Modeling and Analysis of VLSI Interconnects

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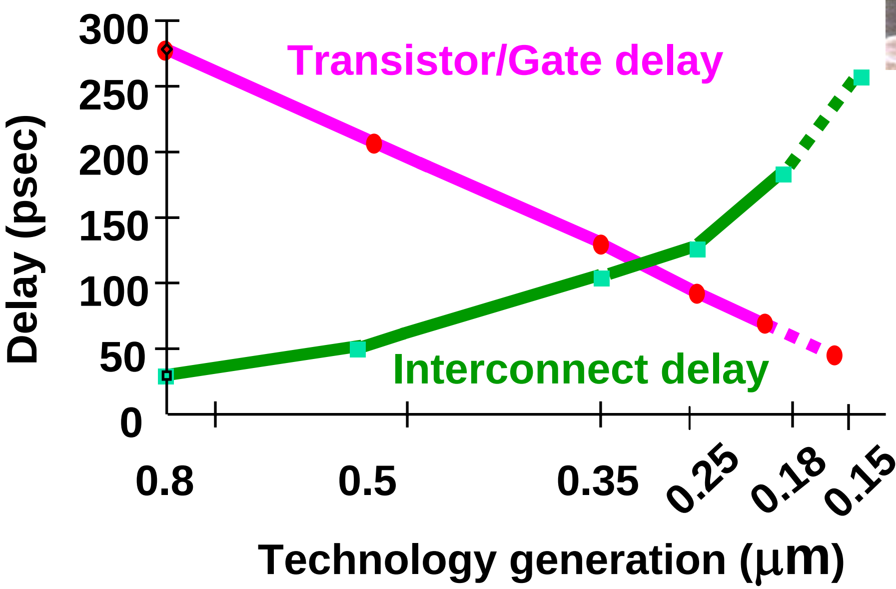
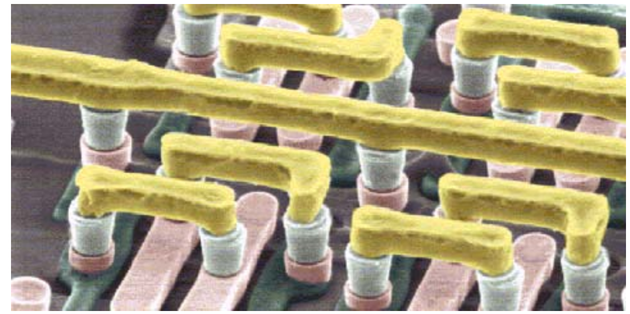
Outline

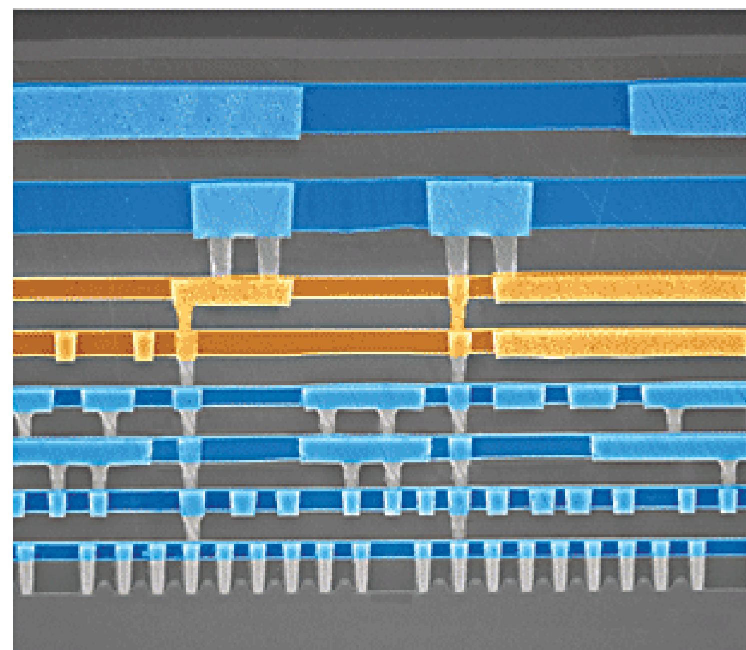
- Introduction
- Capacitance extraction
- Inductance modeling
- Conclusion

Interconnects do not scale well



[Gordon Moore, Chairman Emeritus, Intel Corp.]



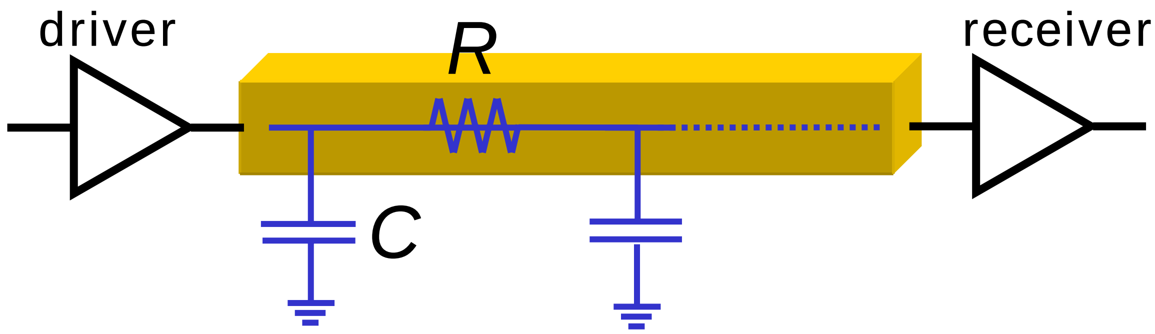


International Technology Roadmap for Semiconductor

Year	2005	2006	2007	2008	2009	2010	2011	2012	2013
Technology (nm)	80	70	65	57	50	45	40	36	32
Max # wiring levels	15	15	15	16	16	16	16	16	17
M1 RC (ps)	440	612	767	1044	1388	1782	2392	2857	3451
M1 resistivity (μΩ-cm)	3.15	3.29	3.47	3.67	3.90	4.08	4.30	4.63	4.83
M1 capacitance (pF/cm)	2.0-2.2	2.0-2.2	1.8-2.0	1.9-2.1	1.8-2.0	1.8-2.0	1.8-2.0	1.6-1.8	1.6-1.8
Global wire RC (ps)	111	165	209	316	410	523	687	787	977
Global wire resistivity (μΩ-cm)	2.53	2.62	2.73	2.87	3.00	3.10	3.22	3.39	3.52
Global wire capacitance (pF/cm)	2.1-2.3	2.1-2.3	1.8-2.0	1.8-2.0	1.7-1.9	1.7-1.9	1.7-1.9	1.5-1.7	1.5-1.7

[ITRS'05 and '06, http://www.itrs.net)]

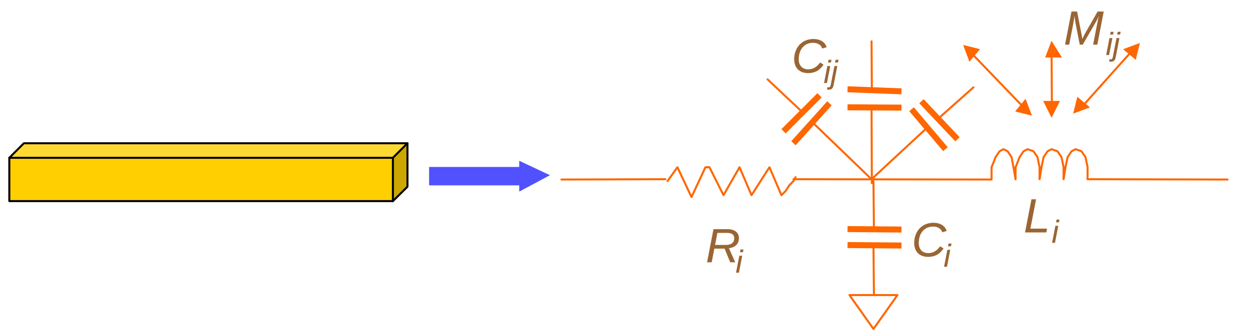
Interconnect Modeling



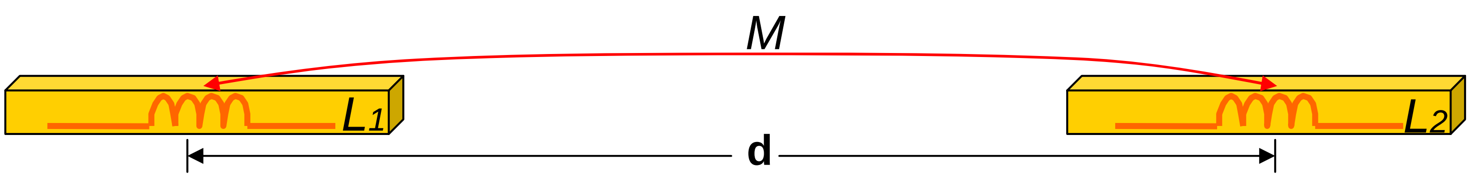
- Capacitance only model
- RC model now widely used
- How about inductance?
  - Ignored at low frequency ($\omega L \ll R$)
- Inductance can no longer be ignored
  - Increasing operation frequency
  - Lower wire resistance

Partial Element Equivalent Circuit

- PEEC Model [A. Ruehli, IBM Journal of R&D, 1972]



- Retardation



$$V_2(t) = L_2 \frac{di_2(t)}{dt} + M \frac{di_1(t - d/v)}{dt}$$

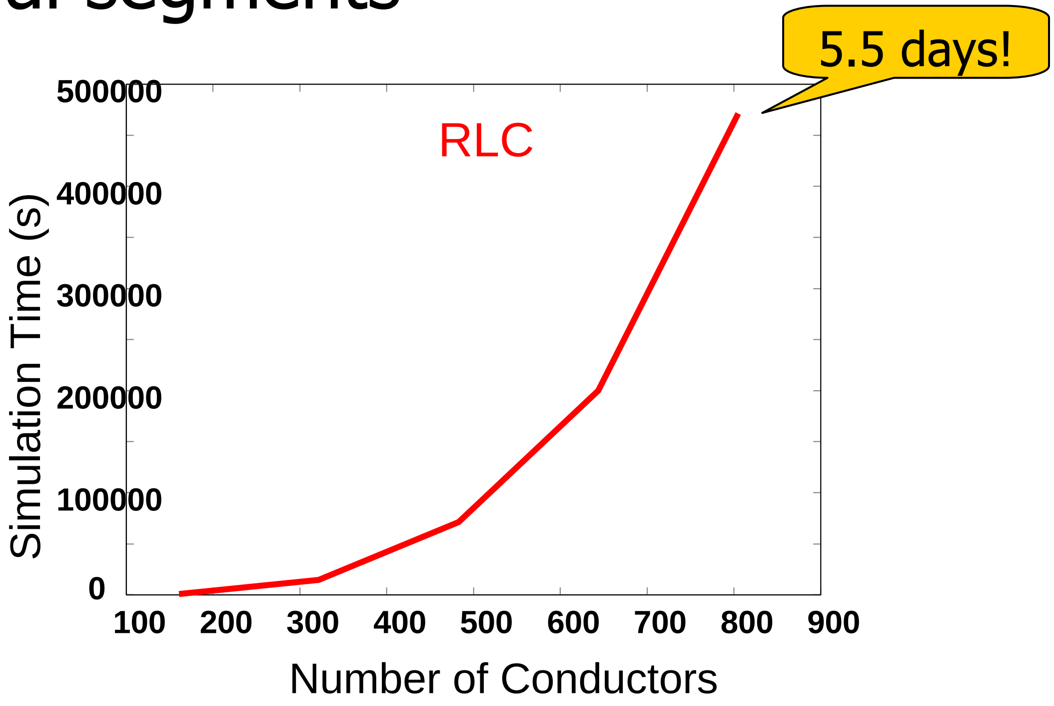
Full-wave PEEC

$$V_2(t) = L_2 \frac{di_2(t)}{dt} + M \frac{di_1(t)}{dt}$$

Quasi-static assumption

Interconnect Modeling Challenge

- Extraction of RLC parameters and simulation of interconnects are large scale problems
- Long interconnects have to be divided into several segments



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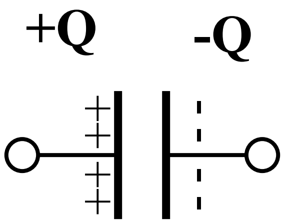
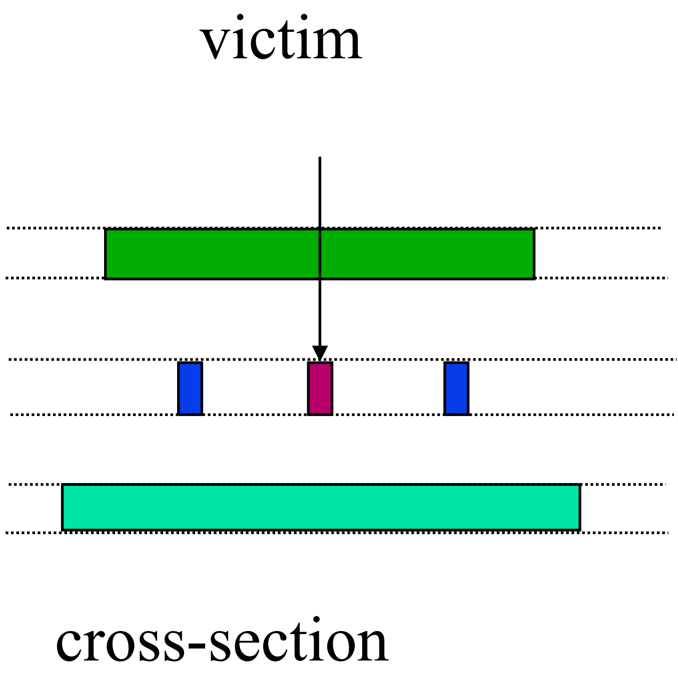
What is capacitance?

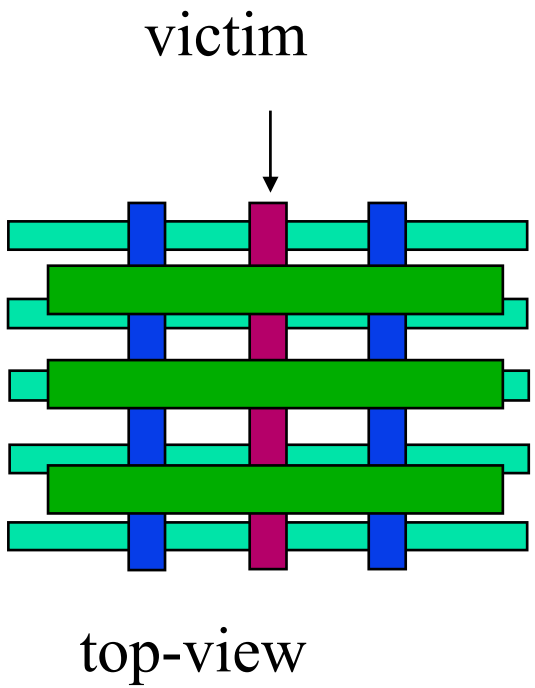


- Simplest model: parallel-plate capacitor
- Two parallel plates and homogeneous dielectric between them
- The capacitance is $C = \varepsilon \frac{A}{d}$
  - ε: permittivity of dielectric
  - A: area of plate
  - d: distance between plates
- The capacitance is the capacity to store charge
  - Total charge at each plate is $Q = CV$
  - One is positive, the other is negative

On-chip interconnects as capacitors

- Conductors: metal wire, via, polysilicon, substrate
- Dielectrics: SiO_2 ,...

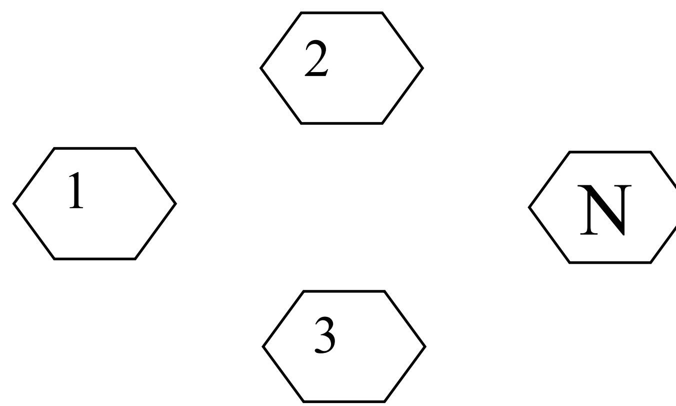




Multiconductor systems

- The presence of charge on any one of the conductors affects the potential of all the others
- Linear proportionality between potential and charge

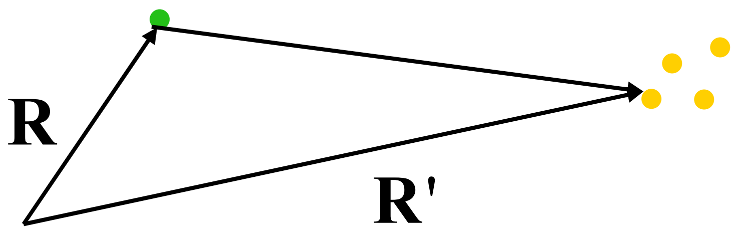
$$
\begin{aligned}
V_1 &= p_{11}Q_1 + p_{12}Q_2 + \cdots + p_{1N}Q_N \\
V_2 &= p_{21}Q_1 + p_{22}Q_2 + \cdots + p_{2N}Q_N \\
&\vdots \\
V_N &= p_{N1}Q_1 + p_{N2}Q_2 + \cdots + p_{NN}Q_N
\end{aligned}
$$



Electric potential

- Electric potential at **R** due to n charged bodies q_1, …, q_n, located at $\mathbf{R}'_1$, …, $\mathbf{R}'_n$

$$V = \frac{1}{4\pi\varepsilon}\sum_{k=1}^{n}\frac{q_k}{\left|\mathbf{R}-\mathbf{R}_k^{'}\right|}$$



- Electric potential due to a continuous distribution of charge confined in a given region:

$$V = \frac{1}{4\pi\varepsilon}\int_{V'}\frac{\rho}{R}dv' \qquad V = \frac{1}{4\pi\varepsilon}\int_{S'}\frac{\rho_s}{R}ds' \qquad V = \frac{1}{4\pi\varepsilon}\int_{L'}\frac{\rho_l}{R}dl'$$

volume   surface   line

(Partial) Capacitance matrix

- Inverting the system of linear equations

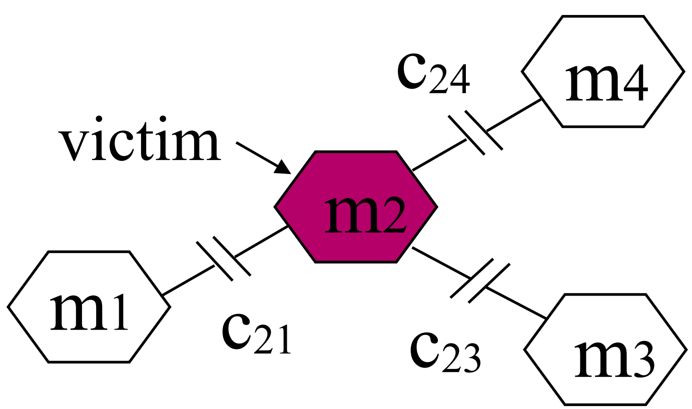
$$
\begin{aligned}
V_1 &= p_{11}Q_1 + p_{12}Q_2 + \cdots + p_{1N}Q_N \\
V_2 &= p_{21}Q_1 + p_{22}Q_2 + \cdots + p_{2N}Q_N \\
&\vdots \\
V_N &= p_{N1}Q_1 + p_{N2}Q_2 + \cdots + p_{NN}Q_N
\end{aligned}
\quad \Rightarrow \quad
\begin{aligned}
Q_1 &= c_{11}V_1 + c_{12}V_2 + \cdots + c_{1N}V_N \\
Q_2 &= c_{21}V_1 + c_{22}V_2 + \cdots + c_{2N}V_N \\
&\vdots \\
Q_N &= c_{N1}V_1 + c_{N2}V_2 + \cdots + c_{NN}V_N
\end{aligned}
$$

- The c_{ii}'s are coefficients of capacitance (self capacitance)
- The c_{ij}'s ($i \neq j$) are coefficients of induction (coupling capacitance)

$O(n^3)$ for Gaussian elimination, $O(n^{2.807})$ [Strassen 1969], $O(n^{2.376})$ [Coppersmith and Winograd 1990]

Framework for numerical method

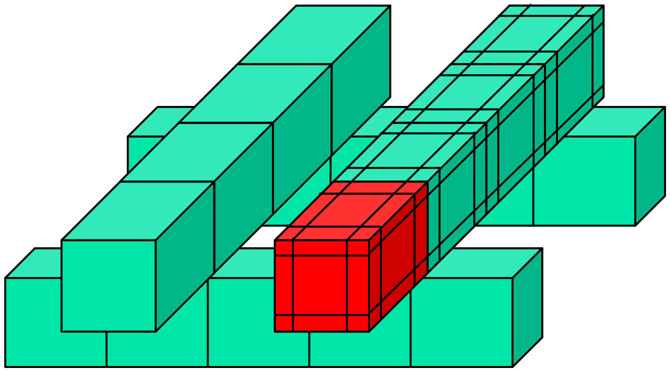
1. Assume voltage [0, 0, …, 1, …, 0], i.e., only conductor i has unit voltage
2. Compute charge q_j for every conductor j
3. Obtain mutual cap $c_{ij} = q_j$, and total cap $c_{ii} = q_i$
4. Iterate through steps 1-3 using different voltage assignments



$$V = \begin{matrix} 0 & 1 & 0 & 0 \end{matrix}$$

$$C = \begin{matrix} 0 & c_{12} & 0 & 0 \\ c_{21} & c_{22} & c_{23} & c_{24} \\ 0 & c_{32} & 0 & 0 \\ 0 & c_{42} & 0 & 0 \end{matrix}$$

Integral equation approach



- Discretize the surfaces of *m* conductors into a total of *n* panels
- Write down potential coefficient matrix and linear system $Pq = v$

$$P_{ij} = \frac{1}{\text{area}(A_i)} \times \int_{x_i \in A_i} \frac{1}{\text{area}(A_j)} \int_{x_j \in A_j} \frac{1}{4\pi\epsilon_0 \|x_i - x_j\|} da_j da_i$$

- Set $v_k = 1$ if panel *k* is on the *j*-th conductor, and $v_k = 0$ otherwise
- Solve for panel charge *q* and sum all the panel charge on the *i*-th conductor $C_{ij} = \Sigma_{k \in \text{conductor } i}\, q_k$

Numerical solution by method of conjugate gradient

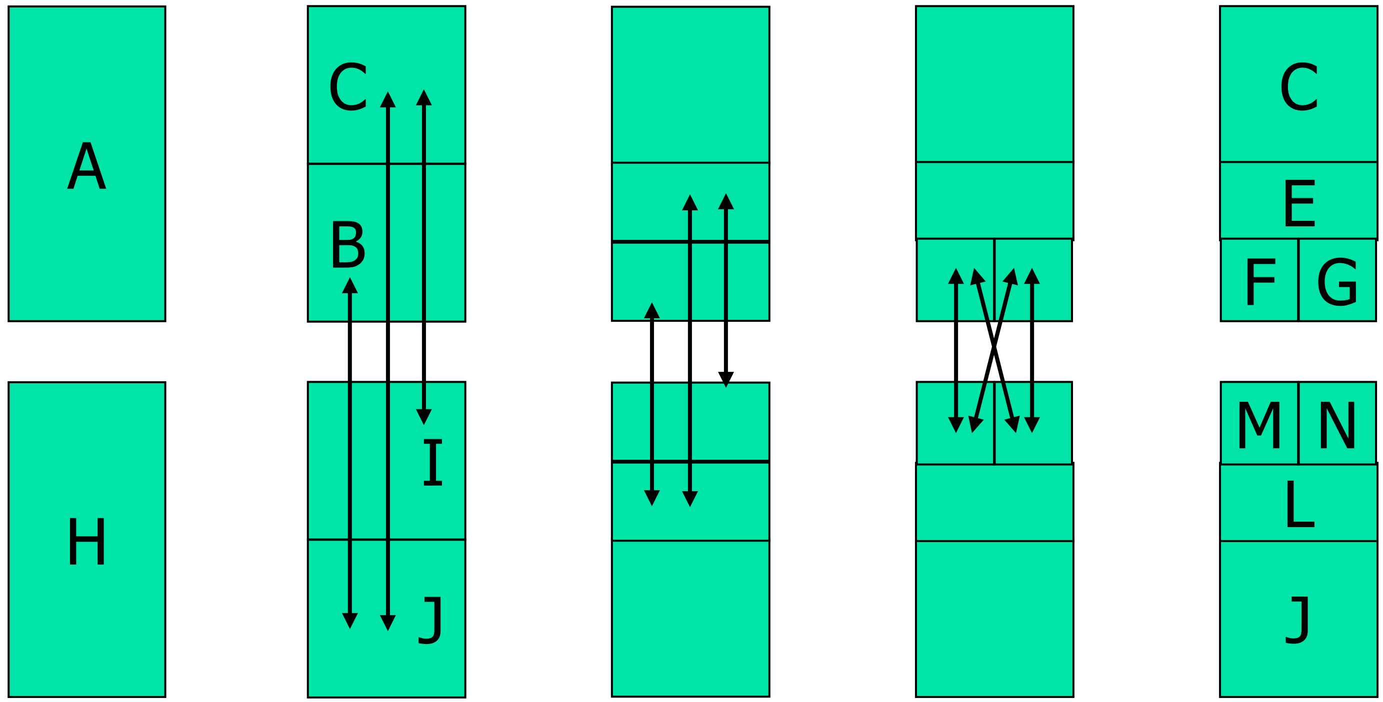
- Initial guess $\mathbf{q}_{(0)} = \mathbf{0}$
- Residual $r_{(i)} = v - Pq_{(i)}$
- Initialize search direction and residual $\mathbf{d}_{(0)} = \mathbf{r}_{(0)} = v$
- Iterate

$$
\begin{aligned}
\alpha_{(i)} &= \frac{\mathbf{r}_{(i)}^T \mathbf{r}_{(i)}}{\mathbf{d}_{(i)}^T \mathbf{P} \mathbf{d}_{(i)}} \\
\mathbf{q}_{(i+1)} &= \mathbf{q}_{(i)} + \alpha_{(i)} \mathbf{d}_{(i)} \\
\mathbf{r}_{(i+1)} &= \mathbf{r}_{(i)} - \alpha_{(i)} \mathbf{P} \mathbf{d}_{(i)} \\
\beta_{(i+1)} &= \frac{\mathbf{r}_{(i+1)}^T \mathbf{r}_{(i+1)}}{\mathbf{r}_{(i)}^T \mathbf{r}_{(i)}} \\
\mathbf{d}_{(i+1)} &= \mathbf{r}_{(i+1)} + \beta_{(i+1)} \mathbf{d}_{(i)}
\end{aligned}
$$

Fast multi-pole method (FMM)

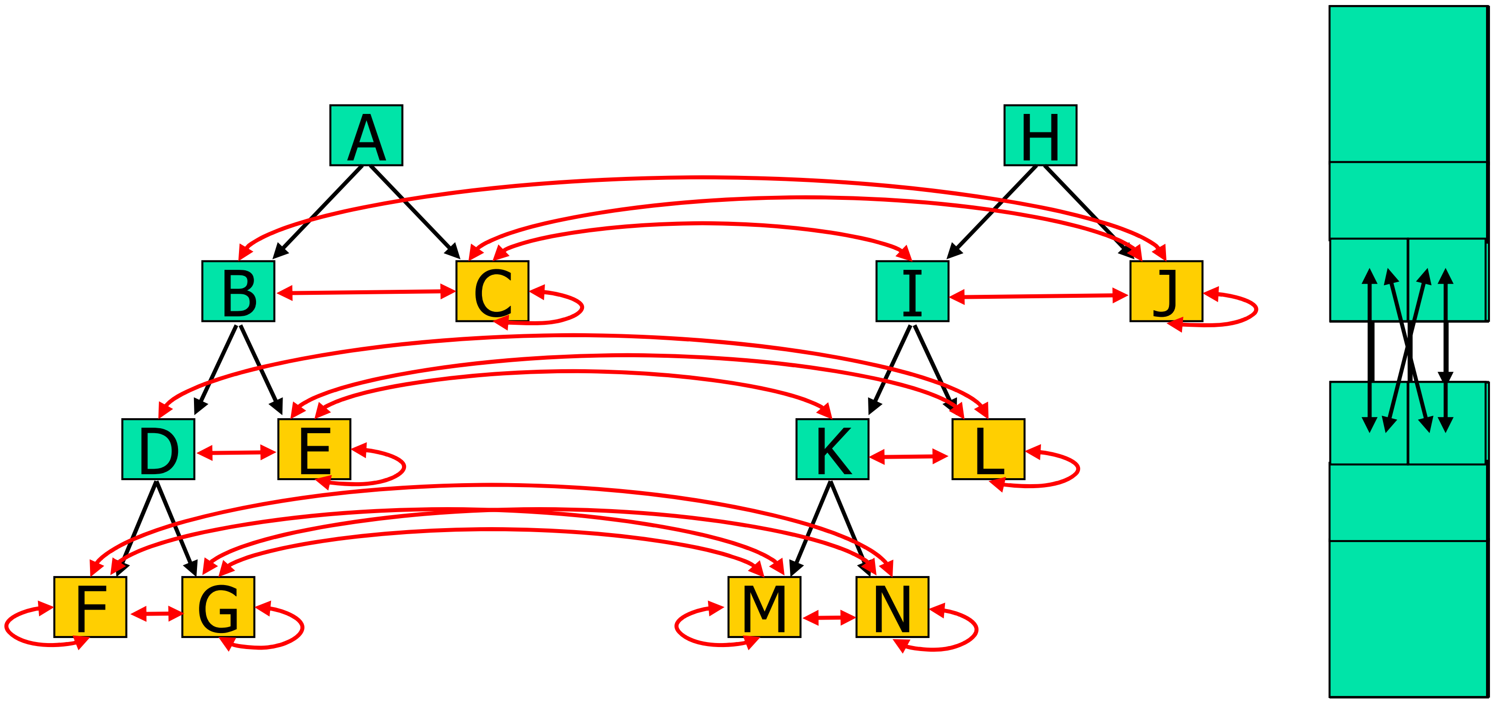
- Bottleneck is in the computation of the potential on all panels, i.e., matrix-vector multiply $Pd_{(i)}$
- Known as the n-body problem, can be solved in O(n) without loss in accuracy (machine precision) using FMM [Greengard-Rohklin] and [Appel]
- Basic idea: Potential due to a cluster of particles at some distance can be approximated with a single term
- FastCap from MIT [Nabors-White] and hierarchical capacitance extraction from Texas-A&M [Shi et al] used different forms of FMM in the iterative solvers

Hierarchical partitioning



- R_i = longest dimension of panel i
- If $P_\varepsilon < P_{ij} \times R_i$ ($\approx R_i / r_{ij}$), subdivide the panel
- Record potential coefficient between two panels otherwise

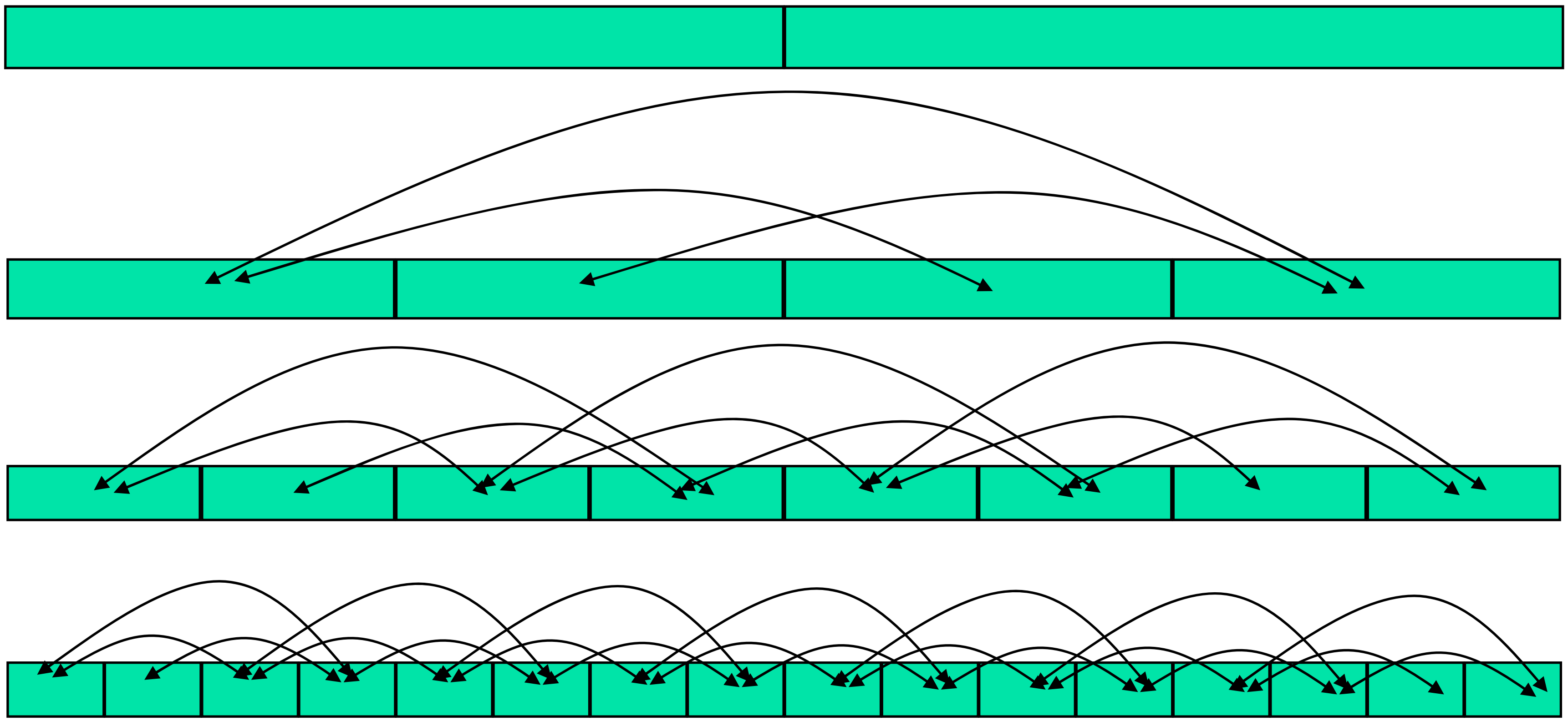
Binary trees with cross links



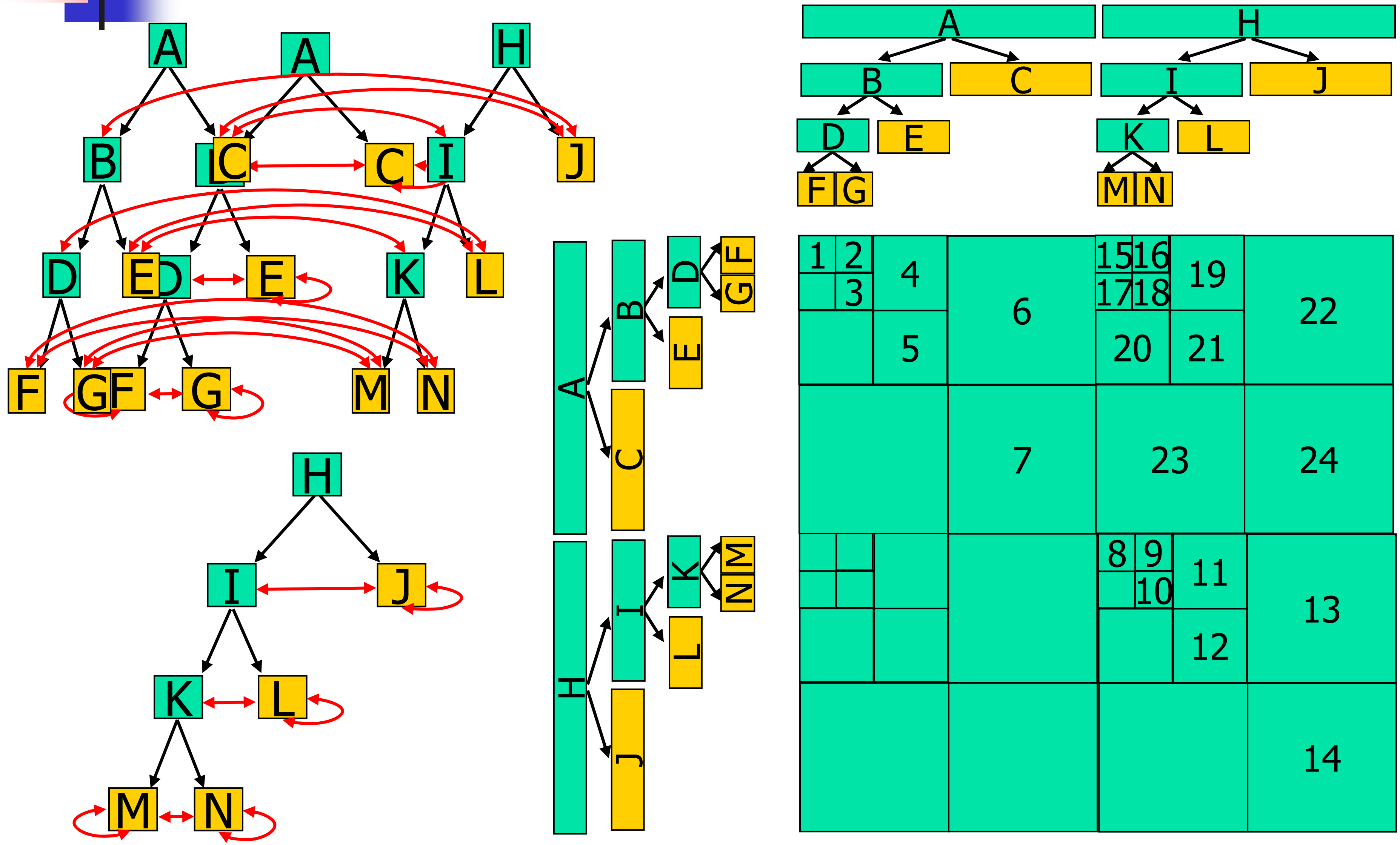
- Cross links record the potential coefficient between two panels

O(1) number of cross links per node

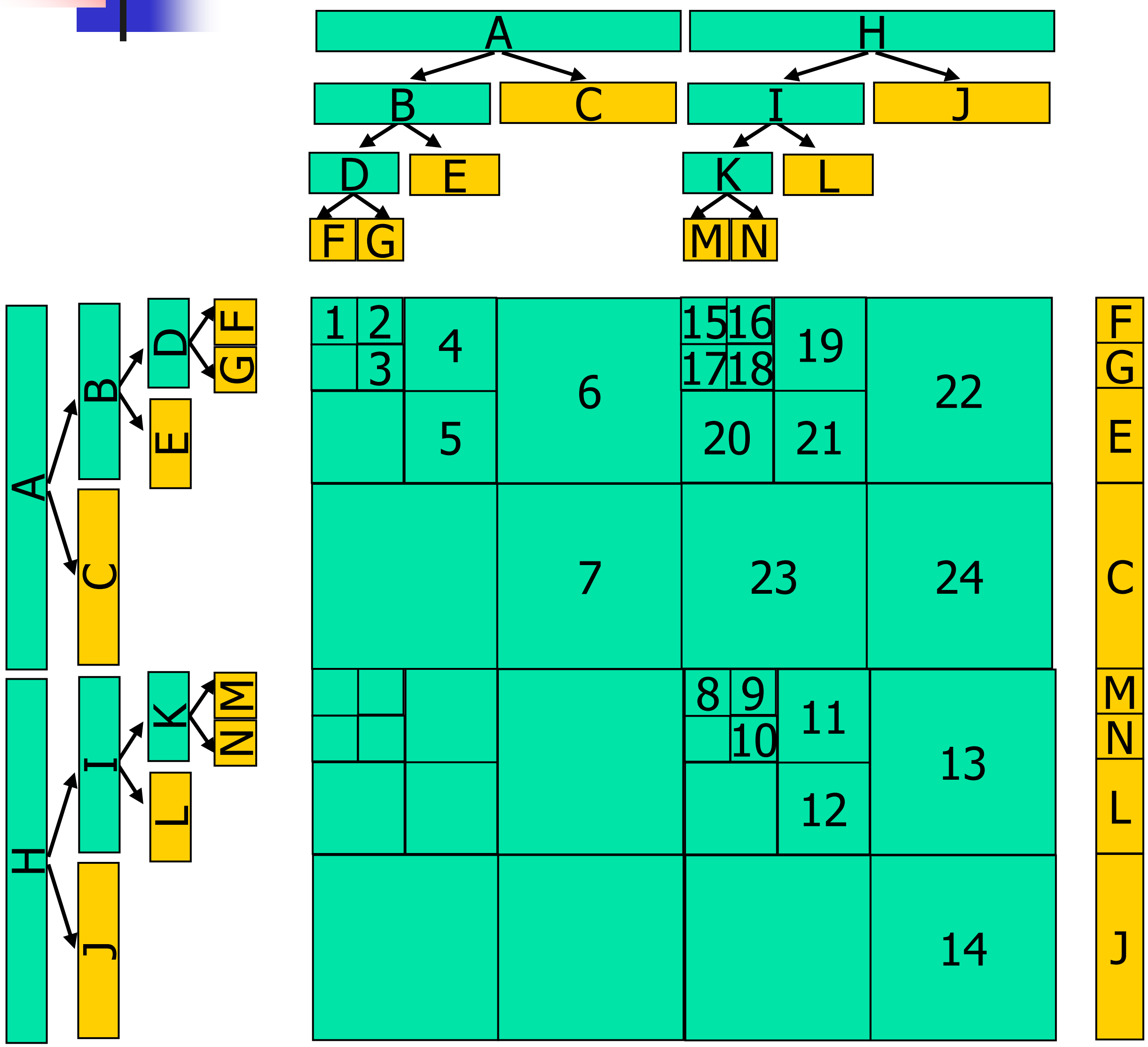
- Two panels interact directly only if $r_{ij} > R_i \times P_\varepsilon$, i.e., there are k panels between them
- Otherwise, parent nodes would have interacted



Equivalent potential coefficient matrix



Matrix-vector multiply: Pq = v

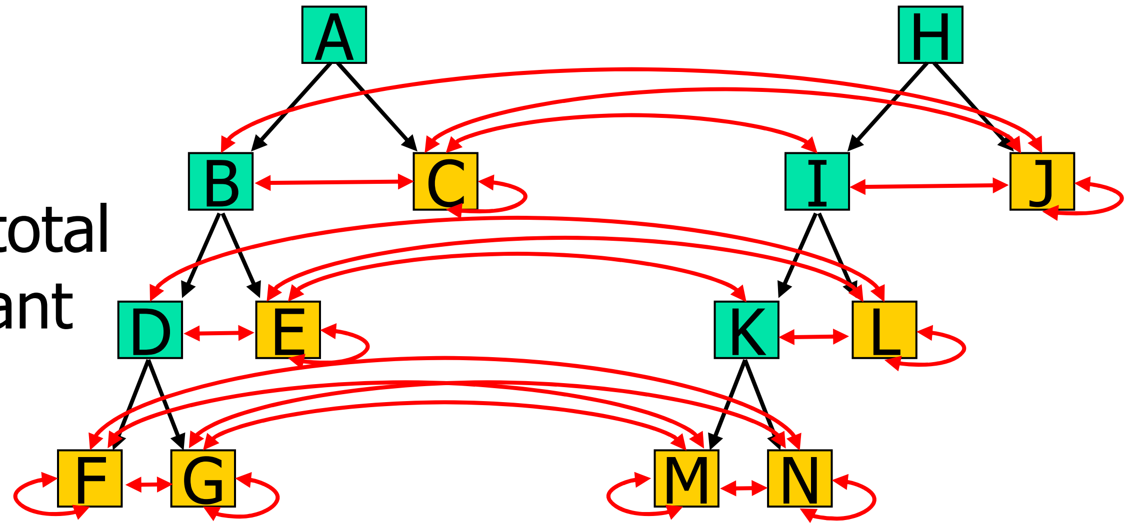


C.potential

$= P_{CB}$F.charge + P_{CB}G.charge + P_{CB}E.charge + P_{CC}C.charge + P_{CI}M.charge + P_{CI}N.charge + P_{CI}L.charge + P_{CJ}J.charge

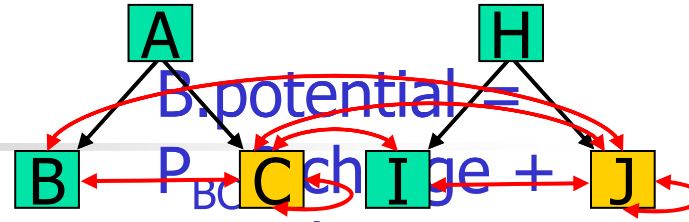
$= P_{CB}$B.charge + P_{CC}C.charge + P_{CI}I.charge + P_{CJ}J.charge

O(n) matrix-vector multiply

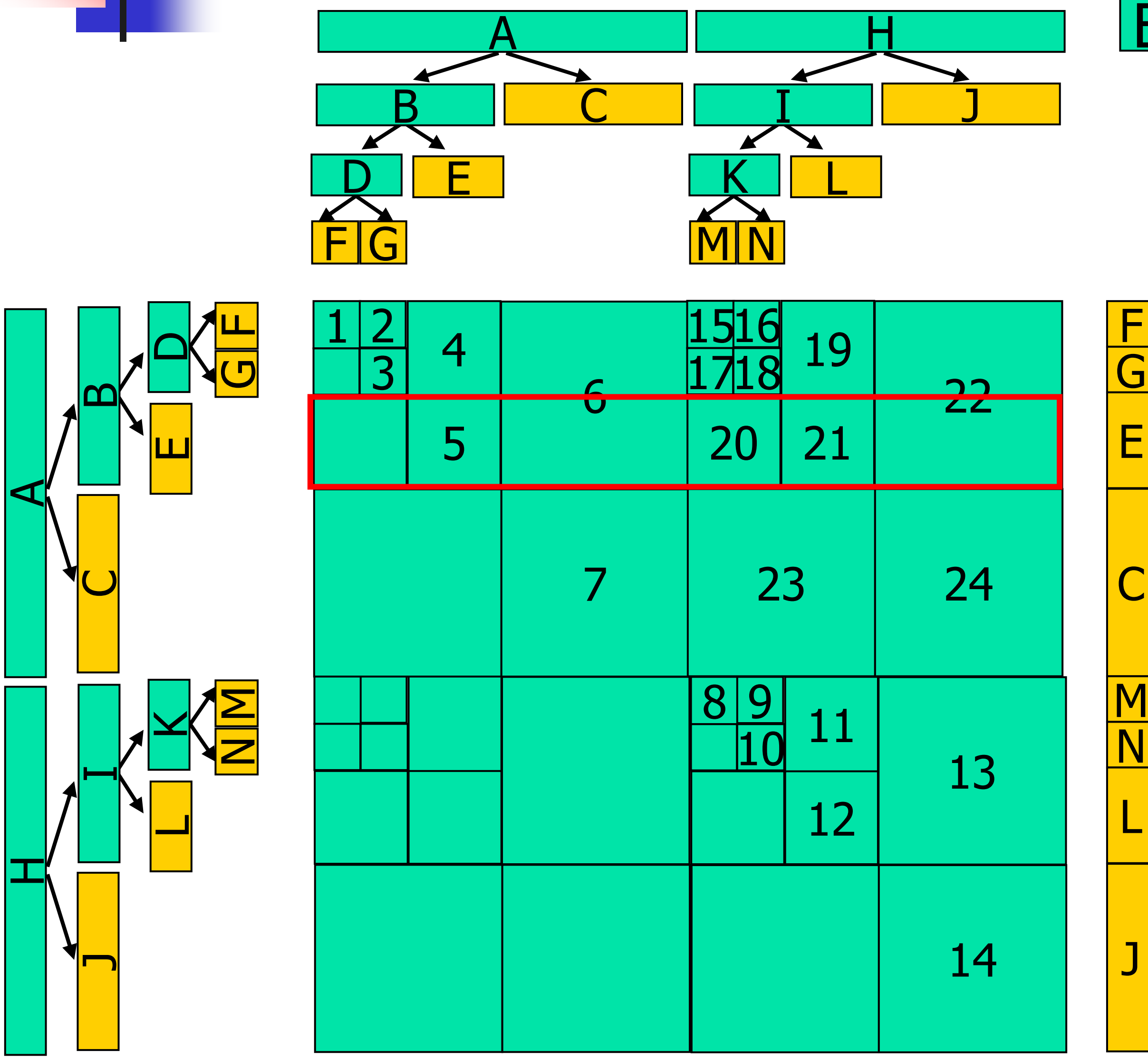


- Collect charge
  - Charge at node = total charge at descendant leaf nodes ($d_{(i)}$)
- Bottom-up traversal of trees, propagate charge at leaf nodes upwards
  - D.charge = F.charge + G.charge = $d_{(i),F} + d_{(i),G}$
  - B.charge = D.charge + E.charge = D.charge + $d_{(i),E}$
  - A.charge = B.charge + C.charge = B.charge + $d_{(i),C}$

Matrix-vector multiply



B.potential = P_{BC}C.charge + P_{BJ}J.charge



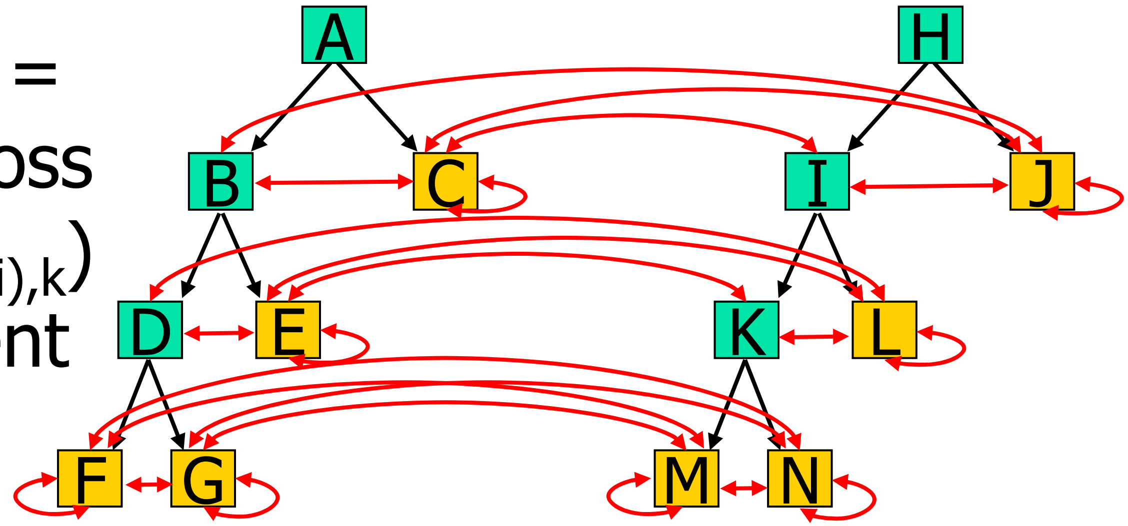
E.potential

= P_{ED}D.charge +
P_{EE}E.charge +
P_{BC}C.charge +
P_{EK}K.charge +
P_{EL}L.charge +
P_{BJ}J.charge

= B.potential +
P_{ED}D.charge +
P_{EE}E.charge +
P_{EK}K.charge +
P_{EL}L.charge

O(n) matrix-vector multiply

- Potential at node j = potential due to cross links at node ($P_{jk}d_{(i),k}$) + potential of parent



- Top-down traversal of trees, propagate potential at root nodes downwards
  - A.potential = 0
  - B.potential = A.potential + P_{BC}C.charge + P_{BJ}J.charge
  - D.potential = B.potential + P_{DE}E.charge + P_{DL}L.charge
  - F.potential = D.potential + $P_{F\{FGMN\}}$\{FGMN\}.charge

Summary

- For *n* panels, *O(n)* fast matrix-vector multiply based on FMM
- Tree structures with cross links to facilitate *O(n)* matrix-vector multiply by bottom-up and top-down tree traversals
- *O(kn)* time complexity for each solve of conductor charge, *k* being the number of iterations
- Assume *m* conductors with *n* panels, overall complexity is *O(kmn)*

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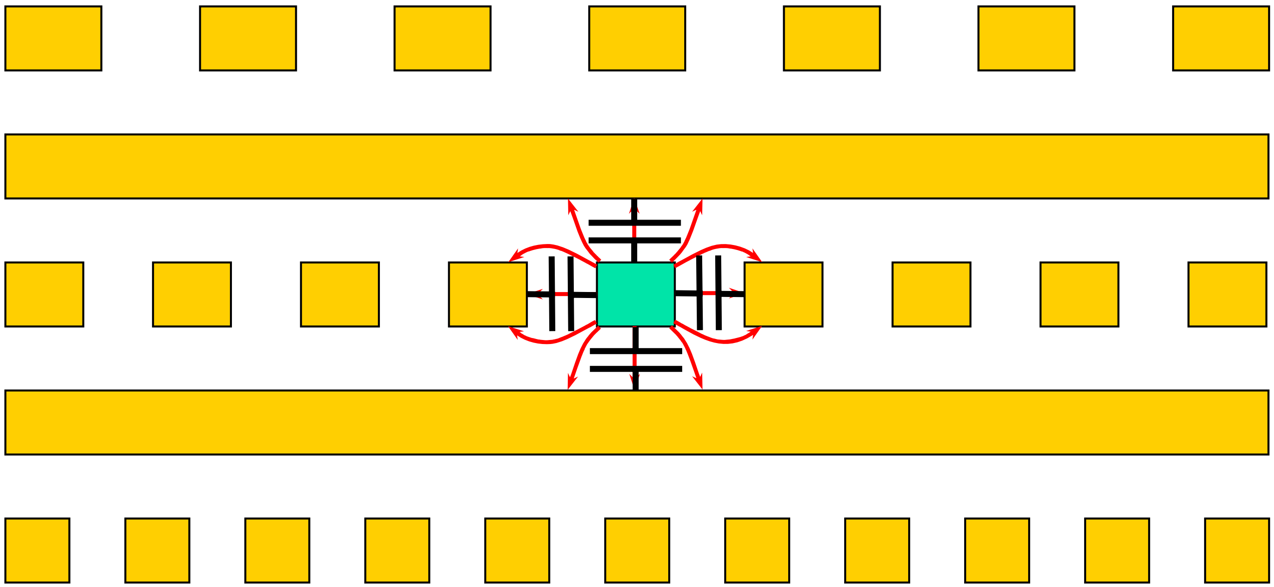
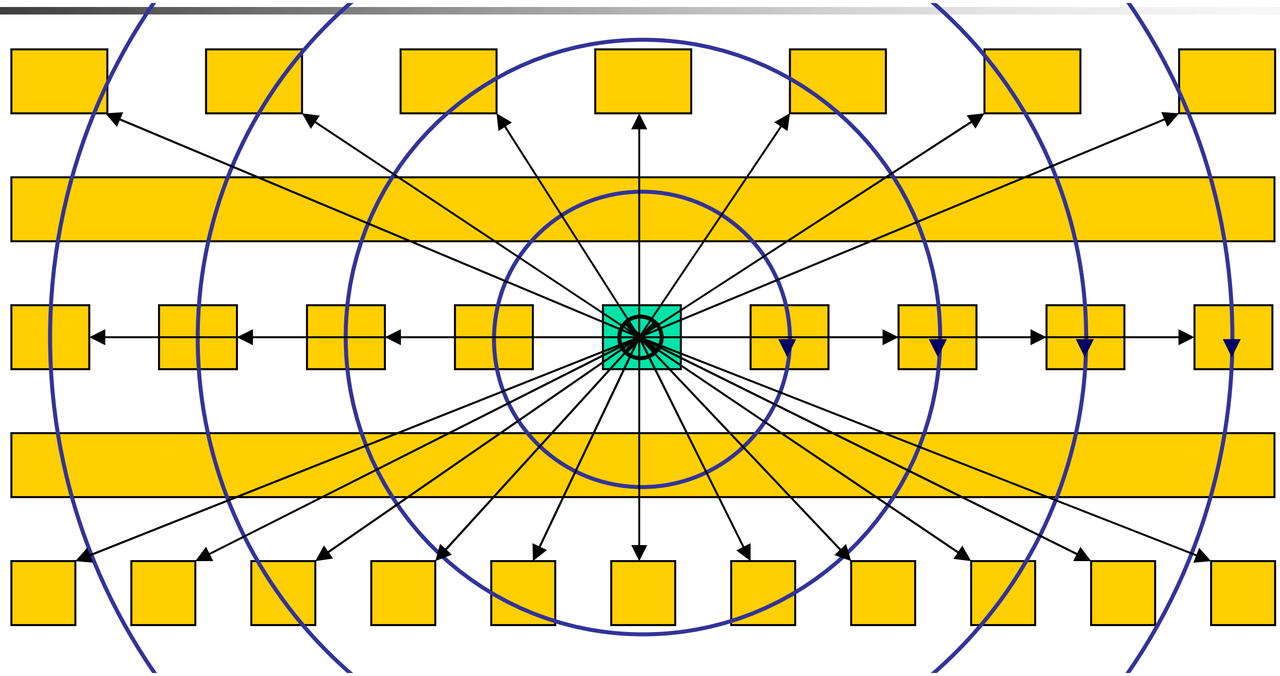
Capacitive Coupling Property



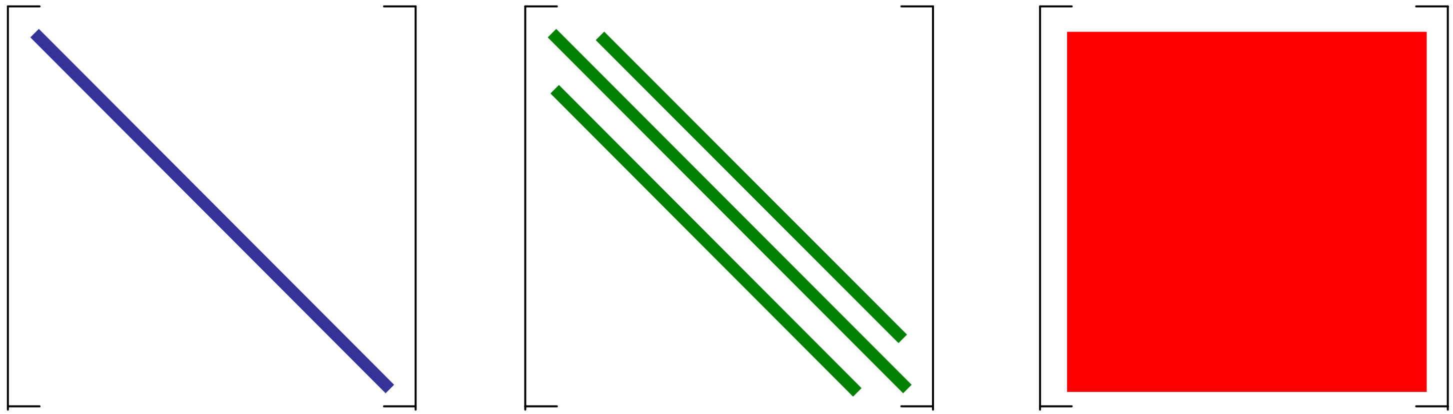
- Coupling capacitance
  - Limited to adjacent neighbors
  - Limited to adjacent layers

Inductive Coupling Property



- Mutual Inductance
  - Not limited to adjacent wires
  - Not limited to adjacent layers
  - Exists among all parallel wires

RLC Matrices

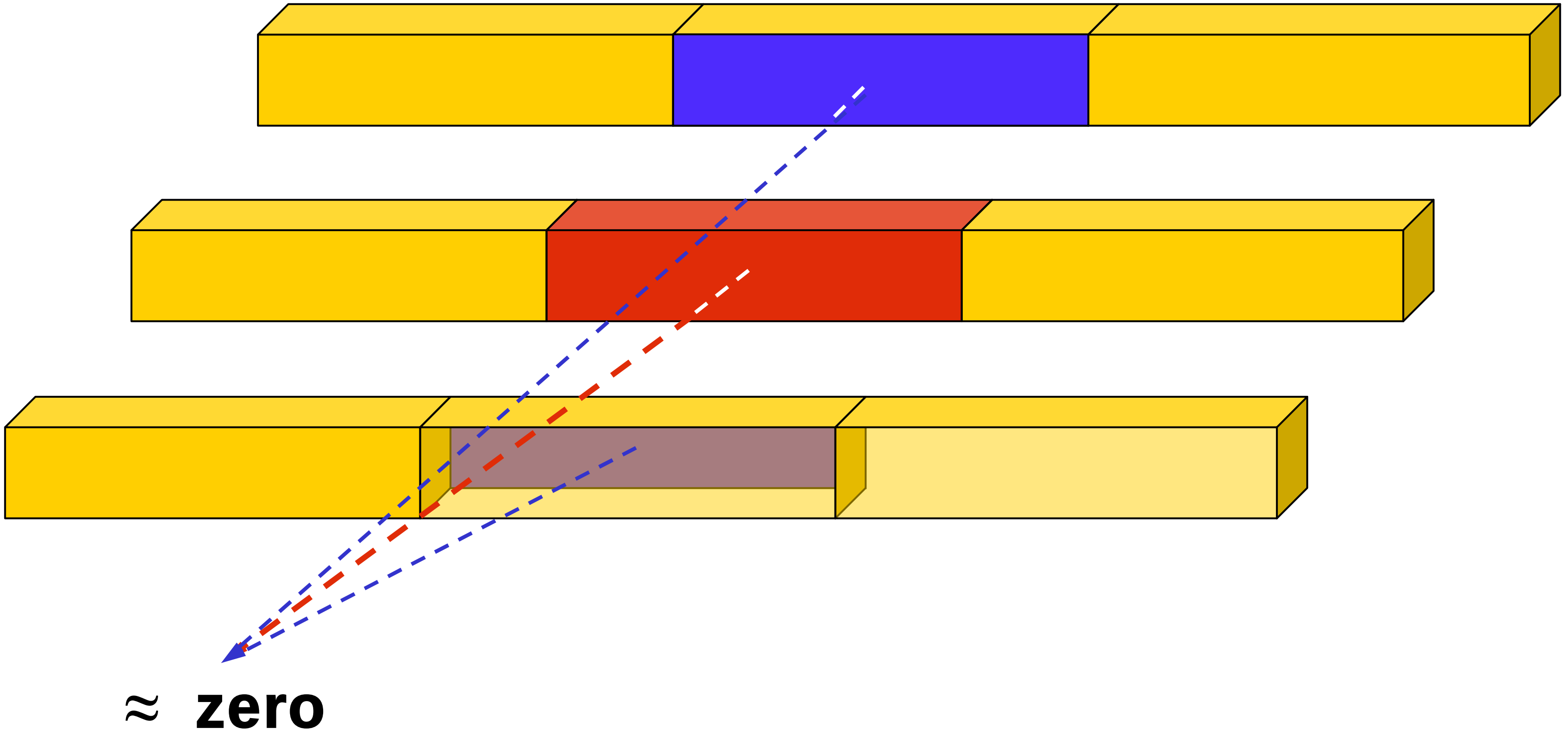


R is diagonal C is tridiagonal L is FULL

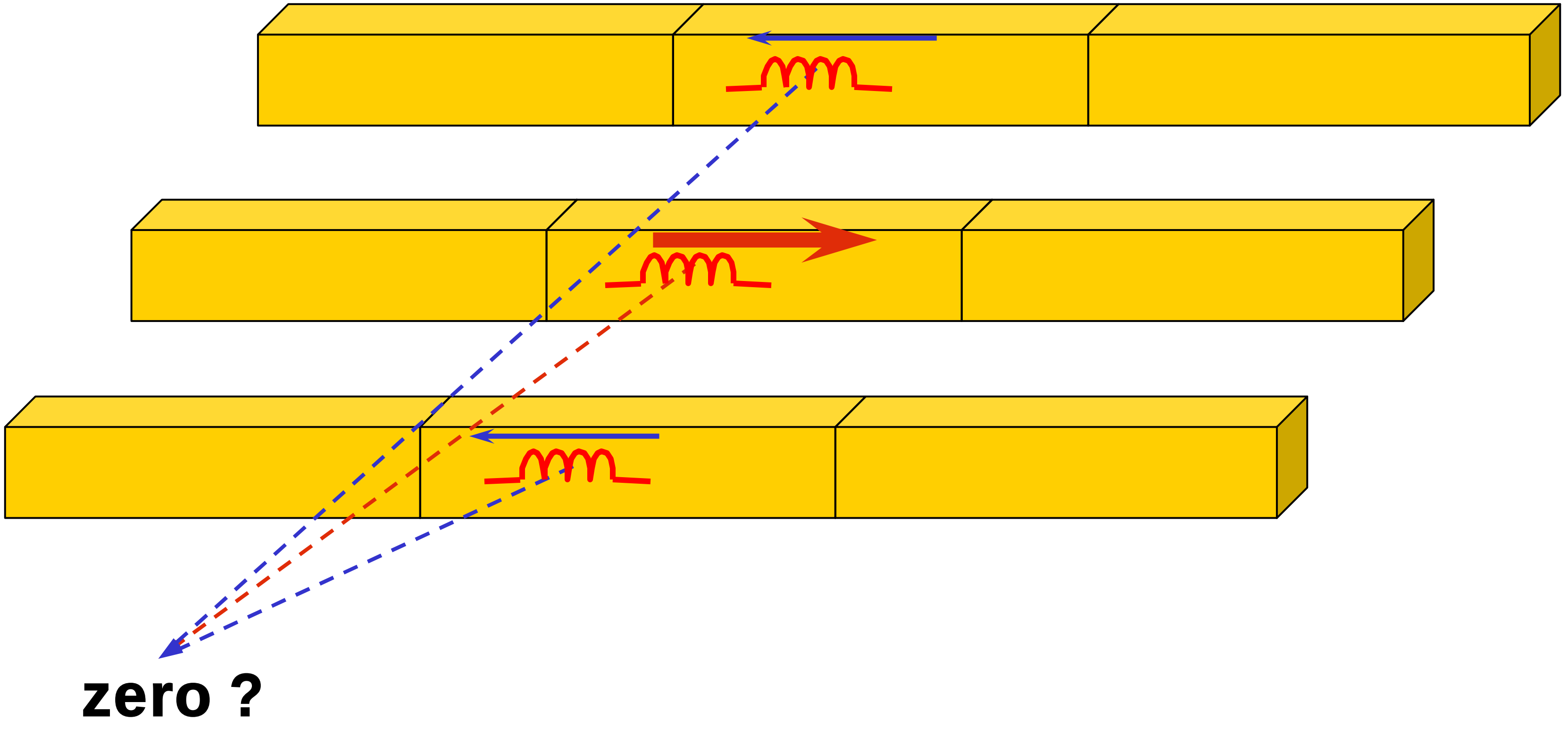
Questions

- Is there sparsity pattern related with L?
- How can we exploit such sparsity pattern in the simulation of RLC on-chip interconnects?

Locality of Capacitive Coupling



Is Inductive Coupling Localized?



Sparsity Resident in L^{-1}

$$L = 10^{-11} \times \begin{bmatrix} 10.80 & 8.51 & 7.22 & 6.45 & 5.90 & 5.47 & 5.13 \\ 8.51 & 10.80 & 8.51 & 7.22 & 6.45 & 5.90 & 5.47 \\ 7.22 & 8.51 & 10.80 & 8.51 & 7.22 & 6.45 & 5.90 \\ 6.45 & 7.22 & 8.51 & 10.80 & 8.51 & 7.22 & 6.45 \\ 5.90 & 6.45 & 7.22 & 8.51 & 10.80 & 8.51 & 7.22 \\ 5.47 & 5.90 & 6.45 & 7.22 & 8.51 & 10.80 & 8.51 \\ 5.13 & 5.47 & 5.90 & 6.45 & 7.22 & 8.51 & 10.80 \end{bmatrix}$$

$$L^{-1} = 10^{10} \times \begin{bmatrix} 2.53 & -1.67 & -0.12 & -0.12 & -0.08 & -0.05 & -0.11 \\ -1.67 & 3.63 & -1.60 & -0.04 & -0.07 & -0.04 & -0.05 \\ -0.12 & -1.60 & 3.64 & -1.59 & -0.04 & -0.07 & -0.08 \\ -0.12 & -0.04 & -1.59 & 3.64 & -1.59 & -0.04 & -0.12 \\ -0.08 & -0.07 & -0.04 & -1.59 & 3.64 & -1.60 & -0.12 \\ -0.05 & -0.04 & -0.07 & -0.04 & -1.60 & 3.63 & -1.67 \\ -0.11 & -0.05 & -0.08 & -0.12 & -0.12 & -1.67 & 2.53 \end{bmatrix}$$

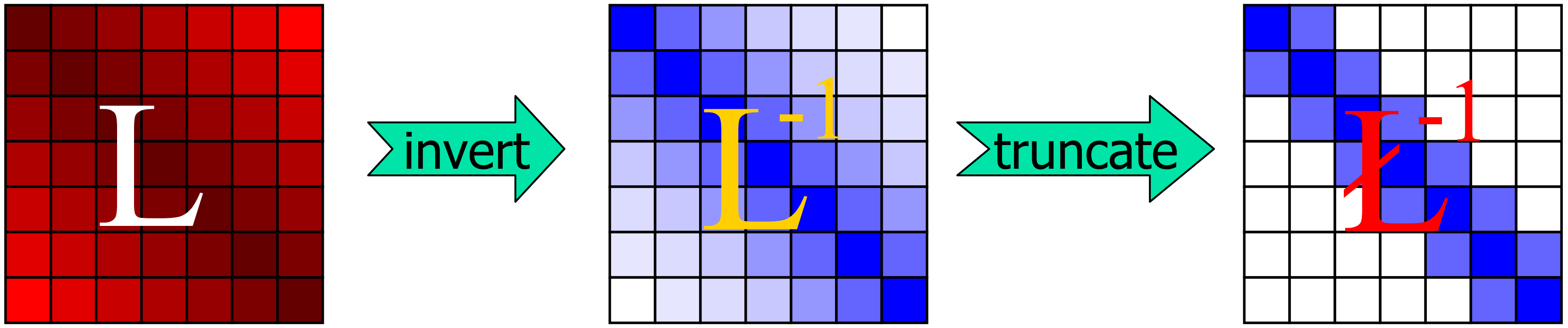
Why sparsity in L^{-1}?

$$M = \frac{\mu}{4\pi A_i A_j}\int_{A_i}\int_{A_j}\int_{L_i}\int_{L_j}\frac{dl_i \cdot dl_j}{R}\,da_j\,da_i$$

Similar to scalar potential coefficients encountered in capacitance extraction

$$P_{ij} = \frac{1}{\text{area}(A_i)} \times \int_{x_i \in A_i} \frac{1}{\text{area}(A_j)} \int_{x_j \in A_j} \frac{1}{4\pi\epsilon_0 \|x_i - x_j\|}\,da_j\,da_i$$

L^{-1} matrix suggests locality



[A. Devgan et al. (ICCAD 2000)]

Direct Truncation

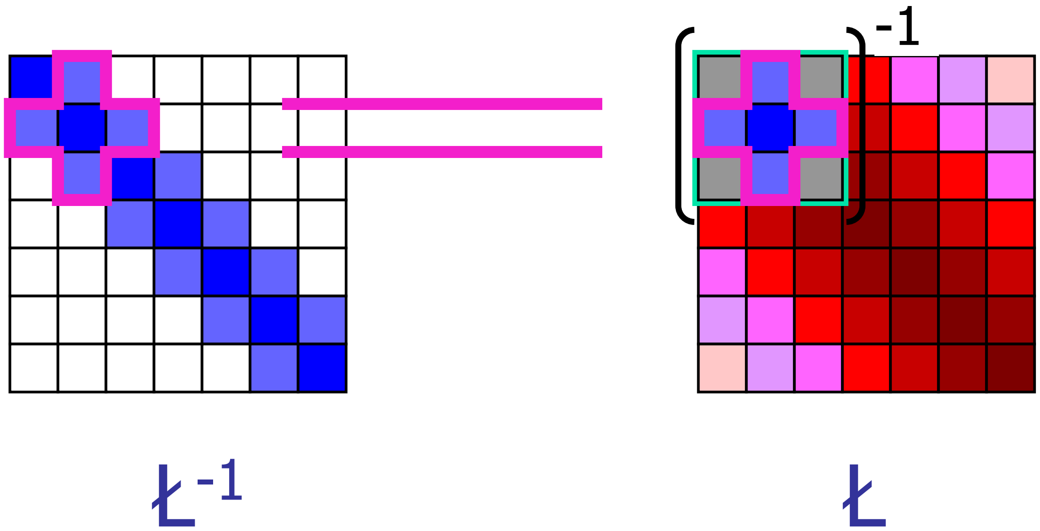
$$L = 10^{-11} \times$$

$$\begin{bmatrix} 10.80 & 8.51 & 7.22 & 6.45 & 5.90 & 5.47 & 5.13 \\ 8.51 & 10.80 & 8.51 & 7.22 & 6.45 & 5.90 & 5.47 \\ 7.22 & 8.51 & 10.80 & 8.51 & 7.22 & 6.45 & 5.90 \\ 6.45 & 7.22 & 8.51 & 10.80 & 8.51 & 7.22 & 6.45 \\ 5.90 & 6.45 & 7.22 & 8.51 & 10.80 & 8.51 & 7.22 \\ 5.47 & 5.90 & 6.45 & 7.22 & 8.51 & 10.80 & 8.51 \\ 5.13 & 5.47 & 5.90 & 6.45 & 7.22 & 8.51 & 10.80 \end{bmatrix}$$

$$Ł^{-1} = 10^{10} \times$$

$$\begin{bmatrix} 2.53 & -1.67 & 0 & 0 & 0 & 0 & 0 \\ -1.67 & 3.63 & -1.60 & 0 & 0 & 0 & 0 \\ 0 & -1.60 & 3.64 & -1.59 & 0 & 0 & 0 \\ 0 & 0 & -1.59 & 3.64 & -1.59 & 0 & 0 \\ 0 & 0 & 0 & -1.59 & 3.64 & -1.60 & 0 \\ 0 & 0 & 0 & 0 & -1.60 & 3.63 & -1.67 \\ 0 & 0 & 0 & 0 & 0 & -1.67 & 2.53 \end{bmatrix}$$

Ł Matrix Contains Redundancy



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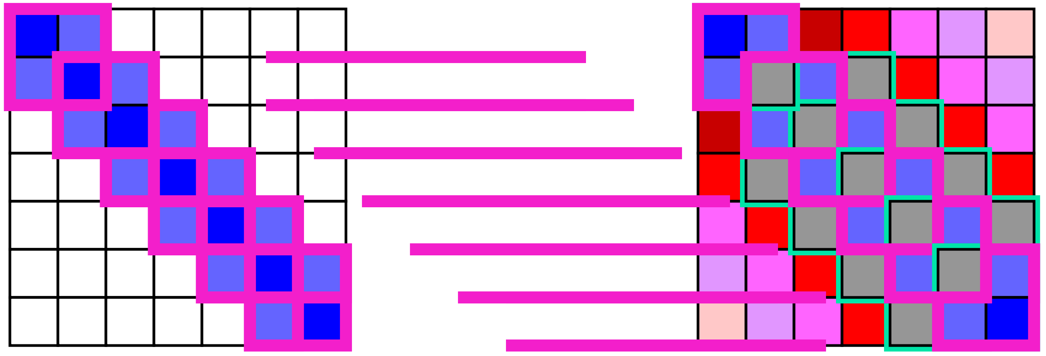

Numerical Example

$$\begin{bmatrix} 1 & -0.5 & & & & & \\ -0.5 & 1 & -0.5 & & & & \\ & -0.5 & 1 & -0.5 & & & \\ & & -0.5 & 1 & -0.5 & & \\ & & & -0.5 & 1 & -0.5 & \\ & & & & -0.5 & 1 & -0.5 \\ & & & & & -0.5 & 1 \end{bmatrix}^{-1} = \begin{bmatrix} 1.75 & 1.50 & 1.25 & 1.00 & 0.75 & 0.50 & 0.25 \\ 1.50 & 3.00 & 2.50 & 2.00 & 1.50 & 1.00 & 0.50 \\ 1.25 & 2.50 & 3.75 & 3.00 & 2.25 & 1.50 & 0.75 \\ 1.00 & 2.00 & 3.00 & 4.00 & 3.00 & 2.00 & 1.00 \\ 0.75 & 1.50 & 2.25 & 3.00 & 3.75 & 2.50 & 1.25 \\ 0.50 & 1.00 & 1.50 & 2.00 & 2.50 & 3.00 & 1.50 \\ 0.25 & 0.50 & 0.75 & 1.00 & 1.25 & 1.50 & 1.75 \end{bmatrix}$$

$$\begin{bmatrix} 1.75 & 1.50 \\ 1.50 & 3.00 \end{bmatrix}^{-1} = \frac{1}{1.75*3-1.5^2} \begin{bmatrix} 3.00 & -1.5 \\ -1.5 & 1.75 \end{bmatrix} = \begin{bmatrix} 1 & -0.5 \\ -0.5 & 0.58 \end{bmatrix}$$

Numerical Example

$$\begin{bmatrix} 1 & -0.5 & & & & & \\ -0.5 & 1 & -0.5 & & & & \\ & -0.5 & 1 & -0.5 & & & \\ & & -0.5 & 1 & -0.5 & & \\ & & & -0.5 & 1 & -0.5 & \\ & & & & -0.5 & 1 & -0.5 \\ & & & & & -0.5 & 1 \end{bmatrix}^{-1} = \begin{bmatrix} 1.75 & 1.50 & 1.25 & 1.00 & 0.75 & 0.50 & 0.25 \\ 1.50 & 3.00 & 2.50 & 2.00 & 1.50 & 1.00 & 0.50 \\ 1.25 & 2.50 & 3.75 & 3.00 & 2.25 & 1.50 & 0.75 \\ 1.00 & 2.00 & 3.00 & 4.00 & 3.00 & 2.00 & 1.00 \\ 0.75 & 1.50 & 2.25 & 3.00 & 3.75 & 2.50 & 1.25 \\ 0.50 & 1.00 & 1.50 & 2.00 & 2.50 & 3.00 & 1.50 \\ 0.25 & 0.50 & 0.75 & 1.00 & 1.25 & 1.50 & 1.75 \end{bmatrix}$$

$$\begin{bmatrix} 3.00 & 2.50 & 2.00 \\ 2.50 & 3.75 & 3.00 \\ 2.00 & 3.00 & 4.00 \end{bmatrix}^{-1} = \begin{bmatrix} 0.75 & -0.5 & \\ -0.5 & 1 & -0.5 \\ & -0.5 & 0.62 \end{bmatrix}$$

Windowing Technique

$$L = 10^{-11} \times \begin{bmatrix} 10.80 & 8.51 & 7.22 & 6.45 & 5.90 & 5.47 & 5.13 \\ 8.51 & 10.80 & 8.51 & 7.22 & 6.45 & 5.90 & 5.47 \\ 7.22 & 8.51 & 10.80 & 8.51 & 7.22 & 6.45 & 5.90 \\ 6.45 & 7.22 & 8.51 & 10.80 & 8.51 & 7.22 & 6.45 \\ 5.90 & 6.45 & 7.22 & 8.51 & 10.80 & 8.51 & 7.22 \\ 5.47 & 5.90 & 6.45 & 7.22 & 8.51 & 10.80 & 8.51 \\ 5.13 & 5.47 & 5.90 & 6.45 & 7.22 & 8.51 & 10.80 \end{bmatrix}$$

$$L(1:2,1:2)^{-1} = 10^{10} \times \begin{bmatrix} \mathbf{2.44} & \boxed{\mathbf{-1.92}} \\ \boxed{\mathbf{-1.92}} & 2.44 \end{bmatrix}$$

$$L(1:3,1:3)^{-1} = 10^{10} \times \begin{bmatrix} 2.48 & \boxed{\mathbf{-1.70}} & -0.31 \\ \boxed{\mathbf{-1.70}} & \mathbf{3.62} & \mathbf{-1.70} \\ -0.31 & \mathbf{-1.70} & 2.48 \end{bmatrix}$$

Windowing Technique

$$L = 10^{-11} \times \begin{bmatrix} 10.80 & 8.51 & 7.22 & 6.45 & 5.90 & 5.47 & 5.13 \\ 8.51 & 10.80 & 8.51 & 7.22 & 6.45 & 5.90 & 5.47 \\ 7.22 & 8.51 & 10.80 & 8.51 & 7.22 & 6.45 & 5.90 \\ 6.45 & 7.22 & 8.51 & 10.80 & 8.51 & 7.22 & 6.45 \\ 5.90 & 6.45 & 7.22 & 8.51 & 10.80 & 8.51 & 7.22 \\ 5.47 & 5.90 & 6.45 & 7.22 & 8.51 & 10.80 & 8.51 \\ 5.13 & 5.47 & 5.90 & 6.45 & 7.22 & 8.51 & 10.80 \end{bmatrix}$$

$$\hat{L}^{-1} = 10^{10} \times \begin{bmatrix} 2.44 & \mathbf{-1.70} & 0 & 0 & 0 & 0 & 0 \\ \mathbf{-1.70} & 3.62 & \mathbf{-1.70} & 0 & 0 & 0 & 0 \\ 0 & \mathbf{-1.70} & 3.62 & \mathbf{-1.70} & 0 & 0 & 0 \\ 0 & 0 & \mathbf{-1.70} & 3.62 & \mathbf{-1.70} & 0 & 0 \\ 0 & 0 & 0 & \mathbf{-1.70} & 3.62 & \mathbf{-1.70} & 0 \\ 0 & 0 & 0 & 0 & \mathbf{-1.70} & 3.62 & \mathbf{-1.70} \\ 0 & 0 & 0 & 0 & 0 & \mathbf{-1.70} & 2.44 \end{bmatrix}$$

[T. H. Chen et al. (ICCAD 2002)]