Thermoelectricity: From Atoms to Systems

Week 4: Thermoelectric Systems

Lecture 4.4: Graded materials, TE leg geometry impact

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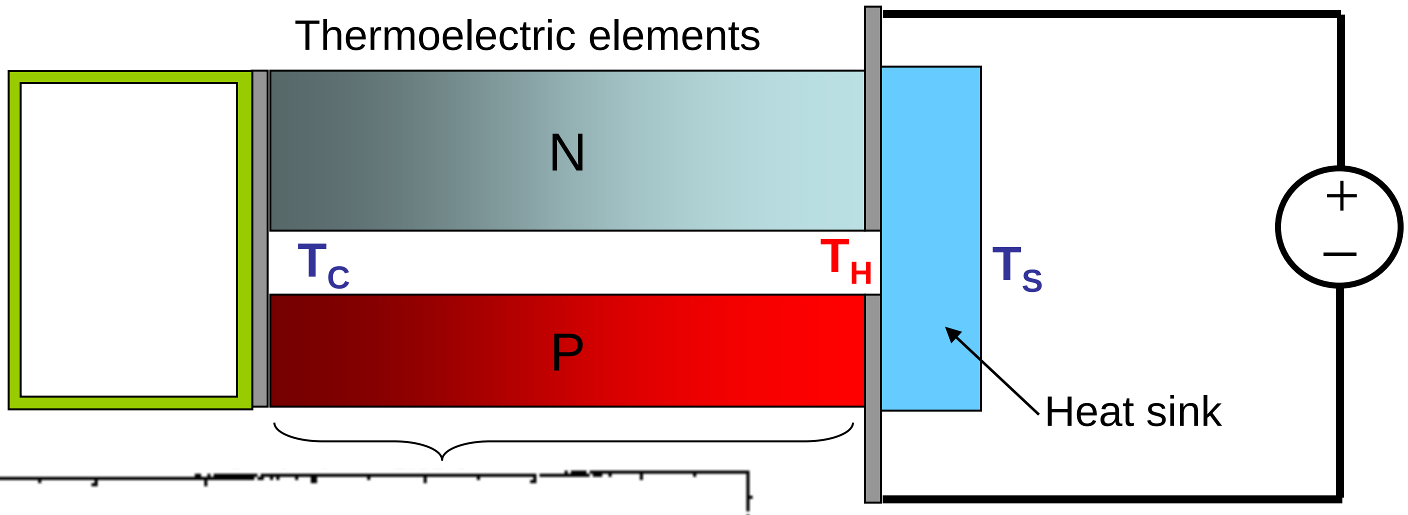
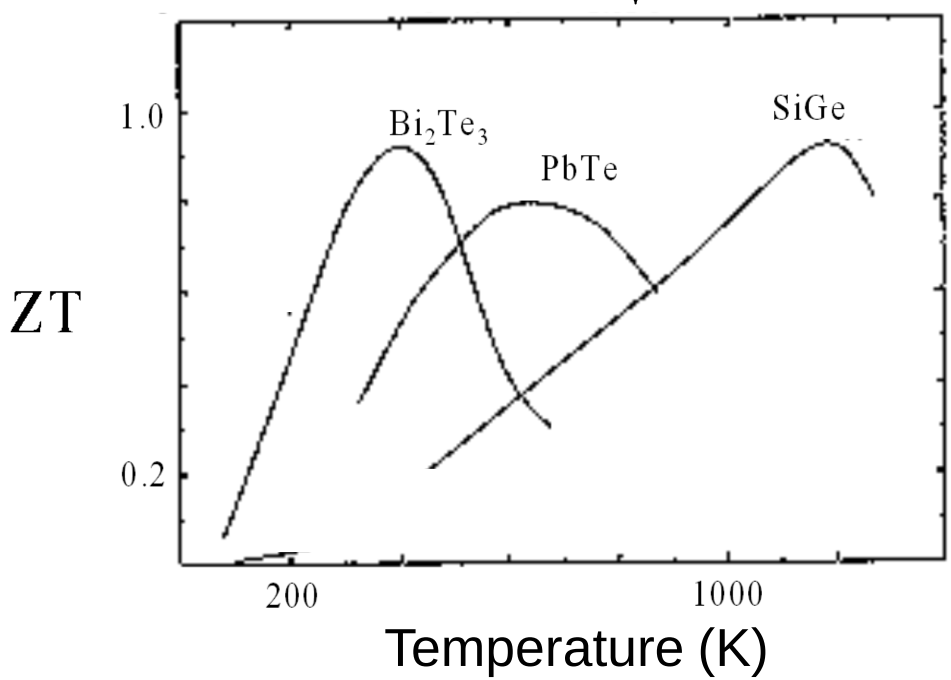
Conventional Functionally Graded or Segmented Thermoelectric Materials





- Optimize material at each location to have highest ZT_{local} at that location.
- Typically for power generation applications (large ΔT)

ZT of Composite Thermoelectrics

Thermoelectric properties of a composite medium

David J. Bergman and Ohad Levy

We study the thermoelectric properties of a composite medium. … We prove that $Z_{effective}$ of the composite can never exceed the largest value of Z in any component.

(rigorous proof for two-component system)

$$Q_{e11}=\frac{1}{V}\int dV\left(Q_{11}E_1^{(1)^2}+2Q_{12}\mathbf{E}_1^{(1)}\cdot\mathbf{E}_2^{(1)}+Q_{22}E_2^{(1)^2}\right)$$

Journal of Applied Physics -- December 1, 1991 – V.70 (11), pp. 6821-6833

Compatibility Factor

VOLUME 91, NUMBER 14 PHYSICAL REVIEW LETTERS

2003

Thermoelectric Efficiency and Compatibility

G. Jeffrey Snyder and Tristan S. Ursell

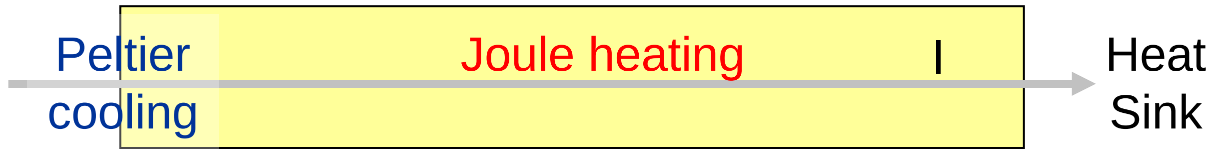
A new materials property $s=(\sqrt{1+zT}-1)/(\alpha T)$, which we call the compatibility factor. Materials with dissimilar compatibility factors cannot be combined by segmentation into an efficient thermoelectric generator. Thus, control of the compatibility factor s is, in addition to z, essential for efficient operation of a thermoelectric device

Application of the compatibility factor to the design of segmented and cascaded thermoelectric generators

G. J. Snyder; Appl. Phys. Lett. 84, 2436 (2004)

… Cascaded generators avoid the compatibility problem.

Maximum cooling of uniform thermoelectrics



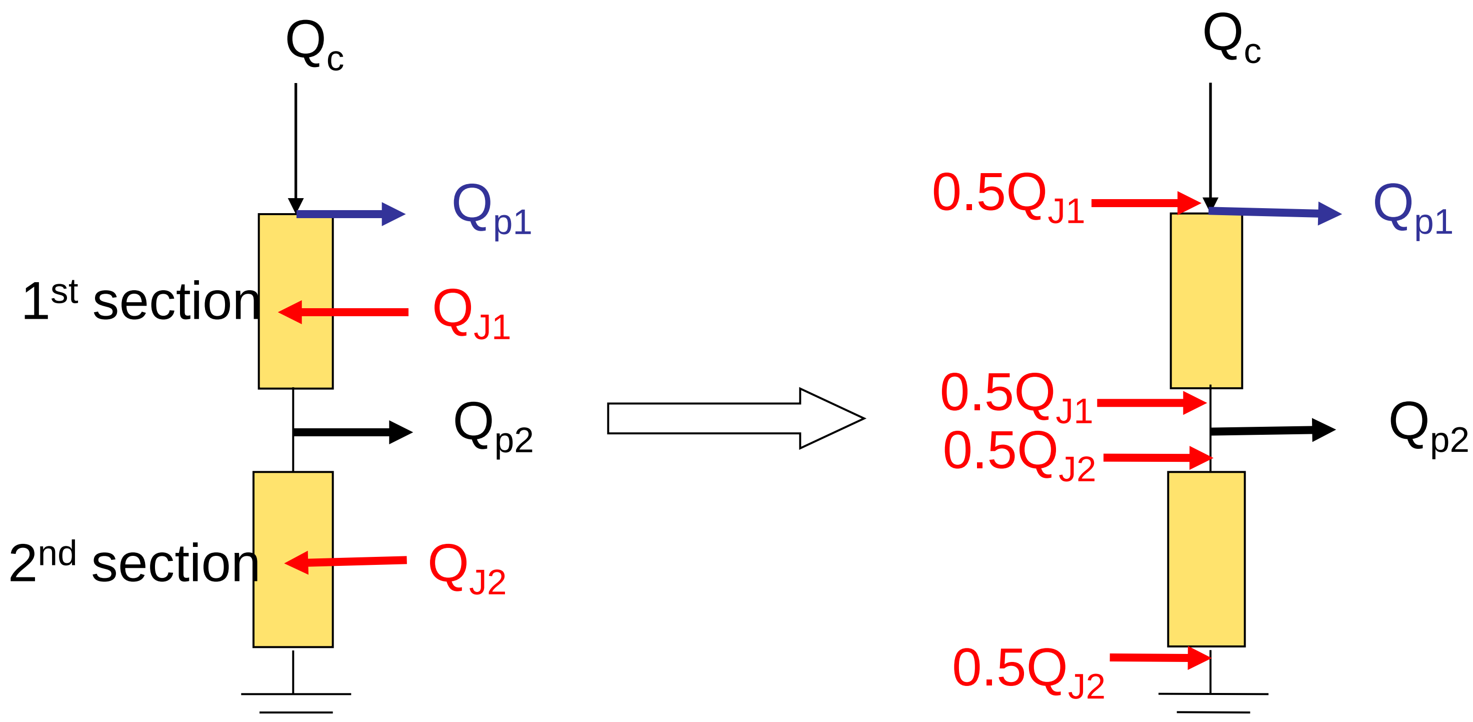
$$Q = IST_c - \frac{1}{2} I^2 R - K\Delta T \xrightarrow{I_{\max} = \frac{ST_c}{R}} \Delta T_{\max} = ZT_c^2 / 2$$

- Maximum cooling is only a function of ZT
- It is geometry independent
- At maximum current, half of the cooling power is cancelled by the Joule heating

Can we beat the ½ ZT_c^2 limit by redistributing the Joule heating in the material?

Thermal circuit analysis

- In analyzing multiple-section thermoelectric materials, we can convert distributed Joule heating in each section to two local heat sources



An intuition to improve maximum cooling in graded thermoelectric materials (Zhixi Bian)

Assumption: power factor and thermal conductivity are constant through the materials, small ZT

$$S^2\sigma = \text{constant}$$

L	L/4	L/9
S	2S	3S
σ	σ/4	σ/9
R	R	R

- If I=ST/R, Joule heating and Peltier cooling are cancelled inside material

$$\Delta T_{max} = \frac{1}{2} Z T_C^{\,2} \sum_{n=1} \frac{1}{n^2}$$

n= number of sections

Maximum cooling can be 33-78% times larger for 2-5 sections (S_{max}/S_{min}~10).

Zhixi Bian et al. Applied Physics Letters 2006

Zhixi Bian

Continuously graded materials

Assumption: power factor and thermal conductivity are constant through the materials, small ZT

$$\frac{d}{dx}\left(K(x)\frac{dT(x)}{dx}\right)=-\frac{J^2}{\sigma(x)}+JT\frac{dS(x)}{dx}$$

$$S(x)^2\sigma(x)=A$$

$$\Delta T_{\max}=\frac{1}{2}ZT^2\frac{\frac{1}{2}\left(\int_0^L S(x)dx\right)^2}{\int_0^L dx\int_0^x S^2(x')dx'}$$

$$\geq\frac{1}{2}ZT^2$$

- If S increases with x monotonically, ΔT_{max} beats uniform materials

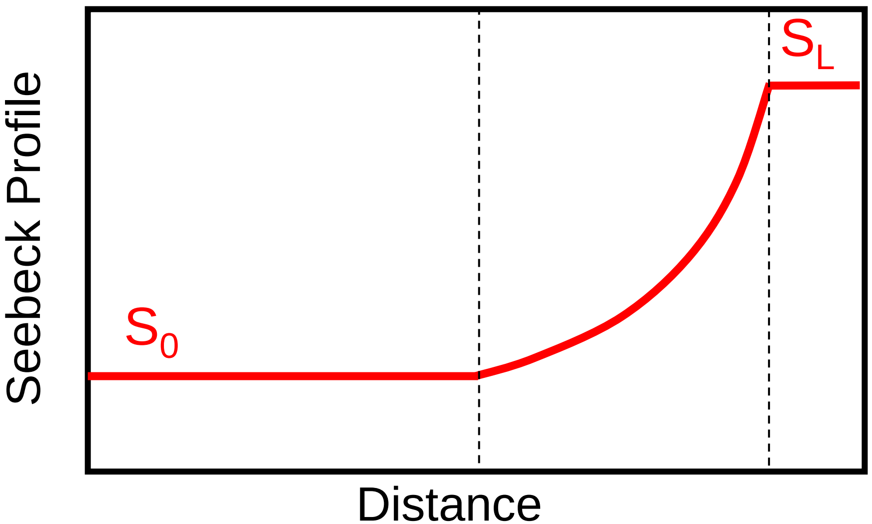
Zhixi Bian, Hongyun Wang et al.; Physical Review B 2007

Analytical solution: **Optimum Seebeck Profile**

$$S(x)=\begin{cases} S_0, & 0<x<L/2 \\ \dfrac{S_0/2}{1-x/L}, & L/2<x<(1-S_0/2S_L)L \\ S_L, & (1-S_0/2S_L)L<x<L \end{cases}$$

$$\boxed{\Delta T_{\max}=\left(1+\frac{1}{2}\ln\left(\frac{S_L}{S_0}\right)\right)\left(\frac{1}{2}ZT^2\right)}$$

For the case $S^2\sigma$=constant, ZT=small, the optimum Seebeck profile and ΔT_{max} can be calculated.

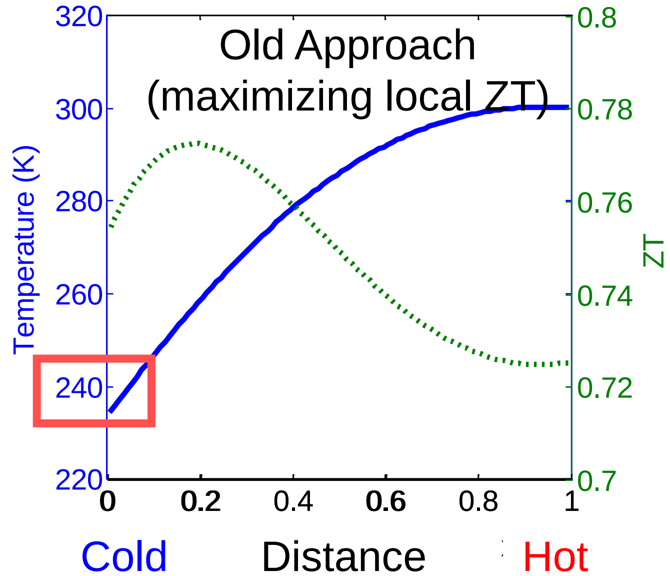


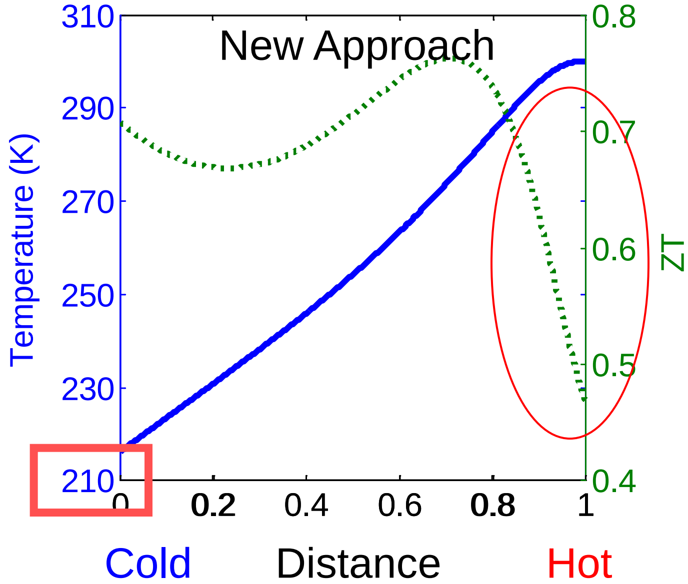
Zhixi Bian, Hongyun Wang et al.; Physical Review B 2007

Intuition: Uniform Efficiency Criterion

Maximum cooling of Bi_2Te_3 Peltier Coolers

- Conventional functionally graded TE materials try to optimize material at each location to have highest ZT_{local} at that location
- ✓ New analysis points to uniform efficiency criterion. This can increase maximum cooling of thermoelectric materials significantly.





Graded Bi_2Te_3 can increase maximum cooling of TE refrigerators from 240K to 210K (without changing max ZT)

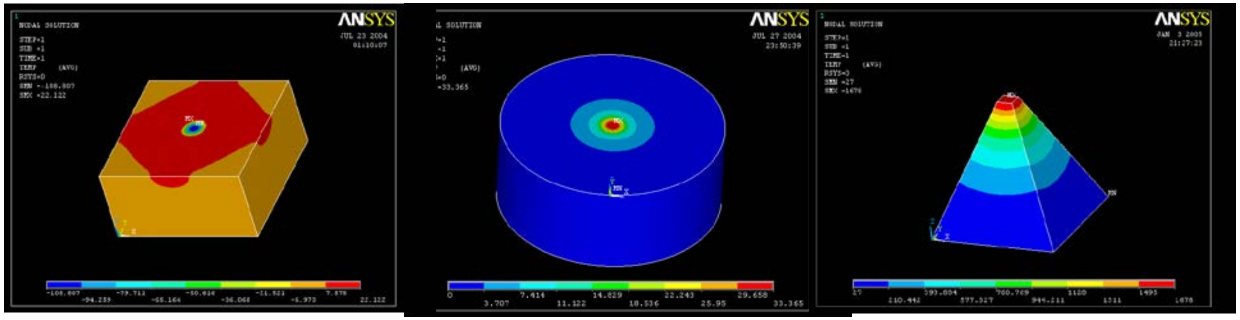
Zhixi Bian et al. PRB 2007

Lecture 4.4 Part I Summary

- Inhomogeneous thermoelectric materials can break the maximum cooling ½ ZT_c^2 (by ~30%)
- The cooling efficiency at large ΔT can also be improved.
- Analytical solution is given for constant power factor approximation.
- Uniform efficiency criterion provides physical insight into the mathematical solution
- Cooling enhancement is independent of material dimensions.

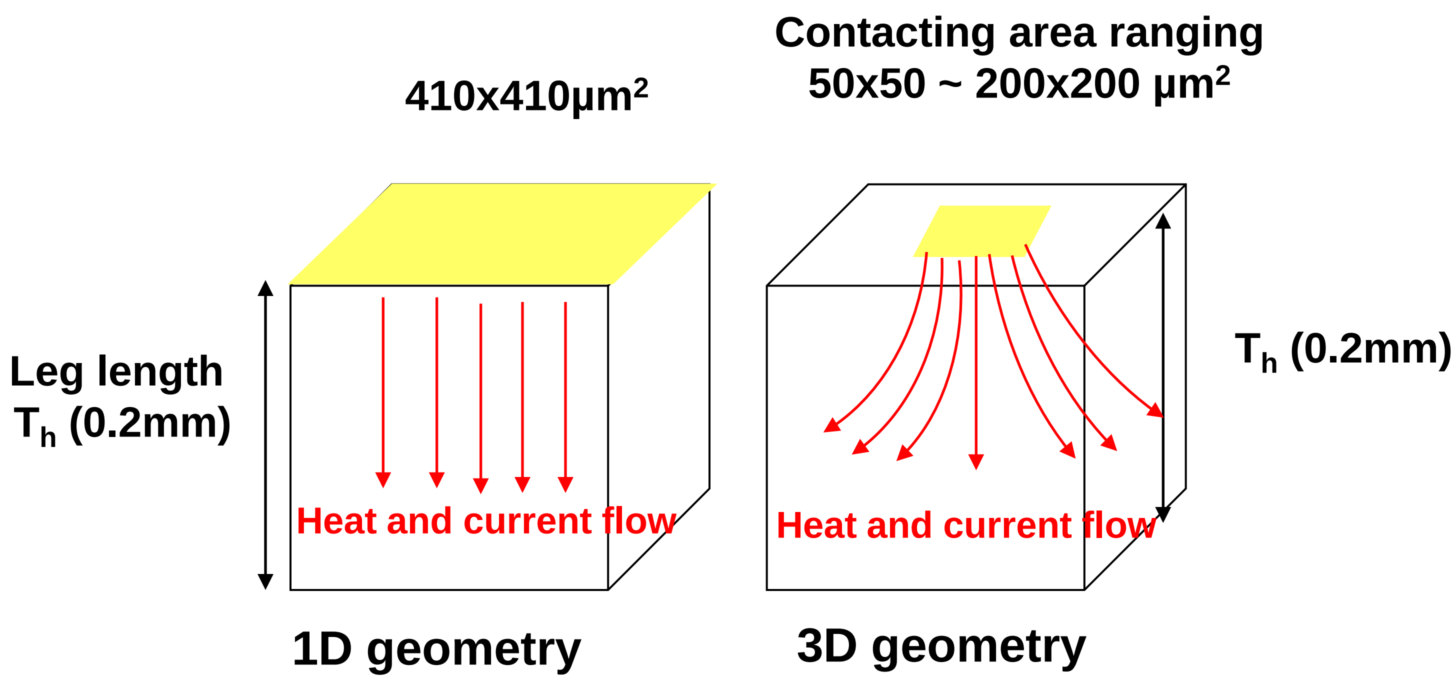
Acknowledgement: ONR-MURI Thermionic Energy Conversion Center

Can Thermoelectric Leg Geometry Improve the Performance?



Yan Zhang, and Ali Shakouri, "**Three-dimensional high cooling power density thermoelectric coolers**", *23rd International conference on Thermoelectrics*, Adelaide, Australia, 2004

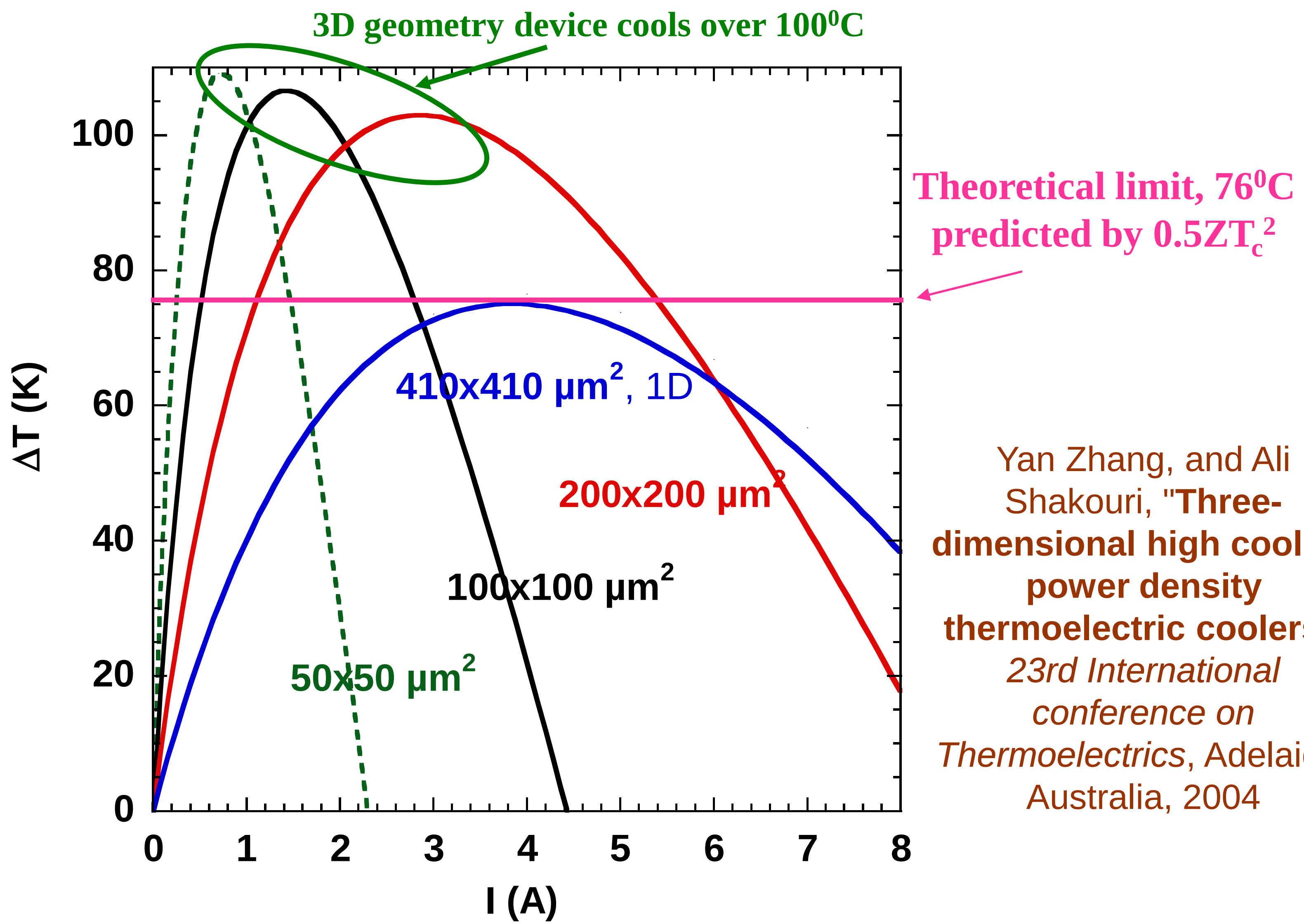
Device Geometries



BiTe material Properties:

- α, Seebeck Coefficient, 205μV/K
- σ, electrical conductivity, 1010 $(\Omega cm)^{-1}$
- κ, thermal conductivity, 1.405 K/W
- Z, figure of merit, $3.02 \cdot 10^{-3}$ K^{-1}

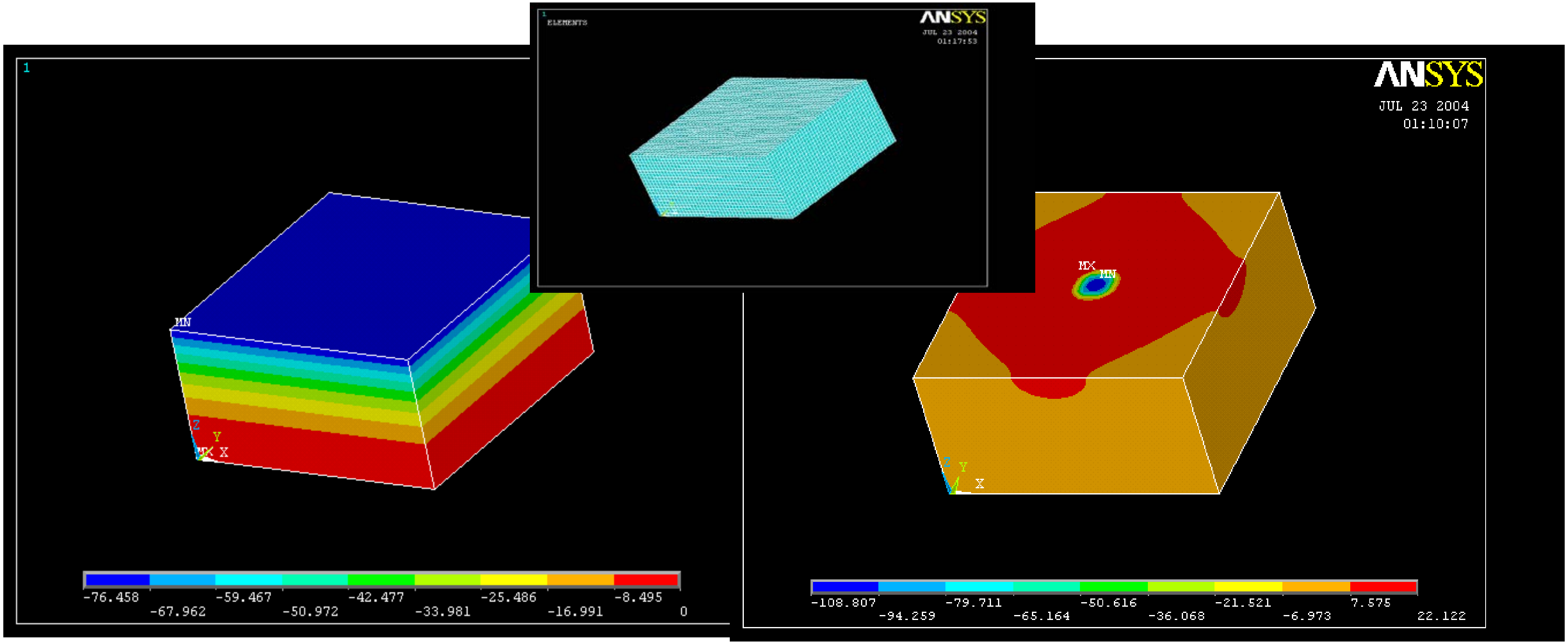
Cooling for various device sizes



Yan Zhang, and Ali Shakouri, "**Three-dimensional high cooling power density thermoelectric coolers**", *23rd International conference on Thermoelectrics*, Adelaide, Australia, 2004

Device cooling distribution

Solid brick meshing



1D device, contact area 410x410µm^2
Max. Cooling, 76.5^{0}C
with supplied current 4A

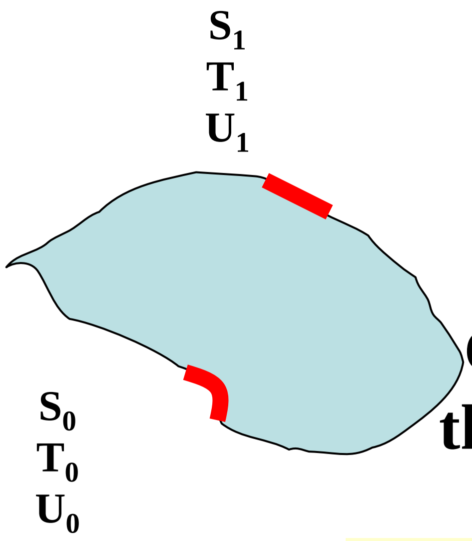
3D device, contact area 50x50µm^2
Max. Cooling, 108.8^{0}C
with supplied current 0.8A

Seemed too good to be true???

- Different opinion
  - V.A Semenyuk, "Efficiency of Cooling Thermoelectric Elements of Arbitrary Shape", Journal of Engineering Physics, Vol.32 (2) p.196, 1978

Poisson Equation

$$\nabla^2 \varphi = -i^2 / \lambda \sigma$$



Two assumptions:

1. No current, heat flux due to T_1-T_0
2. With current but uniform contact T_0

Conclusion: The maximum energy efficiency in the most general case is <u>independent of the shape of the conductor</u>

Assumption: Uniform temperature at contact area

What it really matters?

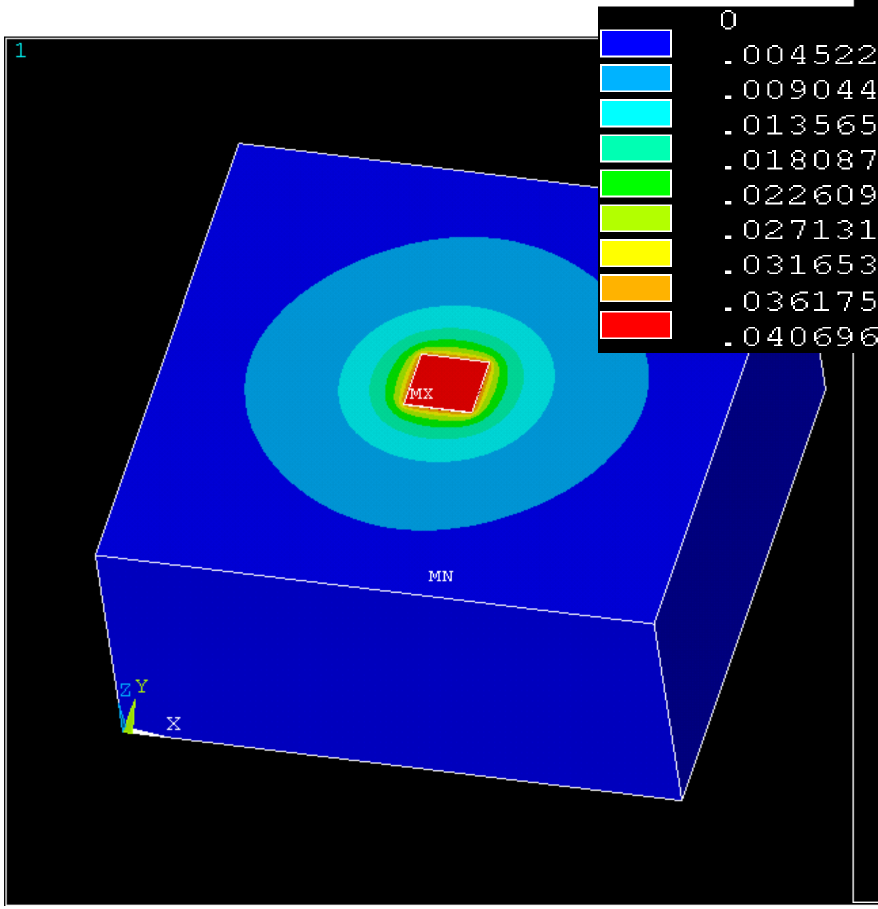
Different Boundary Conditions at contact region:

1. *Uniform Potential;*
2. *Uniform Current Density;*

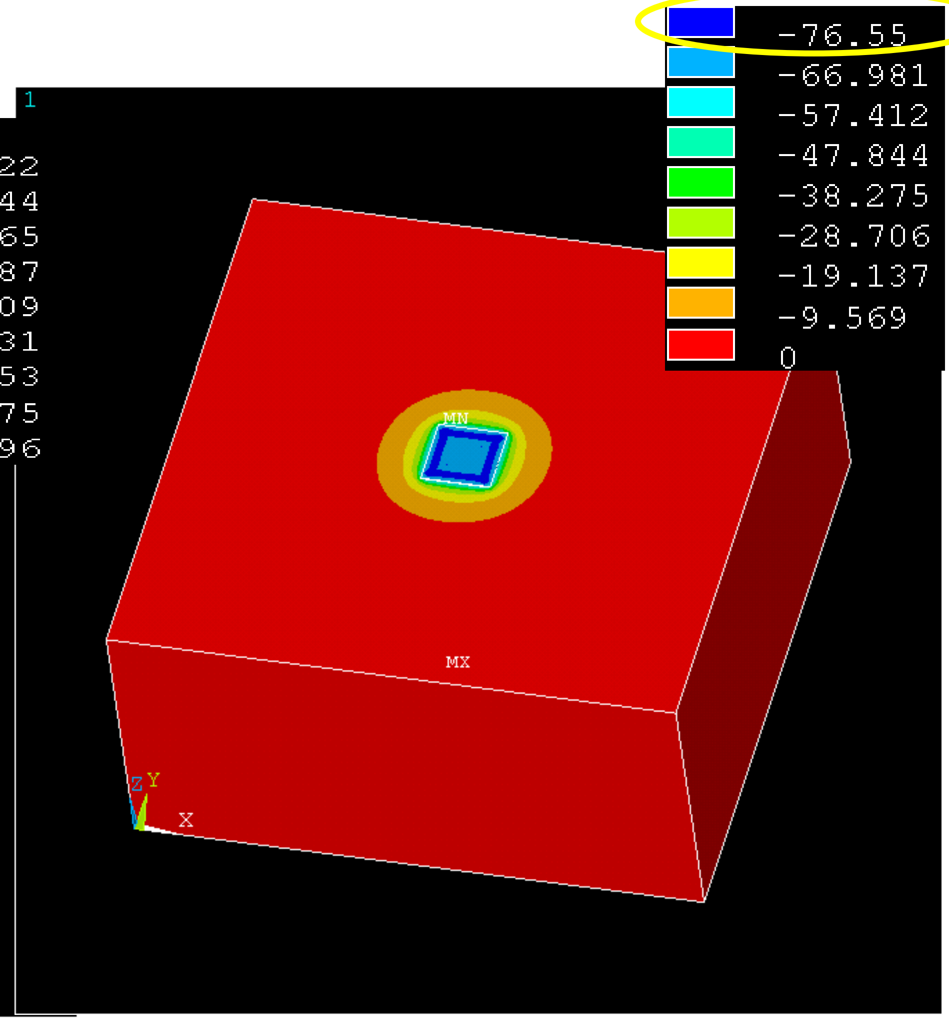
Yan Zhang, Zhixi Bian and Ali Shakouri, **"Improved energy conversion efficiency by optimizing the geometry of thermoelectric leg elements",** *Proceedings of the 24 th International conference on Thermoelectrics*, pp.233-236, June, 2005, Clemson , SC

Yan Zhang , Gehong Zeng, Avram Bar-Cohen and Ali Shakouri, "**Is ZT the main main performance factor for hot spot cooling using 3D microrefrigerators?",** *IMAPS on Thermal Management*, 2005, Palo Alto, CA (Student Competition Award)

Boundary Condition 1: Uniform Potential



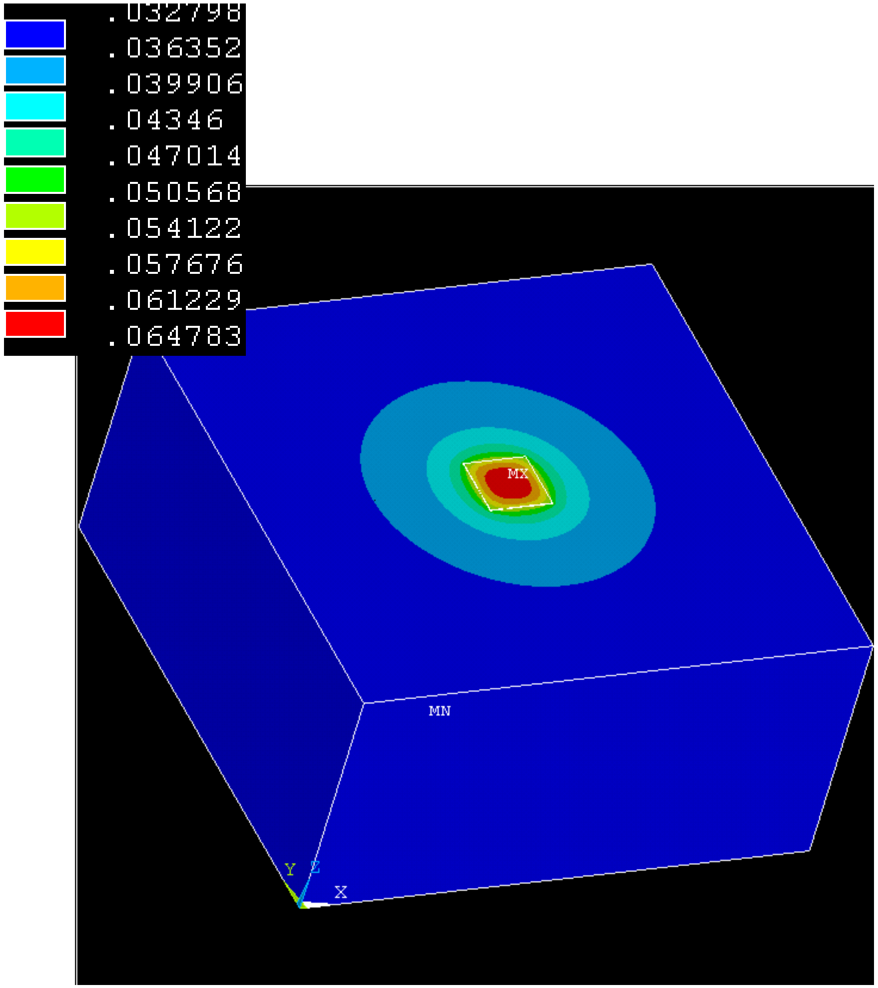
Potential Distribution



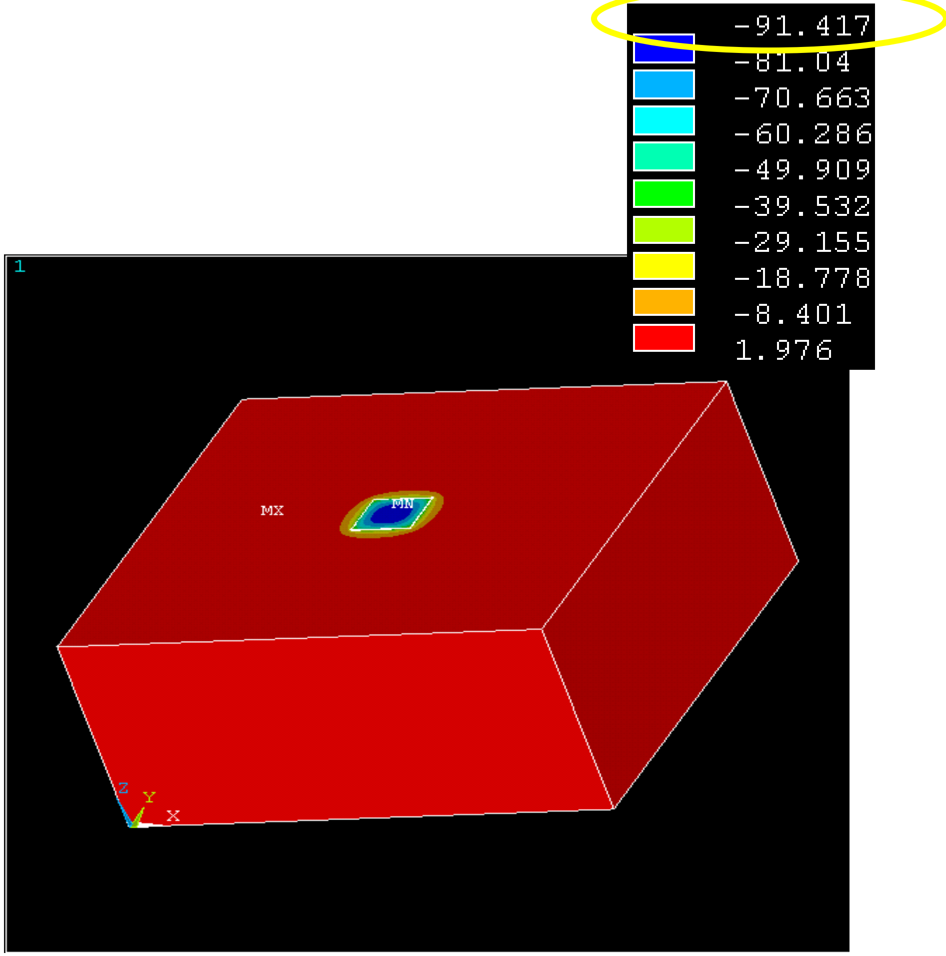
Temperature Distribution

Supplied I=0.6A

Boundary Condition 2: Uniform Current



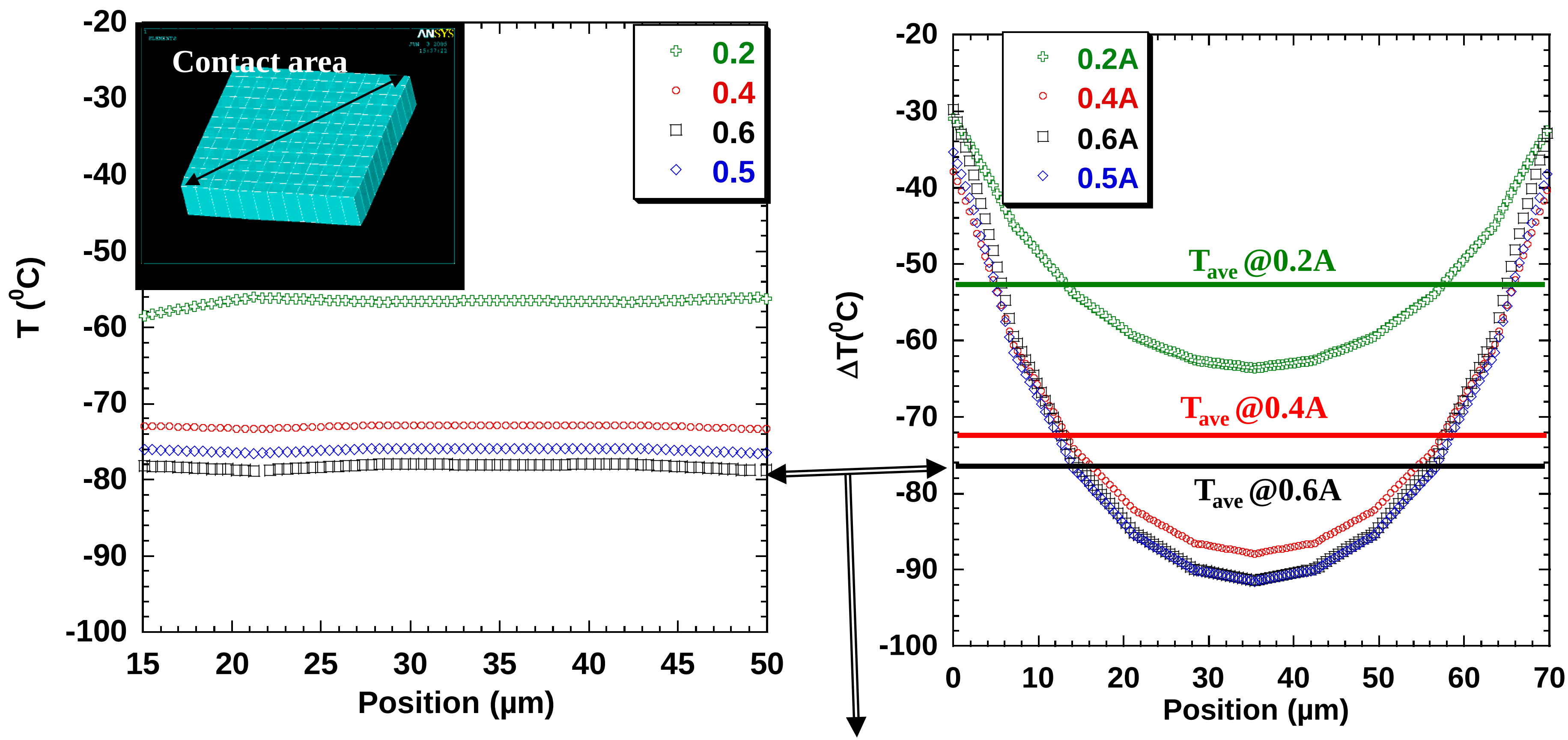
Potential Distribution



Current Distribution

Supplied I=0.6A

Temperature Cross-section under different Boundary Conditions

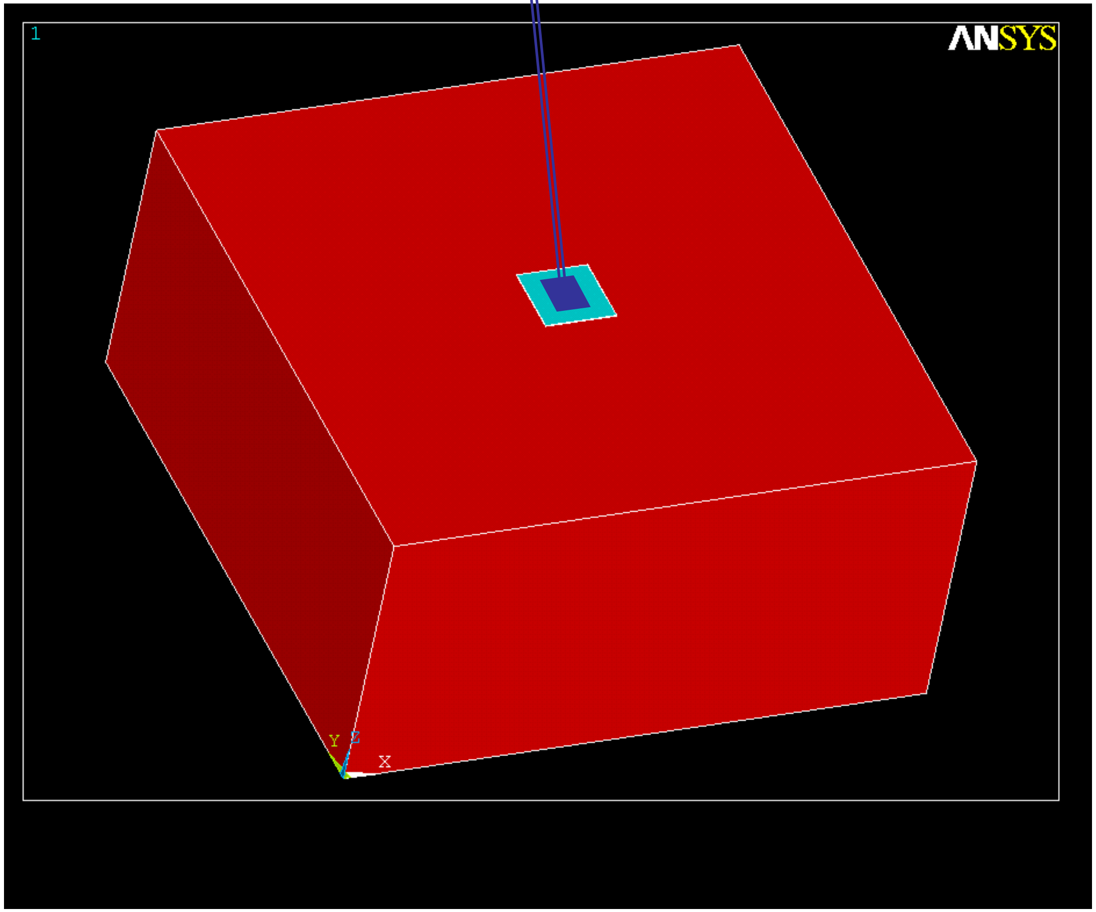


1D theoretical limit

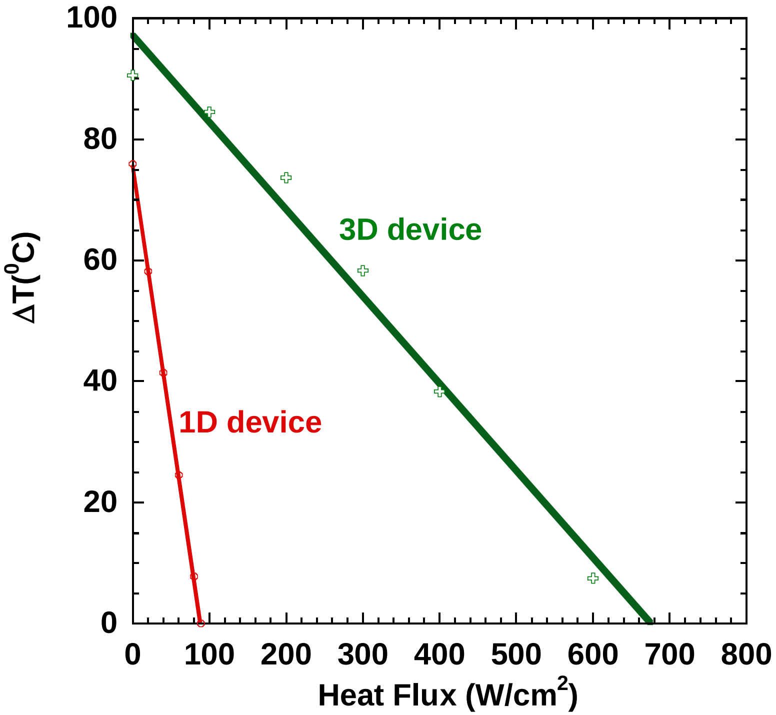
$$\Delta T_{max}=0.5ZT_c^2$$

Benefit of 3D device with uniform current

Center Peak Cooling Region 20x20μm²

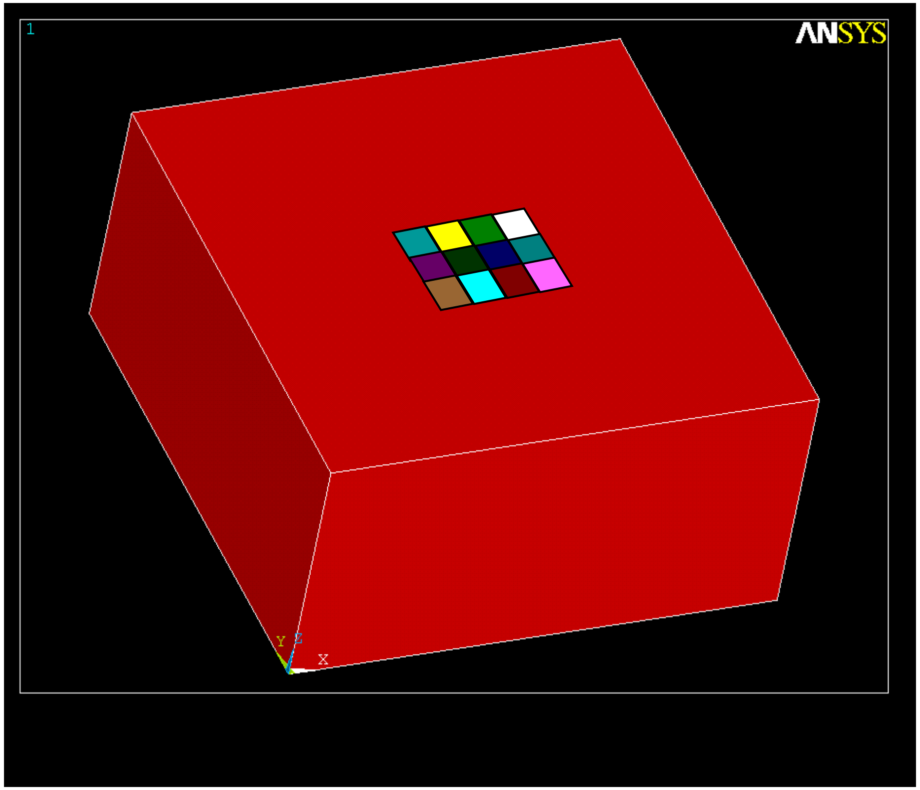


High Cooling Power Density @ center region 655 W/cm²

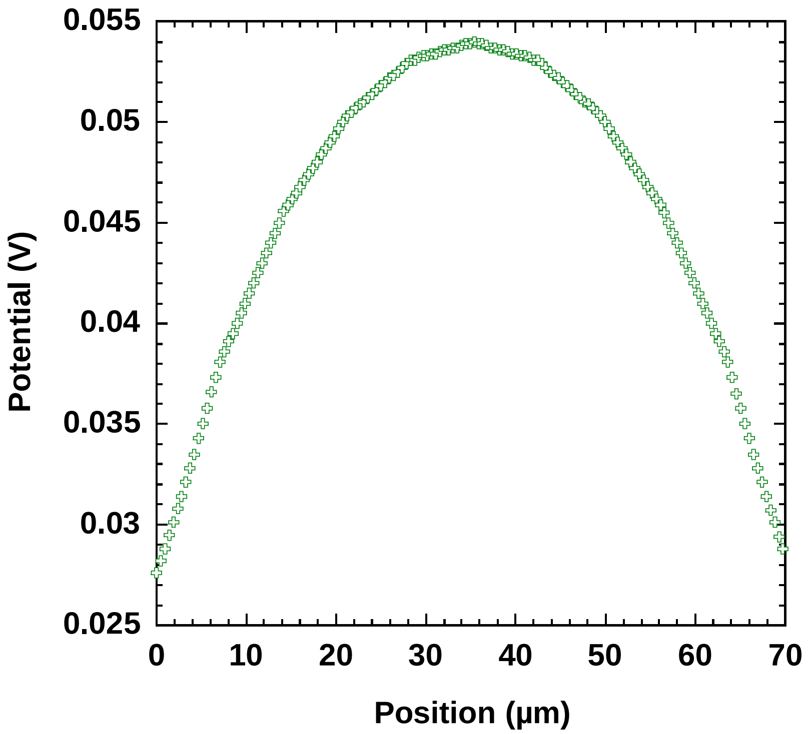


How to Achieve the uniform current injection?

Array Device Solution:
Send different potential to each device to achieve the uniform current profile



Diagonal Potential Distribution @ supplied 0.5A



Lecture 4.4 Part II Summary

- The 3D device can cool better than 1D under the condition of uniform current distribution;
  - The temperature distribution is non-uniform at contact area;
  - The center peak region, 20x20µm^2, could cool over 85^{0}C, or cooling power density of 650 W/cm^2;

- It is possible to create an array structure and manipulating the potential of each device to achieve the maximum cooling in the center;

Yan Zhang, Zhixi Bian and Ali Shakouri, **"Improved energy conversion efficiency by optimizing the geometry of thermoelectric leg elements",** *Proceedings of the 24 th International conference on Thermoelectrics*, pp.233-236, June, 2005, Clemson , SC