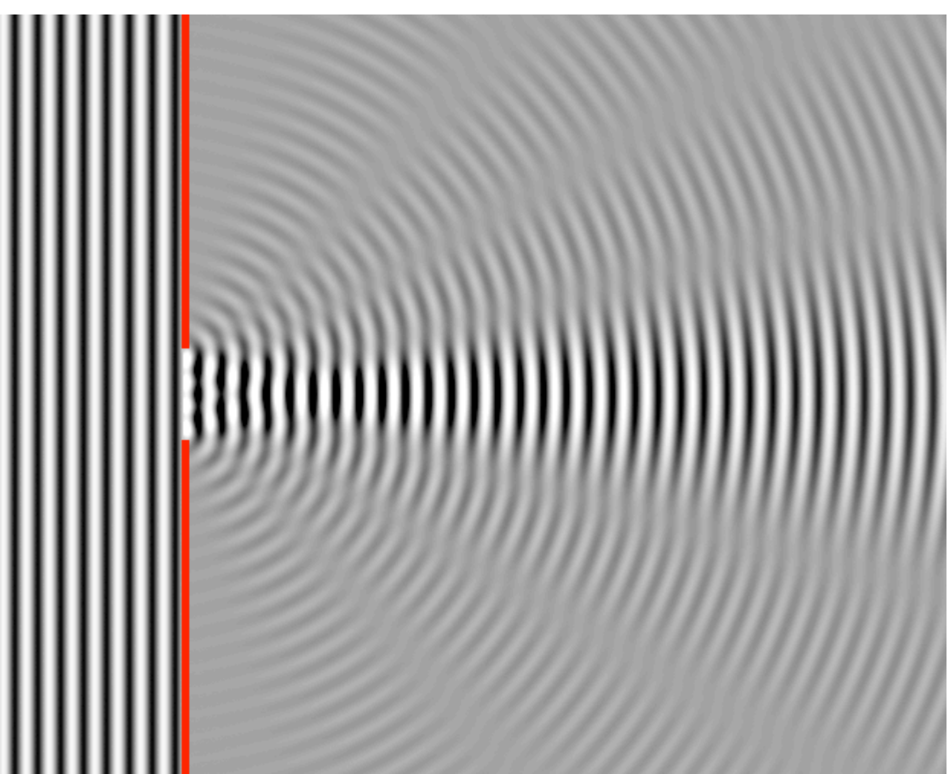
Einstein and Bohr had several discussions over many years. Each man died believing his interpretation of quantum mechanics was right.

Bohr was the champion of the probabilistic interpretation, known as the Copenhagen interpretation.

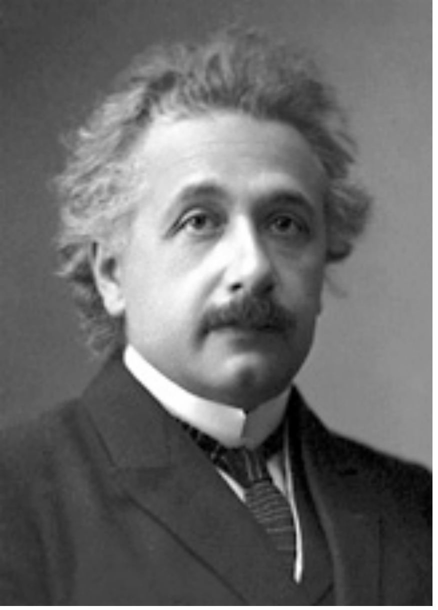


Einstein proposed a series of thought experiments to disprove QM.

"Consider a particle passing through a slit of width d. The slit introduces an uncertainty in momentum of approximately h/d because the particle passes through the wall. But let us determine the momentum of the particle by measuring the recoil of the wall. In doing so, we find the momentum of the particle to arbitrary accuracy by conservation of momentum".



$$\Delta p \Delta x \geq \frac{\hbar}{2} \quad ??$$



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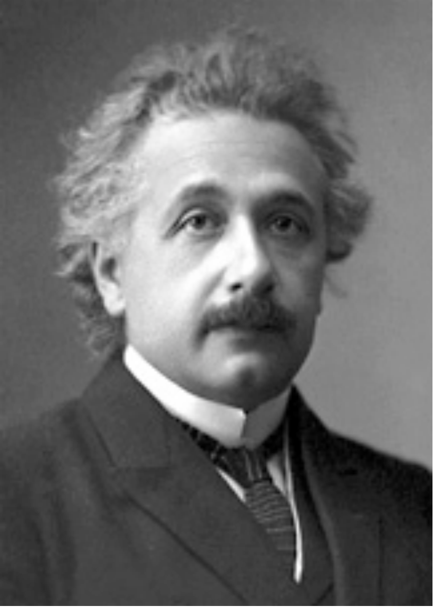
Bohr replied that the wall is quantum mechanical as well. The momentum of the wall must be known with accuracy Δp before the particle passes though it. That introduces an uncertainty in the position of the slit $h/\Delta p$

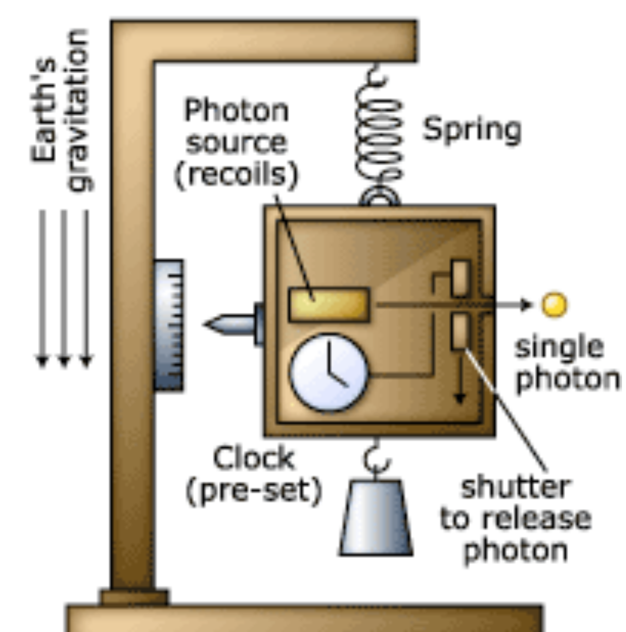


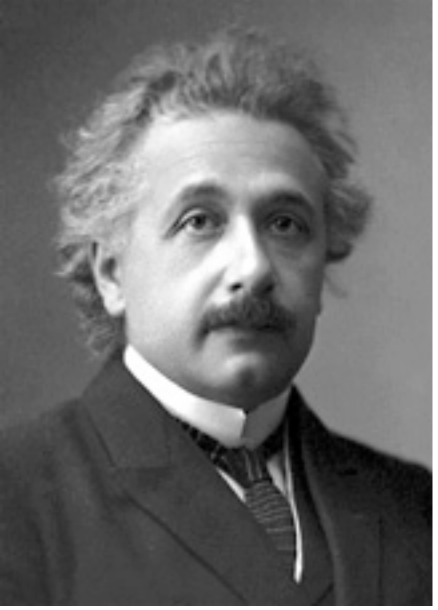
Einstein suggested later another thought experiment:

“Consider an ideal box, lined with mirrors so that it can contain light indefinitely. The box could be weighed before a clockwork mechanism opened an ideal shutter at a chosen instant to allow one single photon to escape. We now know precisely the time at which the photon left the box. Now, weigh the box again. The change of mass tells the energy of the emitted light. In this manner, one could measure the energy emitted and the time it was released with any desired precision, in contradiction to the uncertainty principle."

$$\Delta E \Delta t \geq \frac{\hbar}{2} \quad ??$$

Einstein's Light Box
(after a drawing by Bohr)





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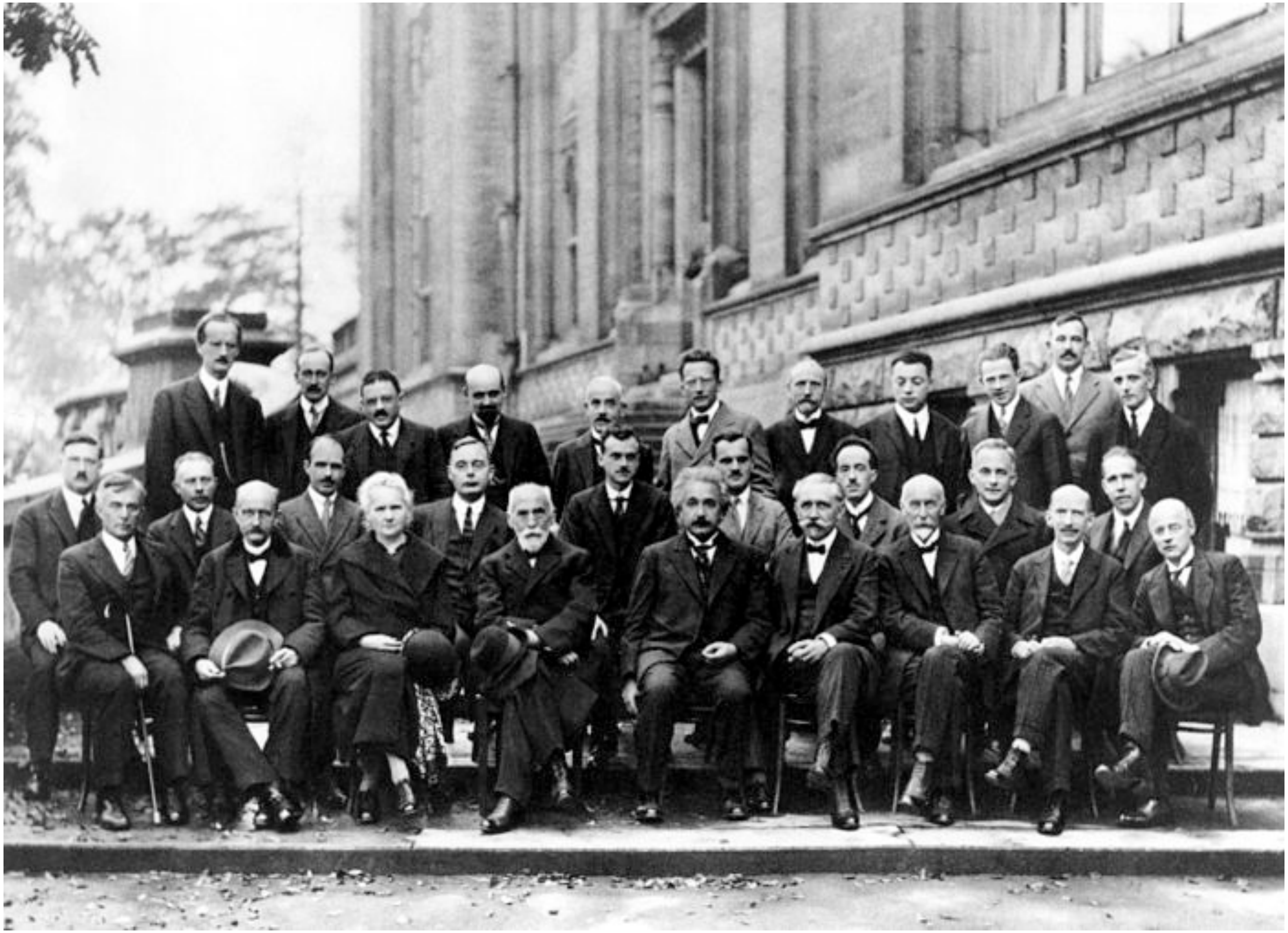
"Consider an ideal box, lined with mirrors so that it can contain light indefinitely. The box could be weighed before a clockwork mechanism opened an ideal shutter at a chosen instant to allow one single photon to escape. We now know precisely the time at which the photon left the box. Now, weigh the box again. The change of mass tells the energy of the emitted light. In this manner, one could measure the energy emitted and the time it was released with any desired precision, in contradiction to the uncertainty principle."

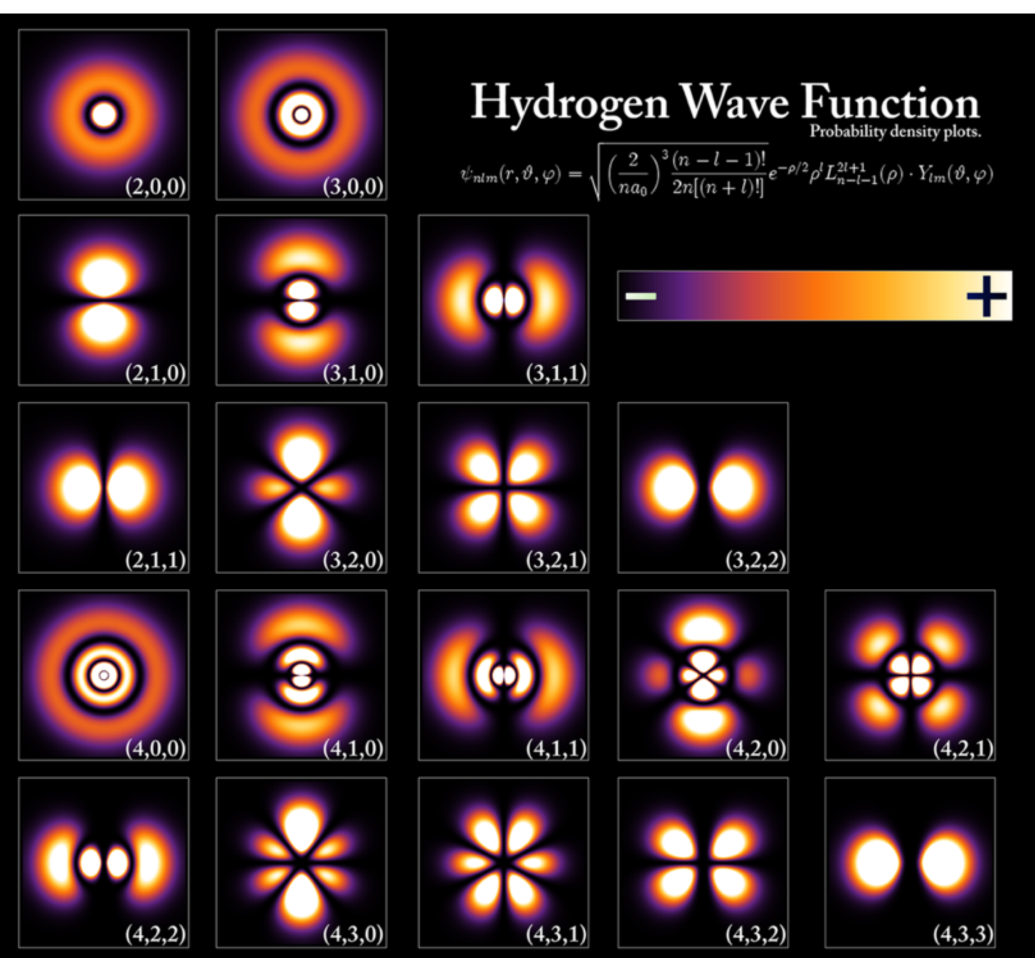
Bohr replied that in the measurement of the mass there would be uncertainty in the velocity of the spring, and hence in the height of the mass. Uncertainty on the height from Earth's surface would produce uncertainty on the clock rate, because of Einstein's general relativity.

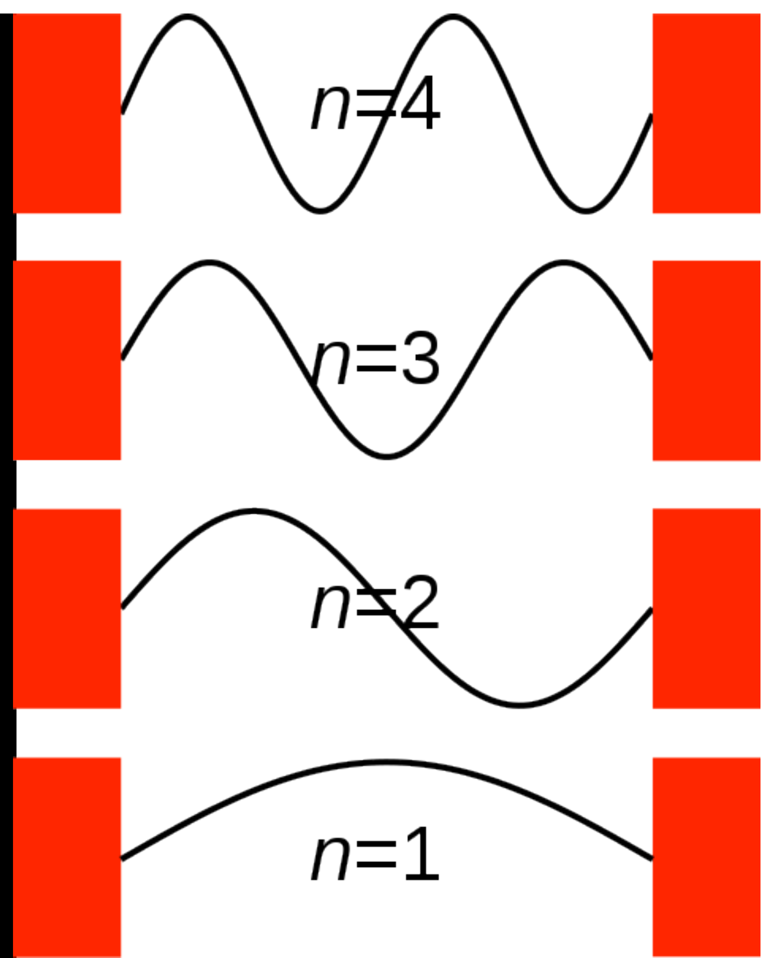


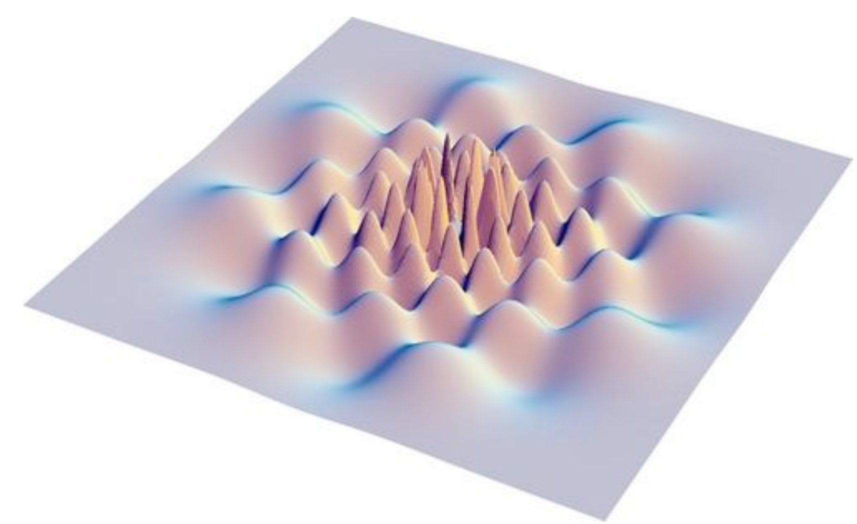
In the fifth Solvey congress (1927), Max Born and Heisenberg gave a presentation declaring quantum mechanics to be a complete theory.

Postulates of QM

I. A pure QM state is completely specified by a normalizable wave-function.







Postulates of QM

2. Correspondence principle:

Every observable in classical mechanics corresponds to a linear, Hermitian operator in QM.

$$A|\psi\rangle = a|\psi\rangle$$

$$\langle\psi|A|\psi\rangle = \langle\psi|a|\psi\rangle = a\langle\psi|\psi\rangle = a \quad \text{since } \langle\psi|\psi\rangle = 1$$

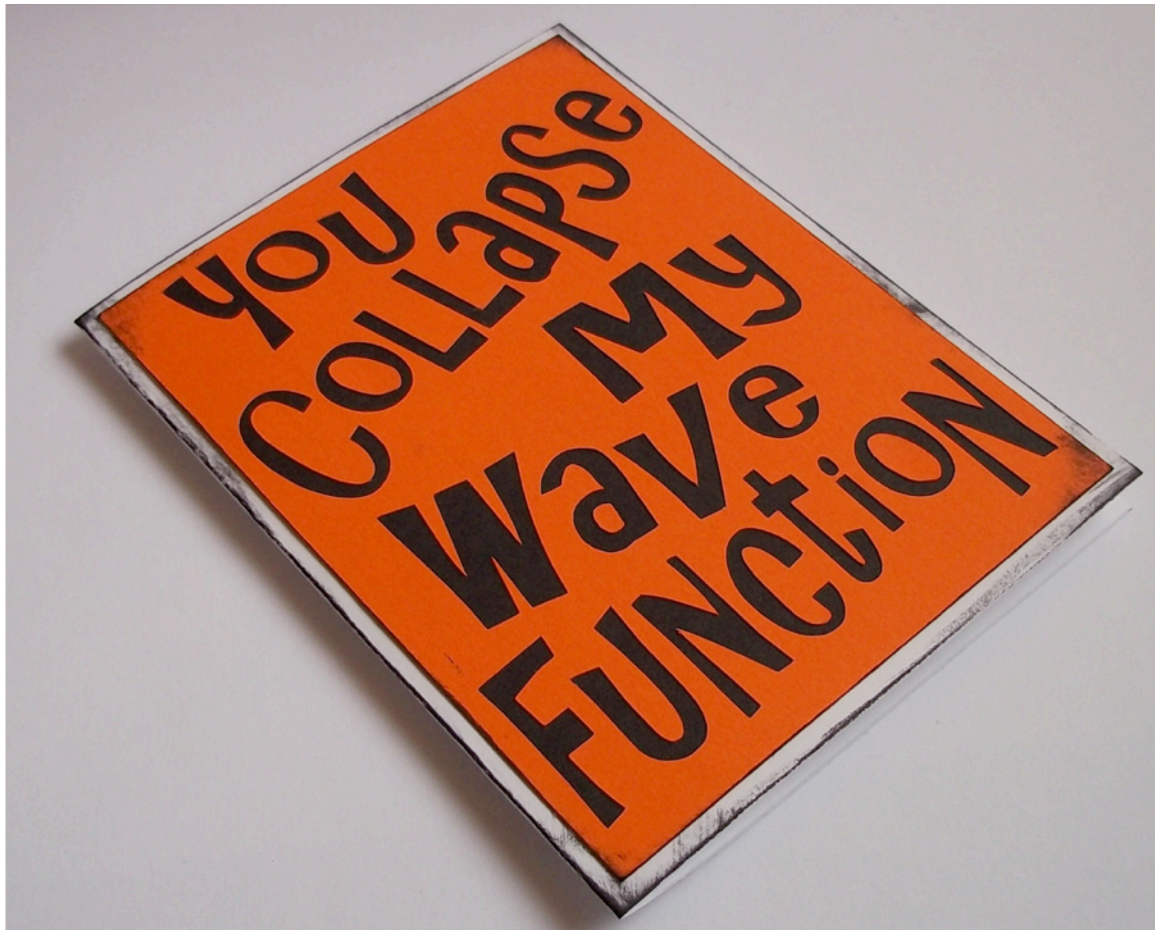
$$\left(\langle\psi|A|\psi\rangle\right)^* = a^*\left(\langle\psi|\psi\rangle\right)^* = a^*$$

$$\langle\psi|A|\psi\rangle = \left(\langle\psi|A|\psi\rangle\right)^* \quad \text{by Hermiticity}$$

$$\therefore a = a^* \quad \text{only true if } a \text{ is real.}$$

Postulates of QM

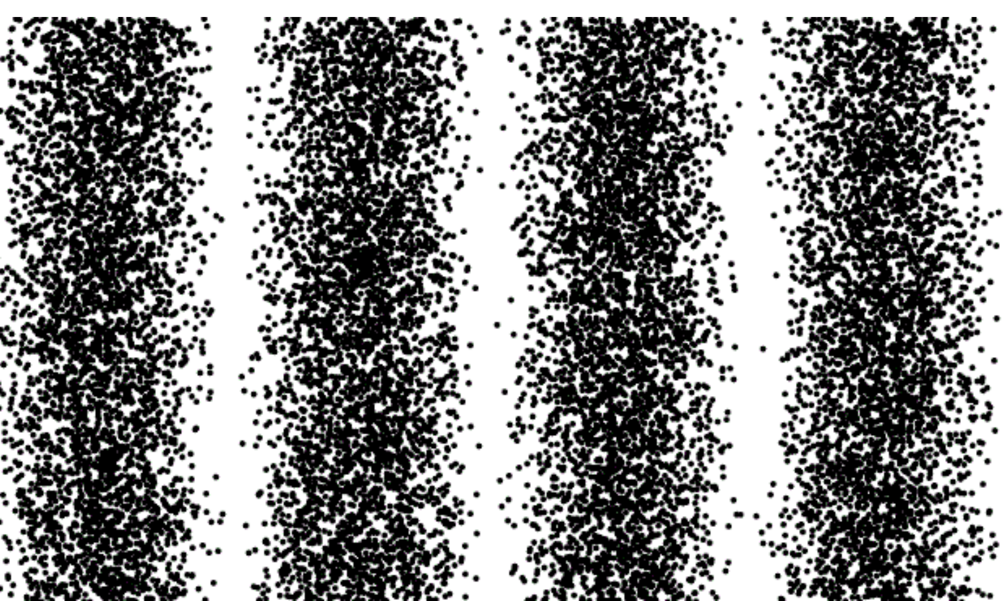
3. For a given observable operator, the only values that can be observed are the eigenvalues.



Postulates of QM

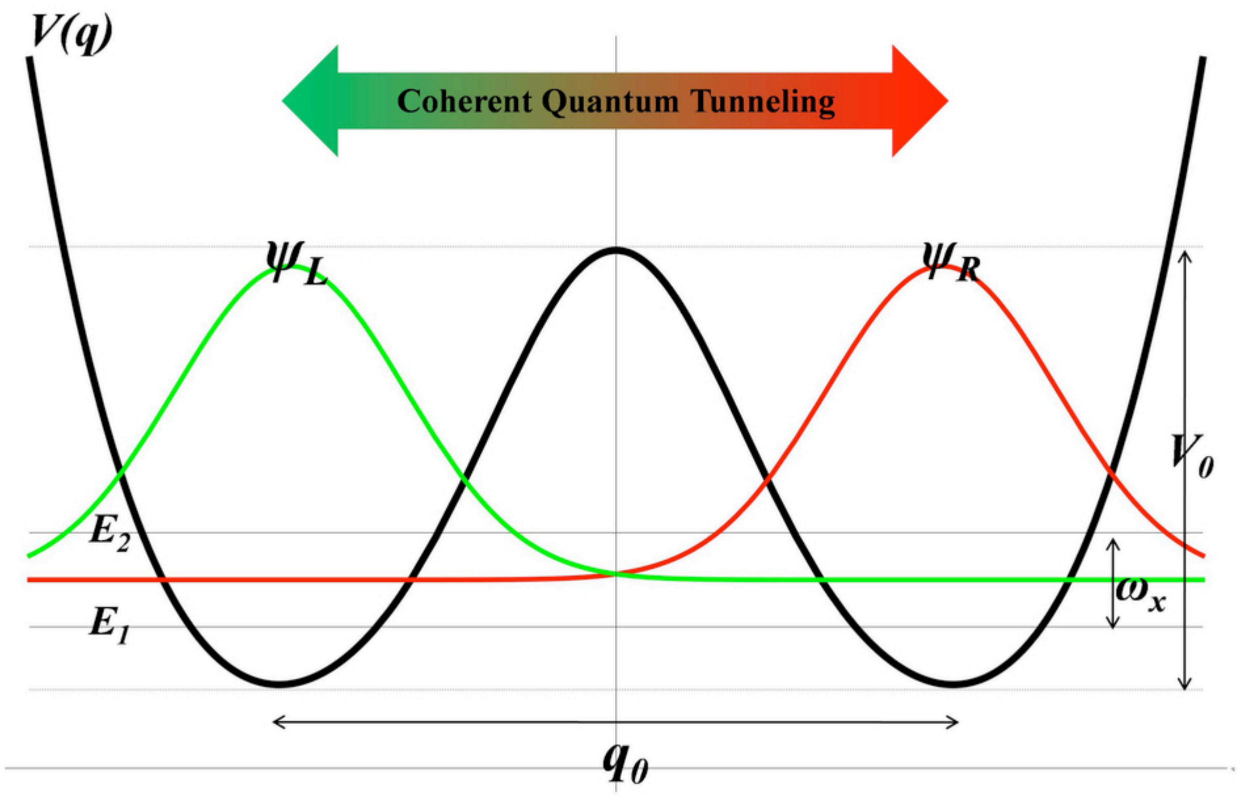
4. The average value of an observable is

$$\langle A \rangle = \langle \Psi | A | \Psi \rangle$$



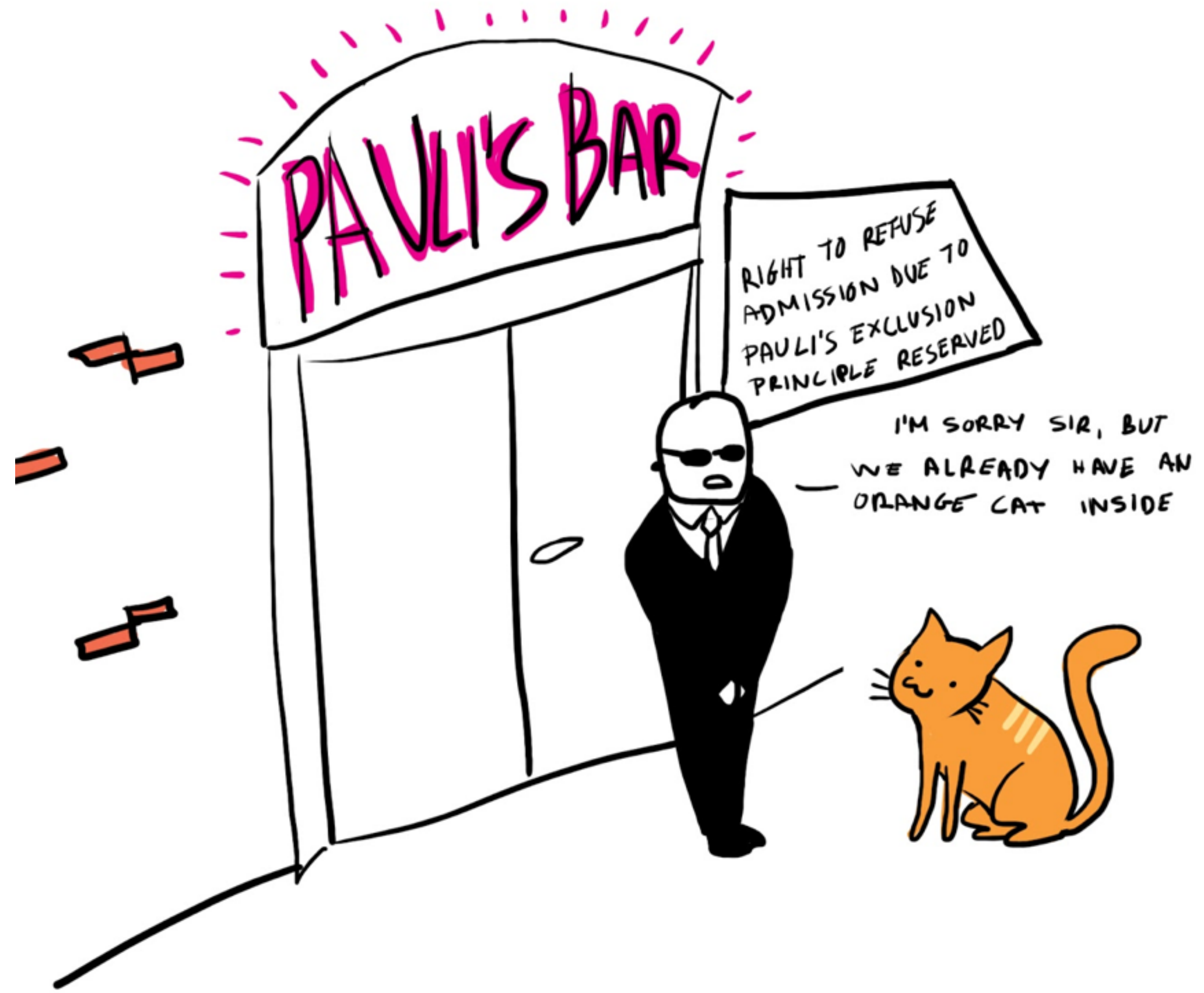
Postulates of QM

5. Schrodinger Equation: $$H(t)|\psi(t)\rangle = i\hbar\frac{\partial}{\partial t}|\psi(t)\rangle$$

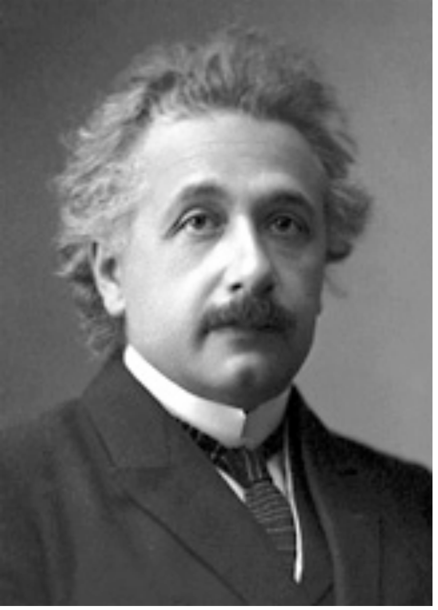


Postulates of QM

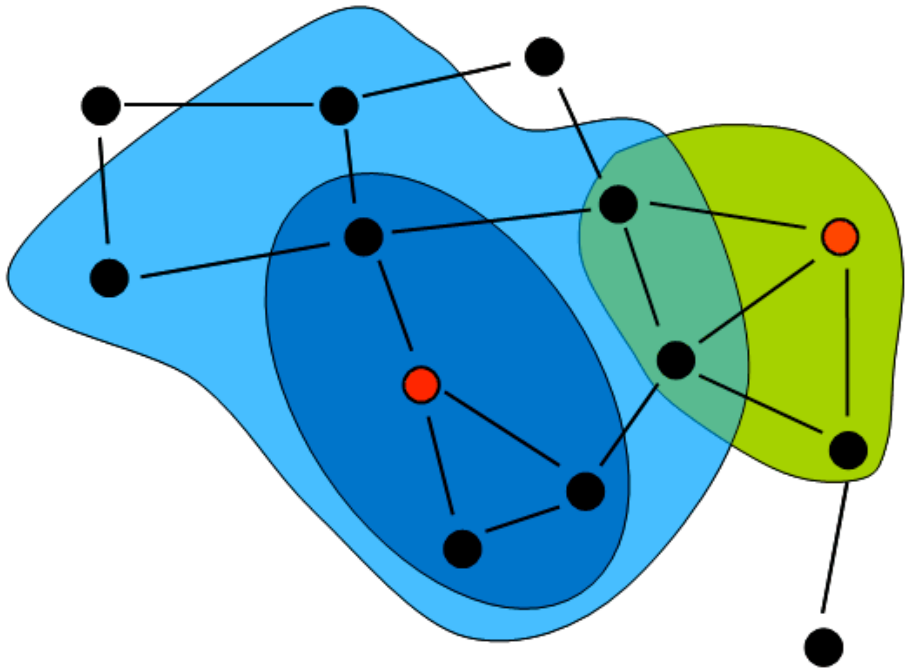
6. Pauli Principle (anti-symmetry of the wavefunction for electrons):



Two electrons cannot occupy the same quantum state.



What Einstein did not like about it is that quantum mechanics violates locality of nature.



Einstein-Podolsky-Rosen Paradox (1935)

Suppose a meson decays in two muons

$$\eta \rightarrow \mu^+ + \mu^-$$

Since the meson spin is zero, the two muons (spin 1/2) form a pure singlet state

$$|\psi\rangle = \frac{1}{\sqrt{2}}\left(|+\rangle|-\rangle - |-\rangle|+\rangle\right)$$

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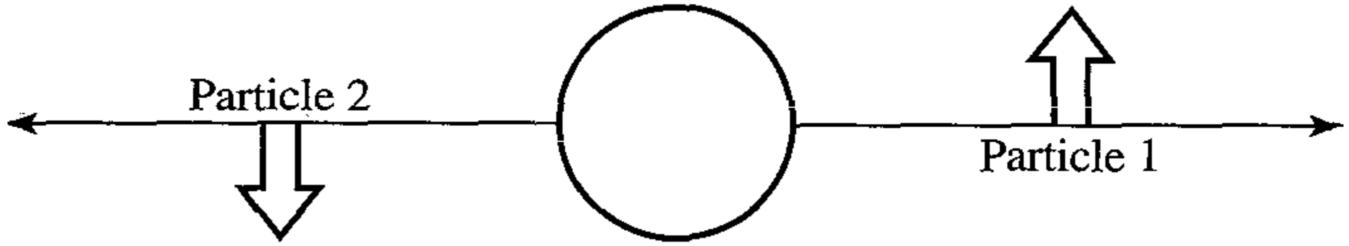


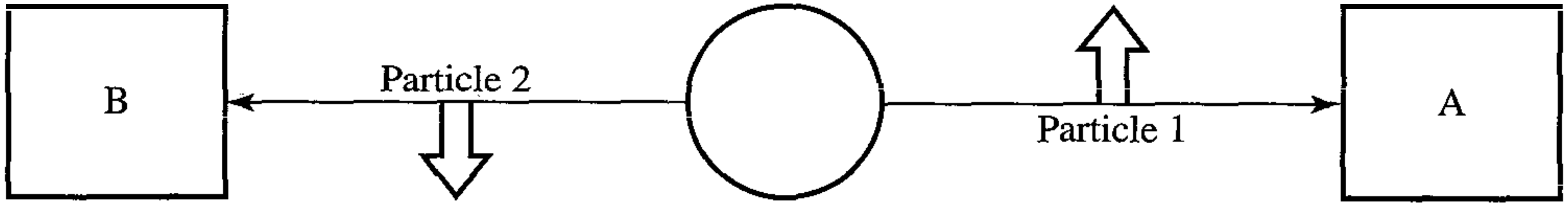
If one particle has spin up the other moving in the opposite direction must have spin down

Einstein-Podolsky-Rosen Paradox (1935)

Suppose a meson decays in two muons

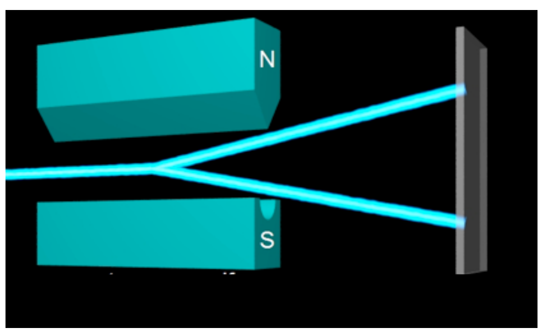
$$\eta \to \mu^+ + \mu^-$$

Suppose now the spin of each particle can be measured by two observers A and B, which are very far apart

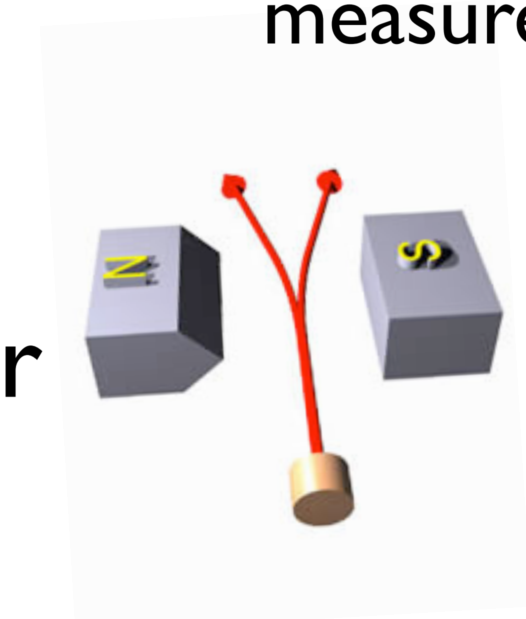


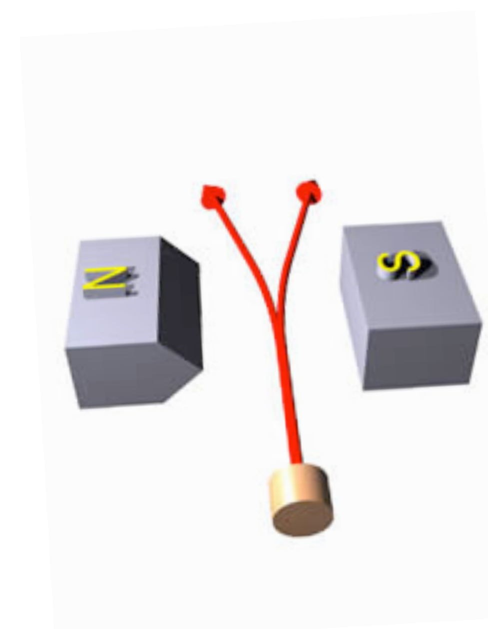
Einstein-Podolsky-Rosen Paradox (1935)

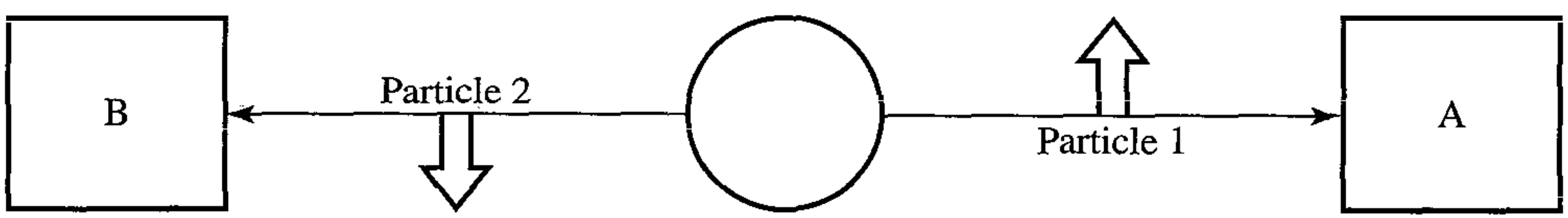
Suppose observer A has the ability to measure either the z or x direction of the spin, while the detector of observer A can only measure the x direction.



or





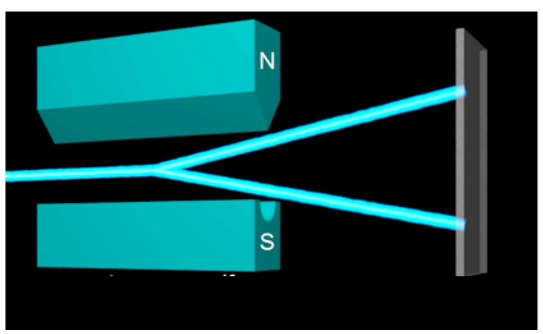


S_z, S_x

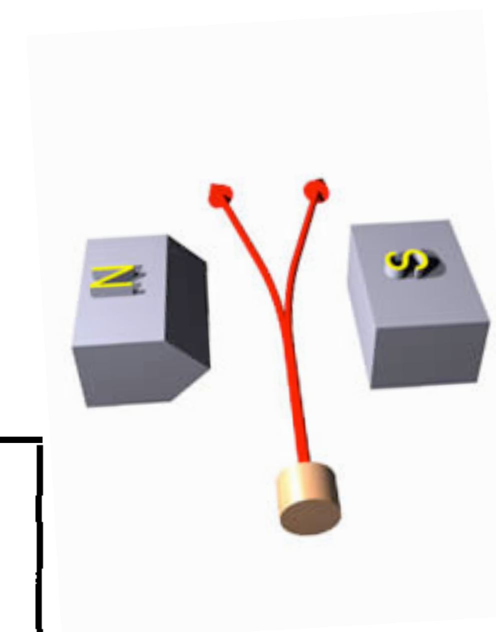
S_x

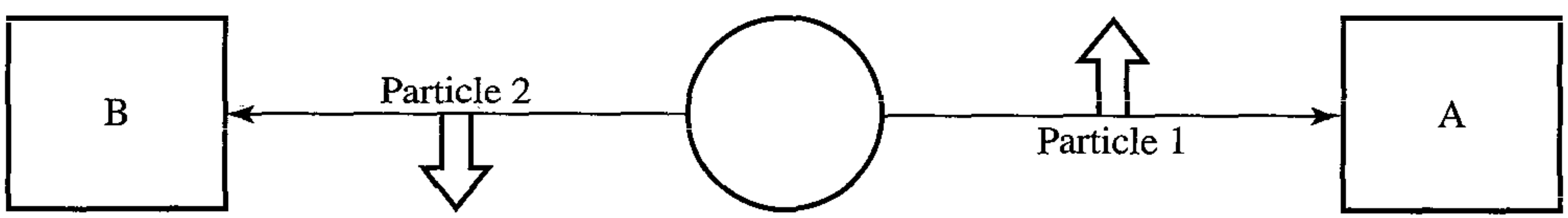
Einstein-Podolsky-Rosen Paradox (1935)

1. If B measures S_z, regardless of the result, A has 50% of measuring "up" or "down".



$$\uparrow |+\rangle \longrightarrow \downarrow |-\rangle = \frac{1}{\sqrt{2}}(|+\rangle_x - |-\rangle_x)$$





S_z, S_x $\qquad$ S_x

Einstein-Podolsky-Rosen Paradox (1935)

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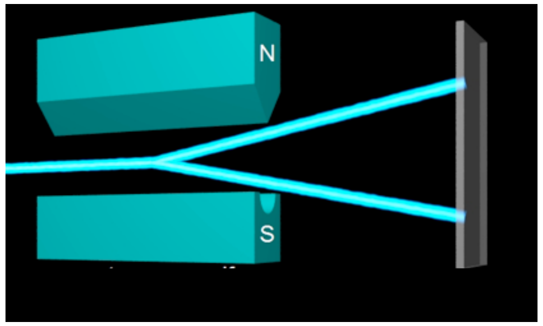
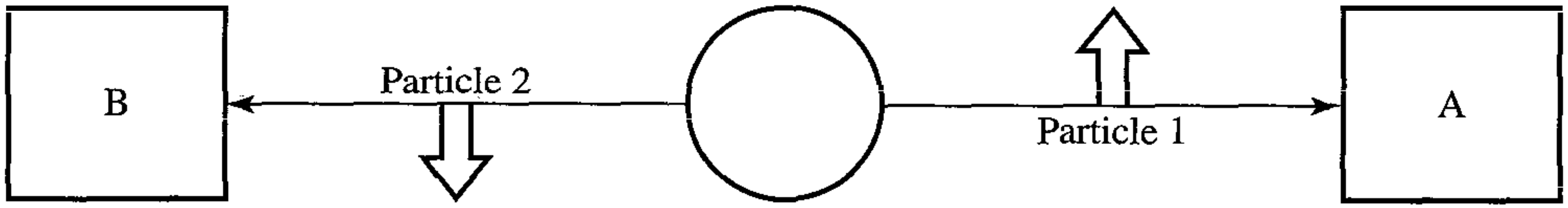


$$\uparrow |+\rangle \longrightarrow \downarrow |-\rangle = \frac{1}{\sqrt{2}}\left(\overset{\rightarrow}{|+\rangle_x} - \overset{\leftarrow}{|-\rangle_x}\right)$$

or

$$\downarrow |-\rangle \longrightarrow \uparrow |+\rangle = \frac{1}{\sqrt{2}}\left(|+\rangle_x + |-\rangle_x\right)$$

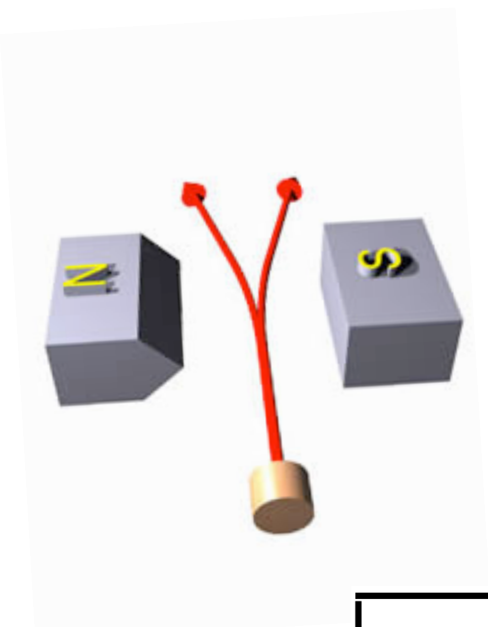


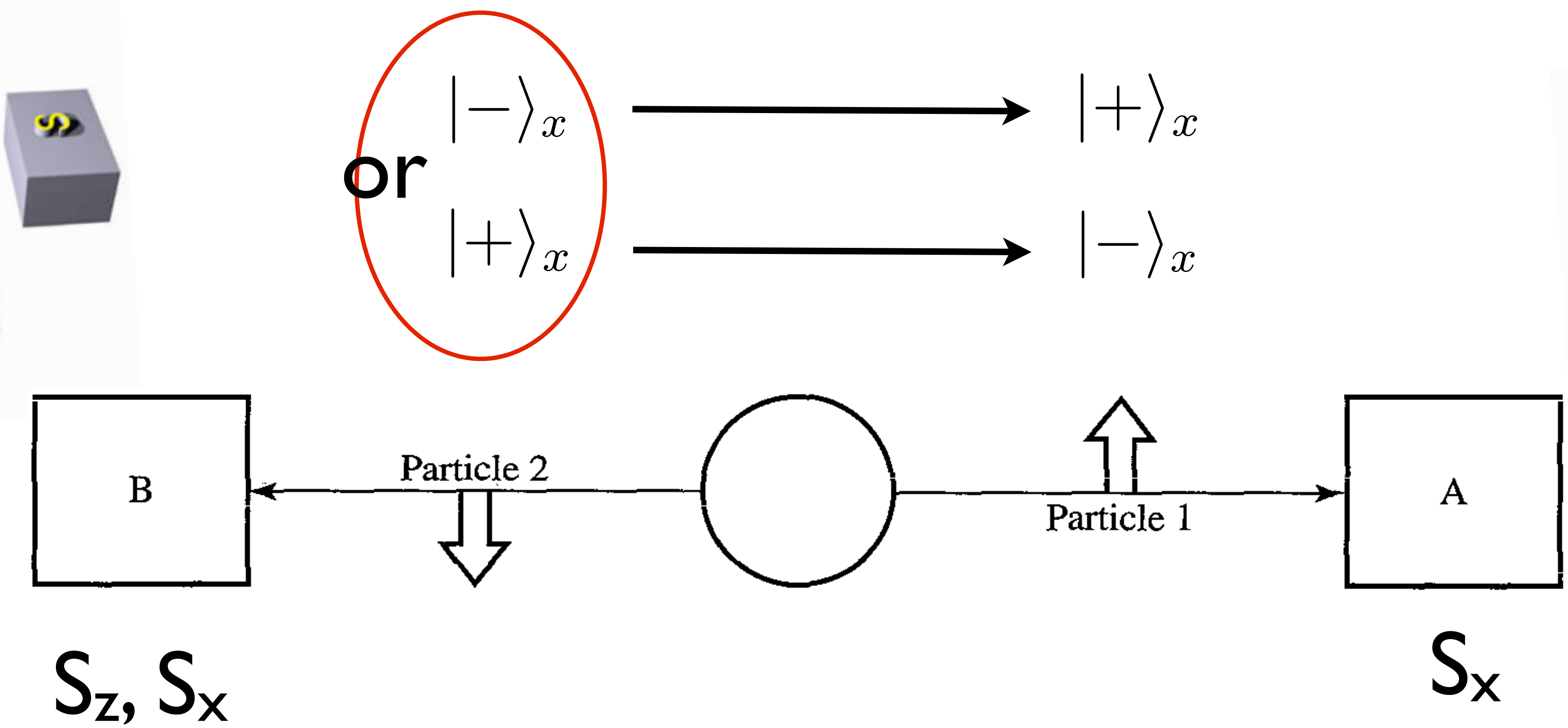
S_z, S_x $\qquad\qquad$ S_x

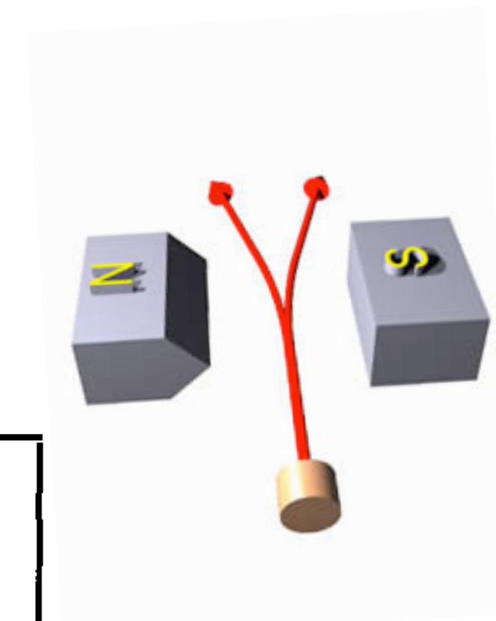
Einstein-Podolsky-Rosen Paradox (1935)

1. If B measures S_z, regardless of the result, A has 50% of measuring + or -

2. If B measures S_x, then A measurement has zero uncertainty.





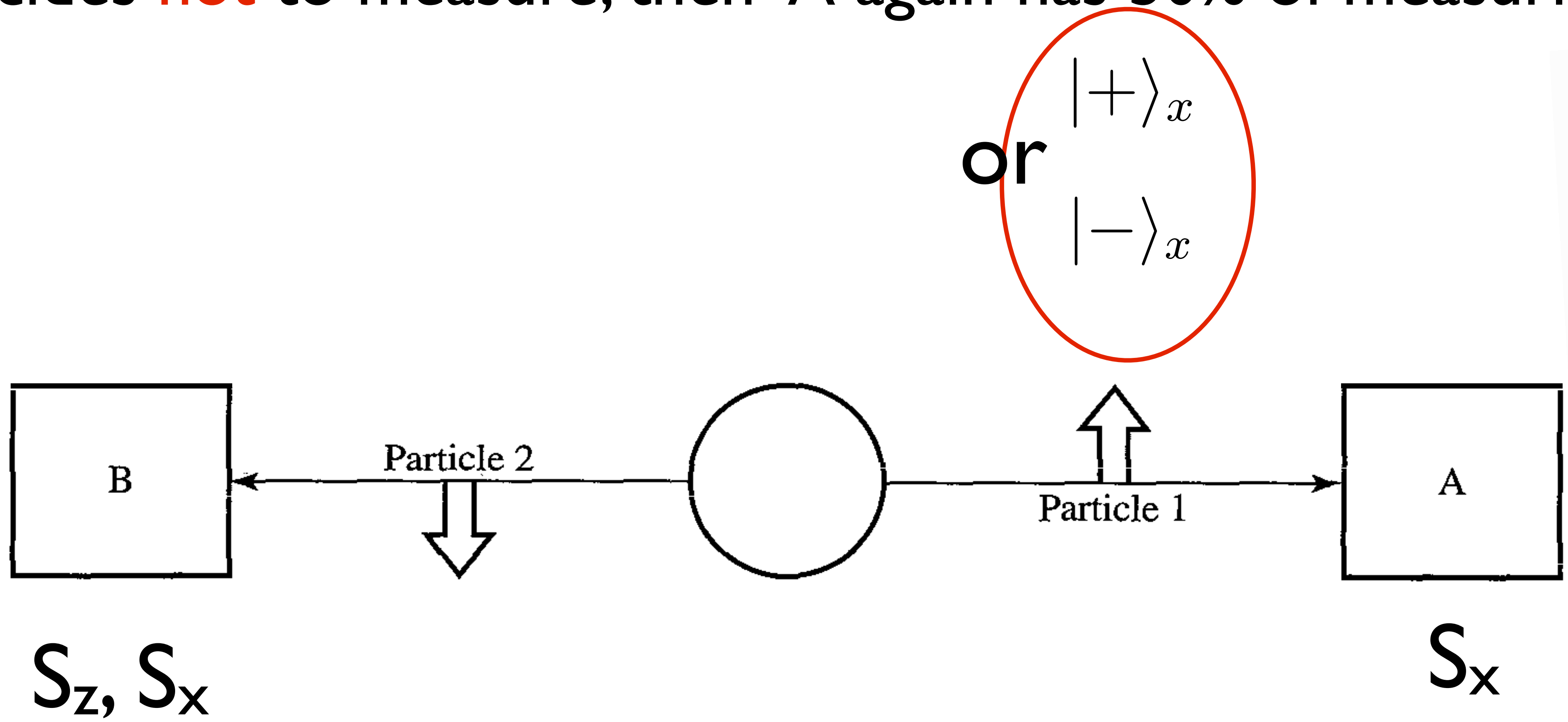


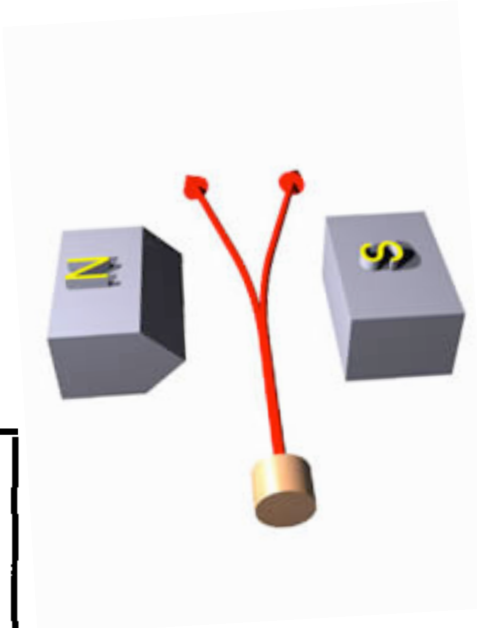
Einstein-Podolsky-Rosen Paradox (1935)

1. If B measures S_z, regardless of the result, A has 50% of measuring + or -

2. If B measures S_x, then A measurement has zero uncertainty.

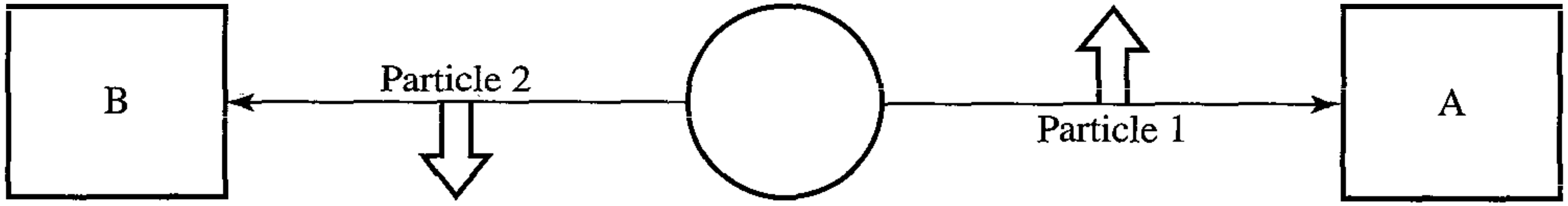
3. If B decides not to measure, then A again has 50% of measuring + or -





Einstein-Podolsky-Rosen Paradox (1935)

The decision of B to measure or not affects the measurement of A on the other side of the universe!



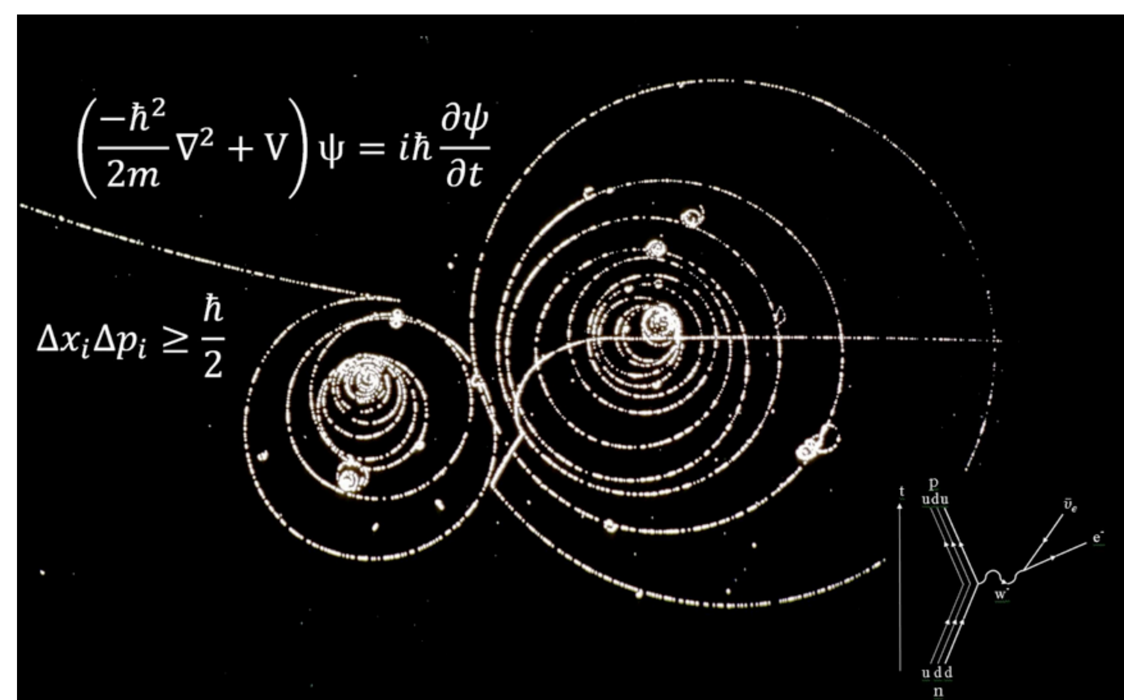
S_z, S_x

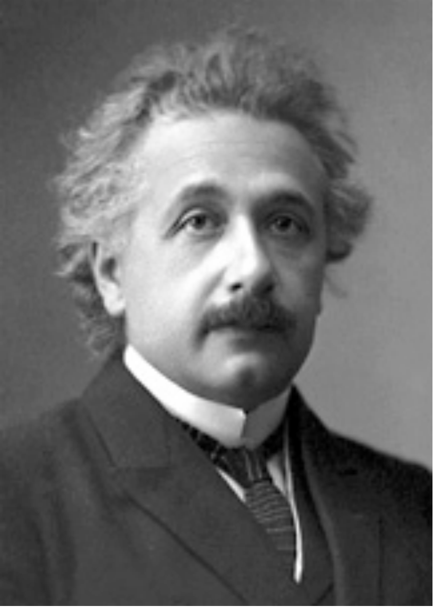
S_x

QM interpretation: when B measures, it is collapsing the wave function of both particles, regardless the distance.

B can either affect the measurement of A or not depending on how B made the measurement.

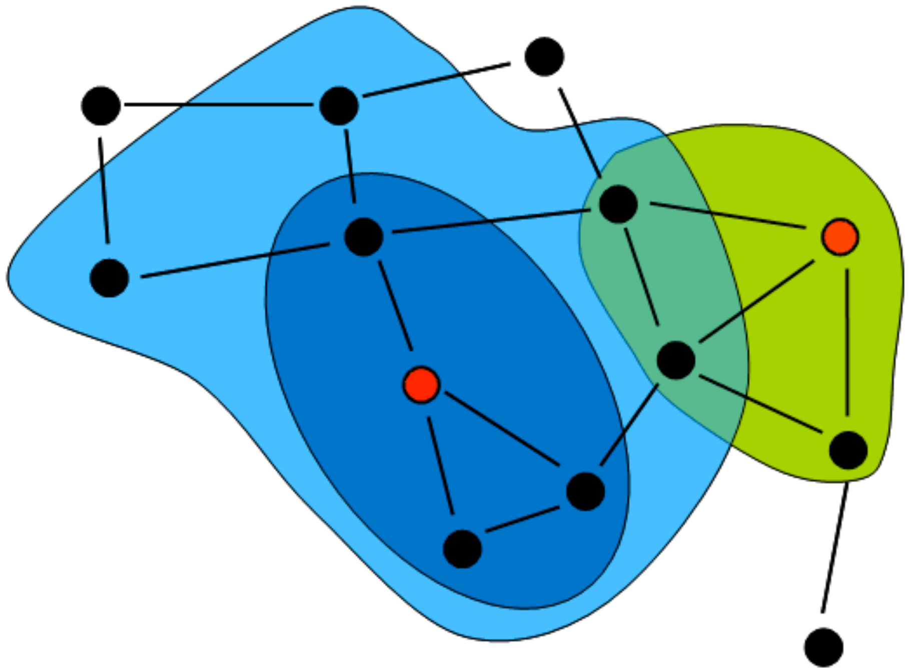
Quantum entanglement!





Einstein's interpretation: Quantum mechanics is an incomplete theory.
The dynamic behavior at the microscopic level appears probabilistic, but in reality is deterministically defined by a set of local hidden variables.

Einstein called quantum entanglement "spooky action at distance". If there were local hidden variables, the measurements of A and B would remain independent, and locality would be restored.



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ON THE EINSTEIN PODOLSKY ROSEN PARADOX*

J. S. BELL†
Department of Physics, University of Wisconsin, Madison, Wisconsin

(*Received* 4 *November* 1964)

I. Introduction

THE paradox of Einstein, Podolsky and Rosen [1] was advanced as an argument that quantum mechanics could not be a complete theory but should be supplemented by additional variables. These additional variables were to restore to the theory causality and locality [2]. In this note that idea will be formulated mathematically and shown to be incompatible with the statistical predictions of quantum mechanics. It is the requirement of locality, or more precisely that the result of a measurement on one system be unaffected by operations on a distant system with which it has interacted in the past, that creates the essential difficulty. There have been attempts [3] to show that even without such a separability or locality requirement no "hidden variable" interpretation of quantum mechanics is possible. These attempts have been examined elsewhere [4] and found wanting. Moreover, a hidden variable interpretation of elementary quantum theory [5] has been explicitly constructed. That particular interpretation has indeed a grossly non-local structure. This is characteristic, according to the result to be proved here, of any such theory which reproduces exactly the quantum mechanical predictions.

Bell's inequality

Suppose two distant observers can perform spin measurements along three different arbitrary directions, **a**, **b**, and **c**.

Define P(**a**,+;**b**,-) as the probability of observer B measure "+" along the **a** direction and observer A to measure "-" along the **b** direction.

Bell showed that any deterministic theory of local hidden variables must satisfy the inequality:

$$P(\hat{\mathbf{a}}, +; \hat{\mathbf{b}}+) \leq P(\hat{\mathbf{a}}, +; \hat{\mathbf{c}}+) + P(\hat{\mathbf{c}}, +; \hat{\mathbf{b}}, +)$$

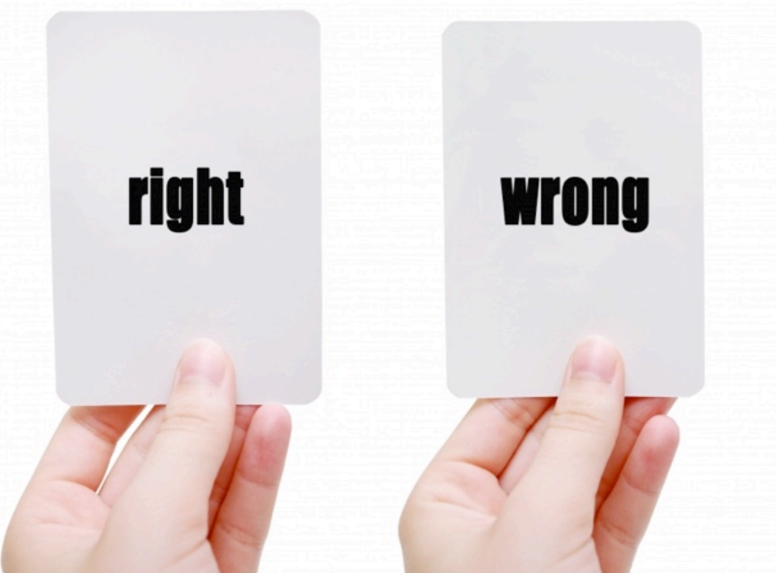
Observer B

Observer A

This inequality violates the predictions of quantum mechanics!

Conclusion: QM is incompatible with a description of local hidden variables.

If one description is right, the other one is wrong.

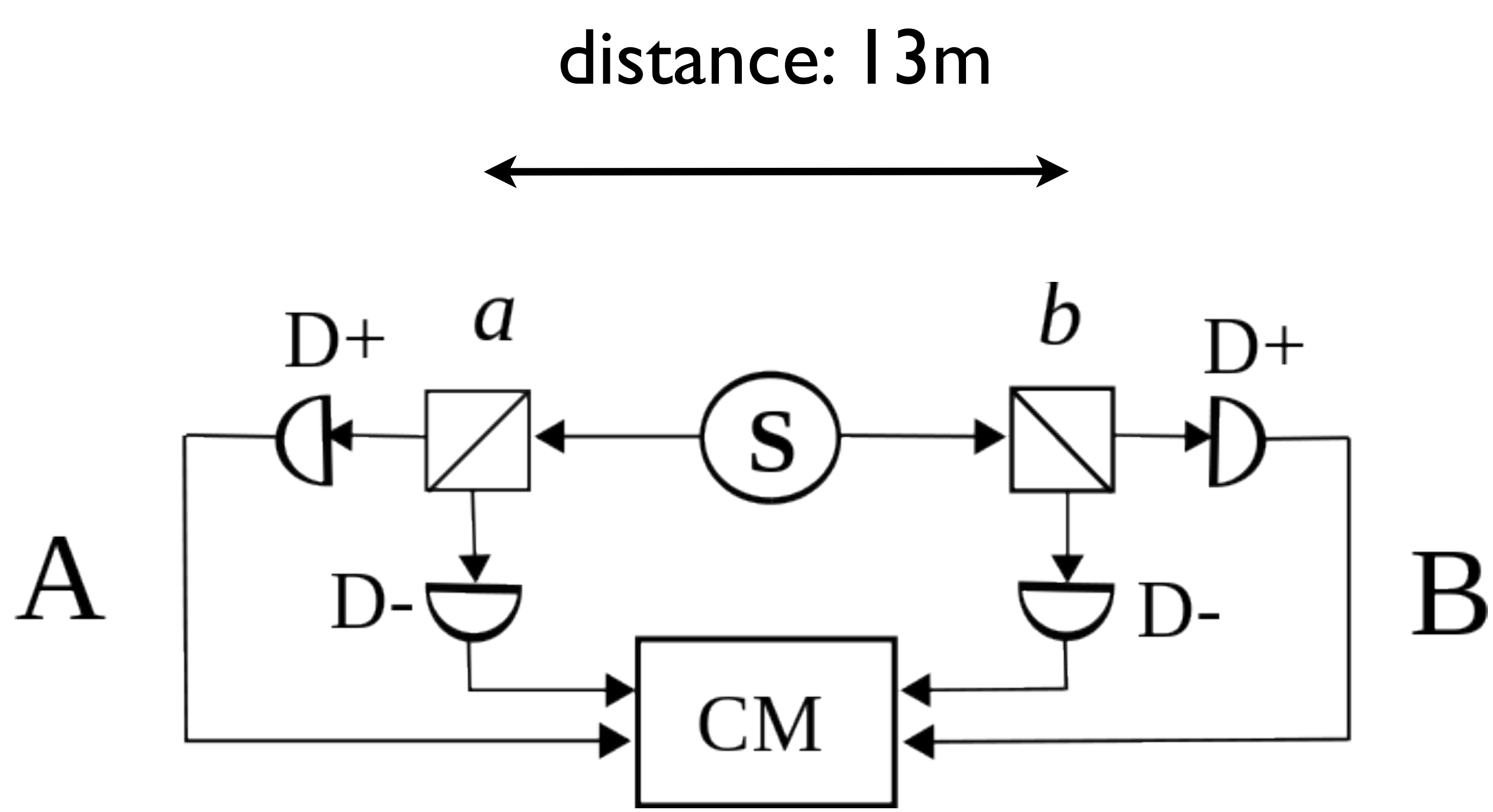


Which one is right?

Experimental Tests of Realistic Local Theories via Bell's Theorem

Alain Aspect, Philippe Grangier, and Gérard Roger
Institut d'Optique Théorique et Appliquée, Université Paris-Sud, F-91406 Orsay, France
(Received 30 March 1981)

We have measured the linear polarization correlation of the photons emitted in a radiative atomic cascade of calcium. A high-efficiency source provided an improved statistical accuracy and an ability to perform new tests. Our results, in excellent agreement with the quantum mechanical predictions, strongly violate the generalized Bell's inequalities, and rule out the whole class of realistic local theories. No significant change in results was observed with source-polarizer separations of up to 6.5 m.



Bell's inequality test: more ideal than ever

Alain Aspect

The experimental violation of Bell's inequalities confirms that a pair of entangled photons separated by hundreds of metres must be considered a single non-separable object — it is impossible to assign local physical reality to each photon.

Newer experiments were able to separate the detectors by 400m and observe the violation of Bell's inequality with 30 standard deviations of certainty!

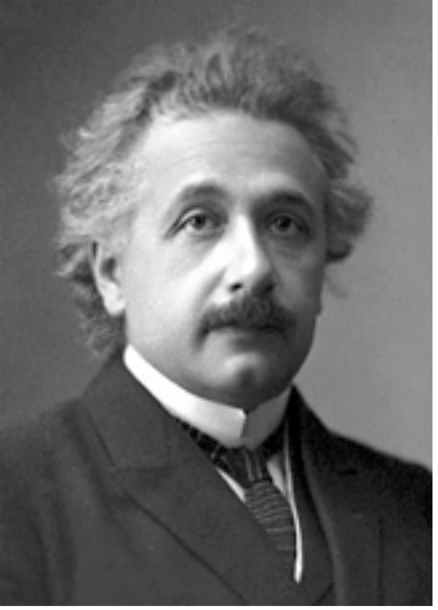
LETTER

doi:10.1038/nature15759

Loophole-free Bell inequality violation using electron spins separated by 1.3 kilometres

B. Hensen[1,2], H. Bernien[1,2]†, A. E. Dréau[1,2], A. Reiserer[1,2], N. Kalb[1,2], M. S. Blok[1,2], J. Ruitenberg[1,2], R. F. L. Vermeulen[1,2], R. N. Schouten[1,2], C. Abellán[3], W. Amaya[3], V. Pruneri[3,4], M. W. Mitchell[3,4], M. Markham[5], D. J. Twitchen[5], D. Elkouss[1], S. Wehner[1], T. H. Taminiau[1,2] & R. Hanson[1,2]

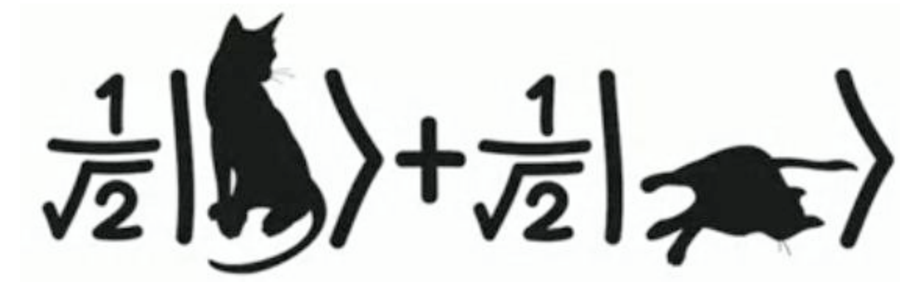
Very recents tests confirmed violation of Bell's inequalities with detectors separated by 1.3km!



Einstein’s locality principle was wrong.

There are no local hidden variables in nature.

Quantum computing



Quantum entanglement has deep implications for quantum computing. The greatest challenge is to manipulate quantum states with long enough decoherence times.