Thermoelectricity: From Atoms to Systems

Week 3: Nanoscale and macroscale characterization
Tutorial 3.1 Homework solutions, problems 1-6

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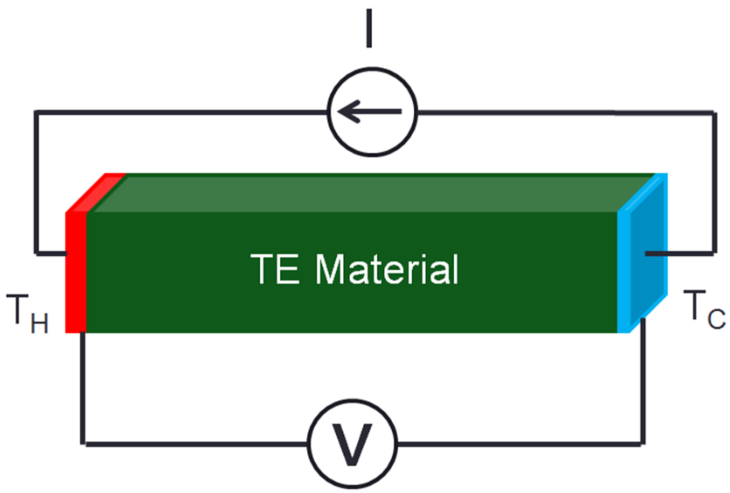
Prob. 1. Transient Harman method



Heat balance equation under adiabatic condition at the cold side (T_H=constant/reservoir):

$$Q = STI - \frac{1}{2} I^2 R - K\Delta T = 0$$

$$\therefore \; \Delta T = \frac{ST_C I}{K} - \frac{I^2 R}{2K}$$

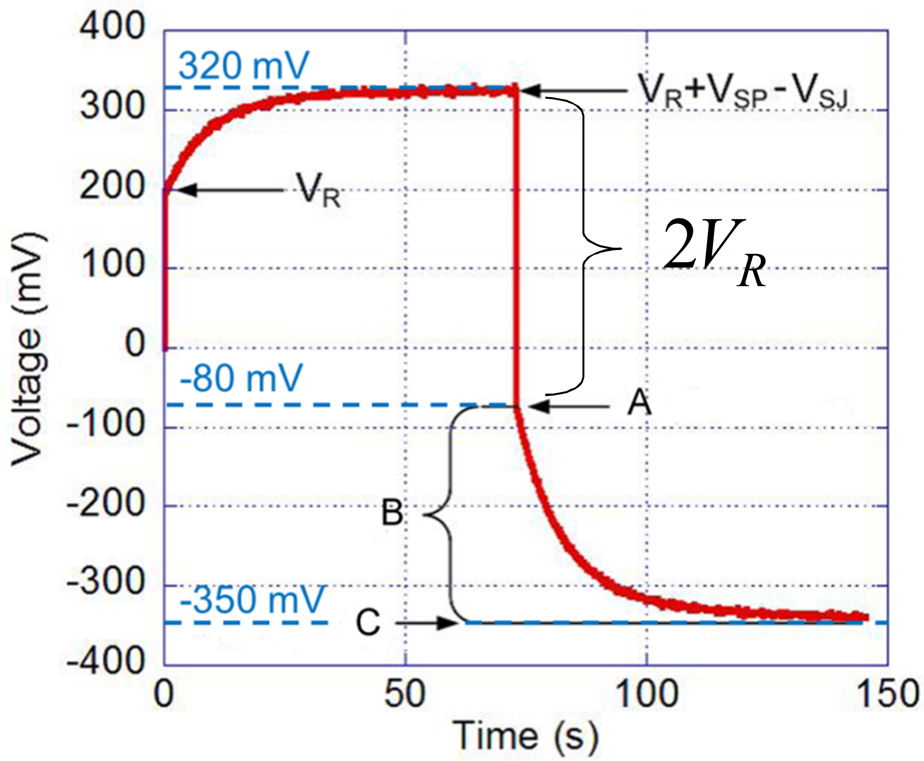
What we measure is the total voltage:

$$V_T = IR + S\Delta T = IR + \frac{S^2 T_C I}{K} - \frac{SI^2 R}{2K}$$

1-1. $V_R = IR$ $\quad V_{SP} = \frac{S^2 TI}{K}$ $\quad \Rightarrow \quad \frac{V_{SP}}{V_R} = \frac{S^2 TI}{RKI} = ZT$

Prob. 1. Transient Harman method

1-2.



At t = 70s - ε

$$V_T(+I) = IR + \frac{S^2 T_C I}{K} - \frac{SI^2 R}{2K}$$

At t = 140s

$$V_T(-I) = -IR - \frac{S^2 T_C I}{K} - \frac{SI^2 R}{2K}$$

$$A = -V_R + V_{SP} - V_{SJ}$$

$$B = 2V_{SP}$$

$$C = V_T(-I) = A - B$$

$$= -V_R - V_{SP} - V_{SJ}$$

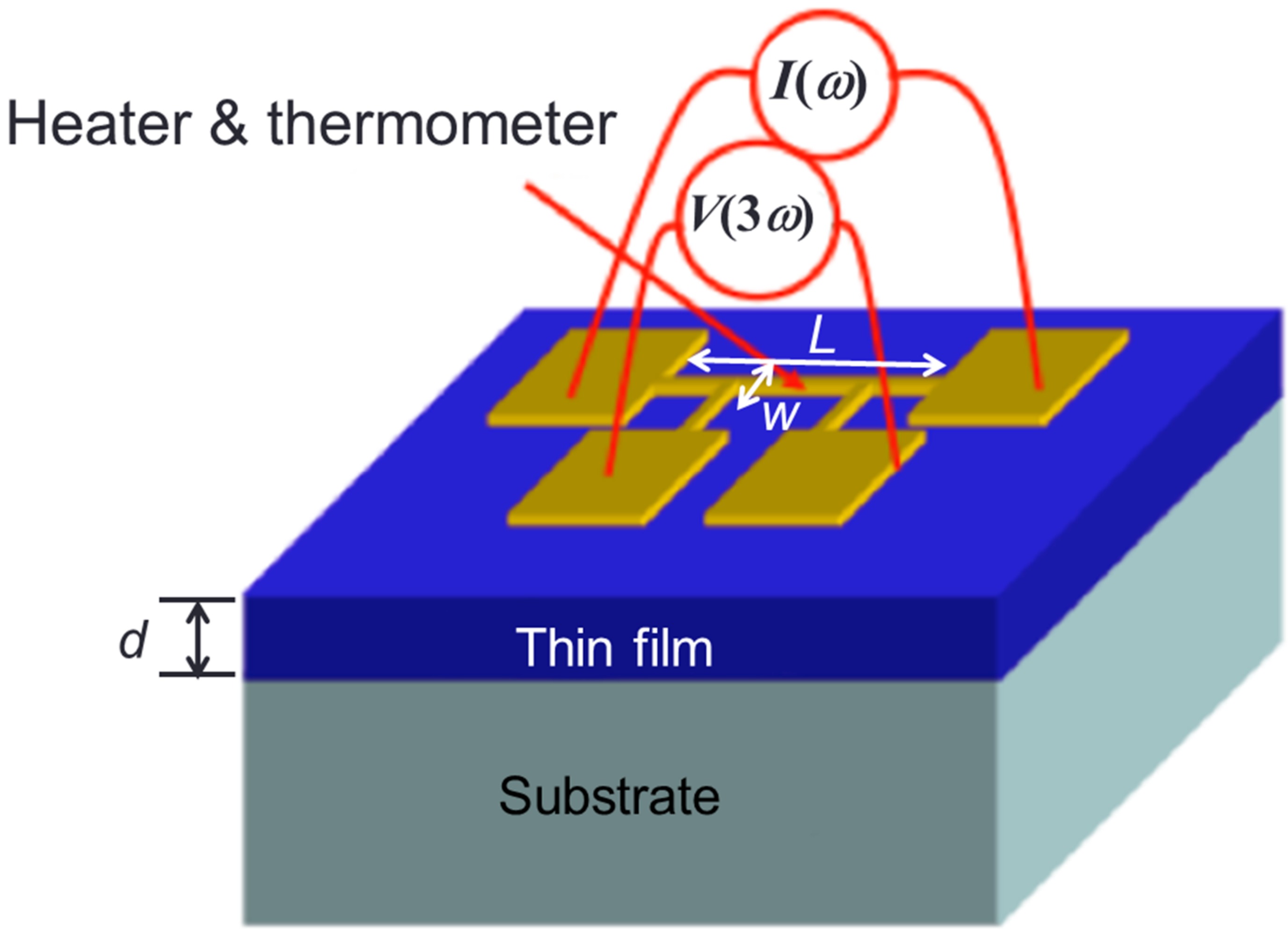
1-3.

$$2V_R = 320 - (-80) = 400 \text{ mV}$$

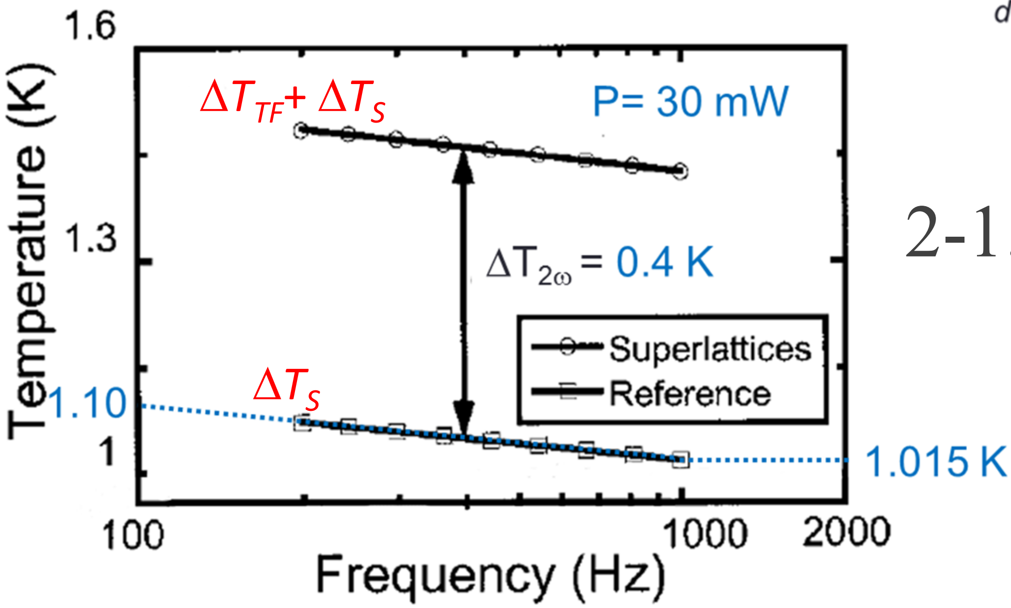
$$B = 2V_{SP} = -80 - (-350) = 270 \text{ mV}$$

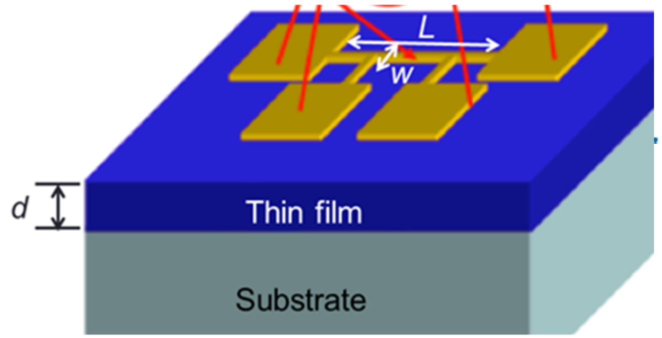
$$\therefore ZT = \frac{V_{SP}}{V_R} = \frac{270/2}{400/2} = 0.675$$

Prob. 2. 3ω method



Prob. 2. 3ω method





2-1. From the left figure, the temperature drop across thin film

$$\Delta T_{TF} = 0.4\ \mathrm{K}.$$

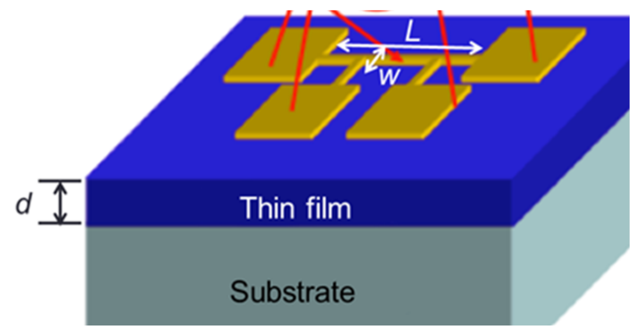
Assuming 1D heat flow from heater through thin film,

$$P = \kappa \frac{wL}{d} \Delta T_{TF}$$

$$\therefore\ \kappa_{TF} = \frac{Pd}{wL\Delta T_{TF}} = \frac{(0.03\ \mathrm{W})(1\times 10^{-6}\ \mathrm{m})}{(25\times 10^{-6}\ \mathrm{m})(1\times 10^{-3}\ \mathrm{m})(0.4\ \mathrm{K})} = 3\ \mathrm{W/mK}$$

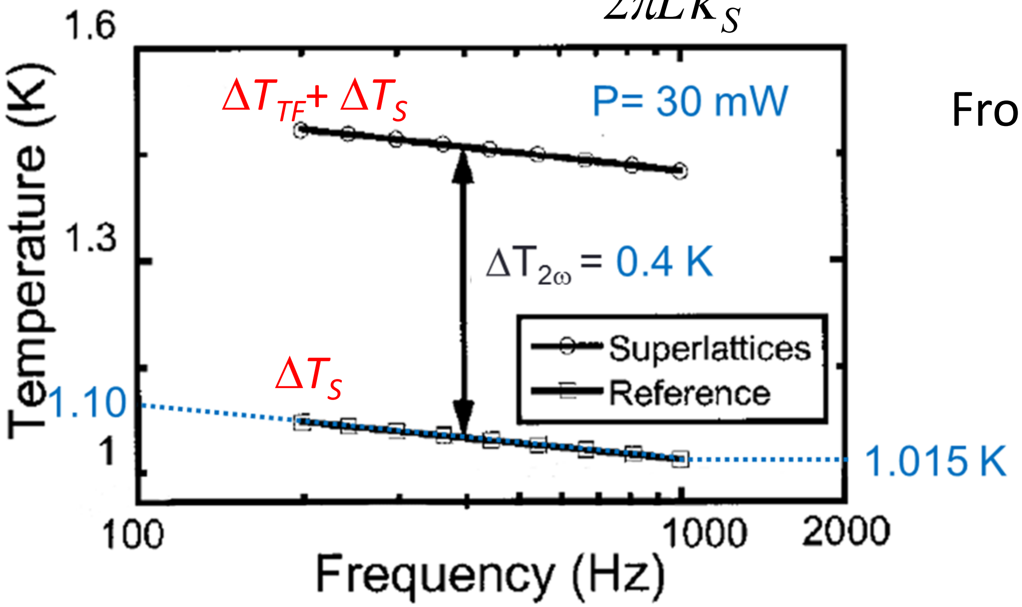
Prob. 2. 3ω method

2-2. $$\Delta T_S = \frac{P}{\pi L \kappa_S}\left[\frac{1}{2}\ln\left(\frac{4D_S}{w^2}\right) + \eta - \frac{1}{2}\ln(2\omega) - i\frac{\pi}{4}\right]$$



Take the derivatives with respect to ω

$$d(\Delta T_S) = -\frac{P}{2\pi L \kappa_S} d(\ln(\omega))$$



From the ΔT_S curve in the plot,

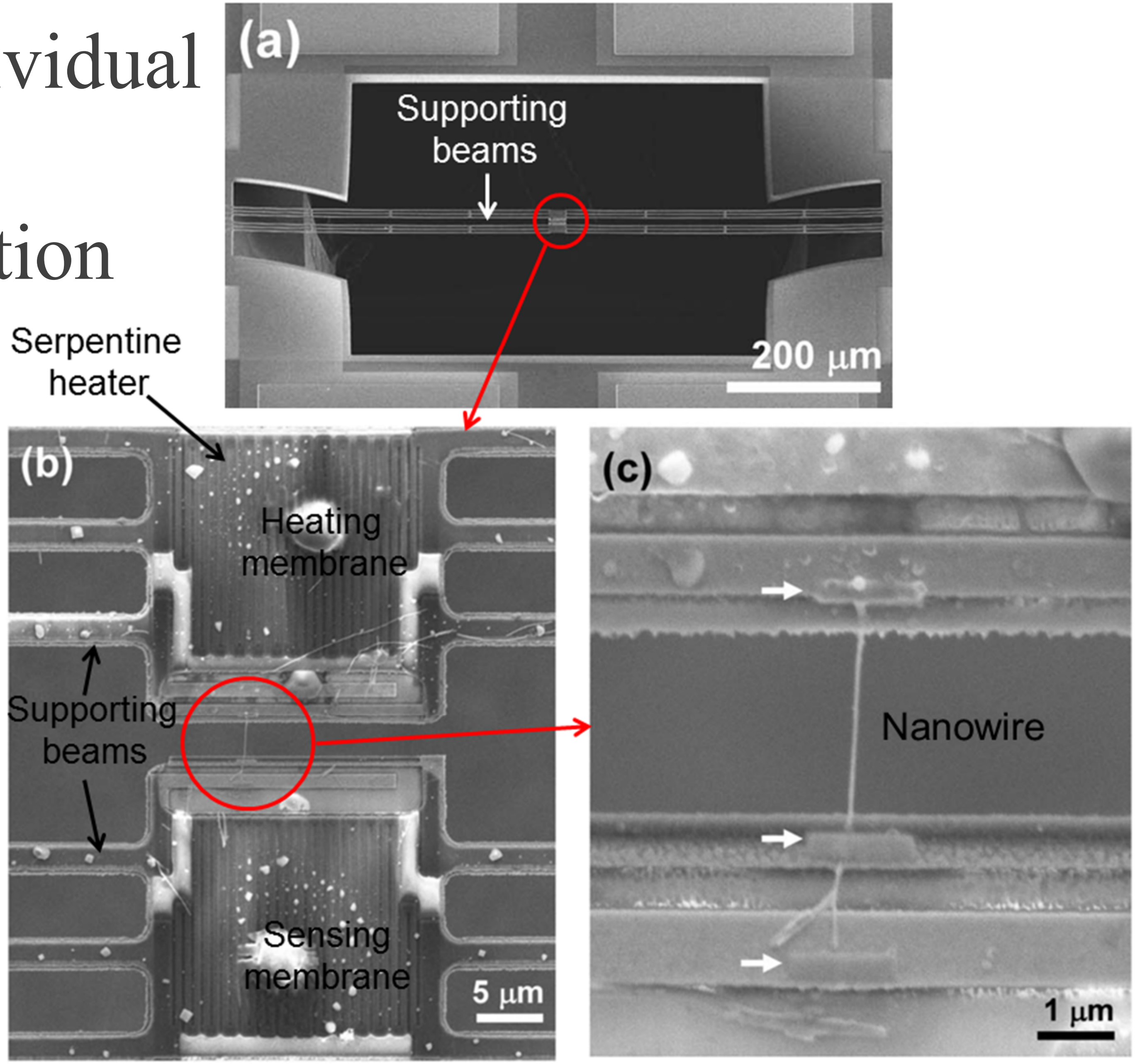
$$\frac{d\left(\Delta T_s\right)}{d(\ln(\omega))} = \frac{d\left(\Delta T_s\right)}{d(\log(\omega))}\log(e)$$

$$= (1.015 - 1.10) * \log(e)$$

$$= -0.0369$$

$$\therefore \kappa_S = -\frac{P}{2\pi L}\left(\frac{d(\Delta T_S)}{d(\ln(\omega))}\right)^{-1} = -\frac{0.03\text{ W}}{2\pi(1\times 10^{-3}\text{ m})(-0.0369\text{ K})} = 129.4\text{ W/mK}$$

Prob. 3. Individual nanowire characterization



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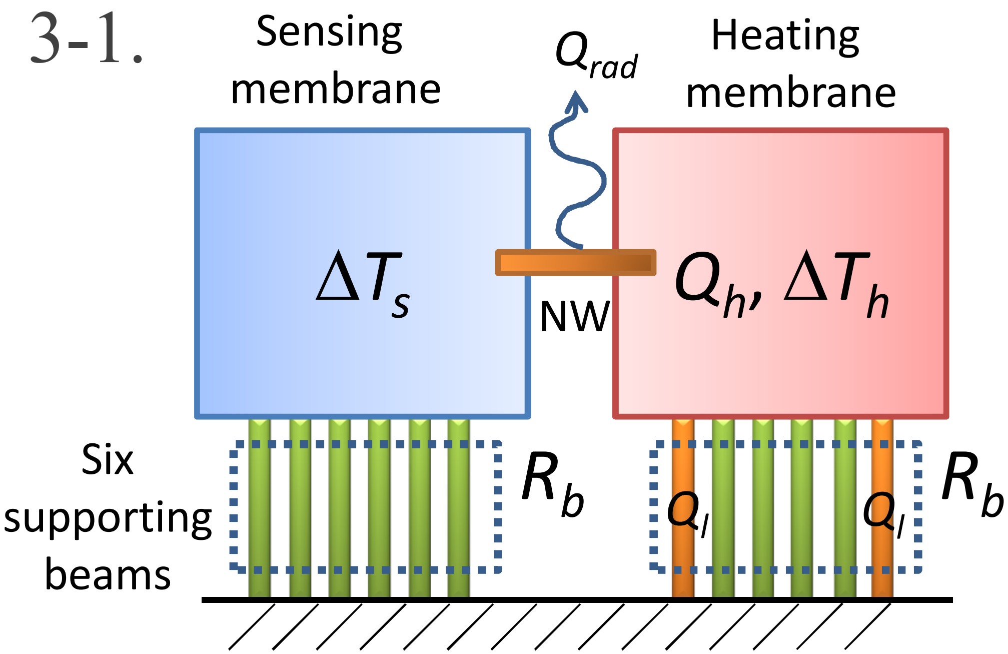
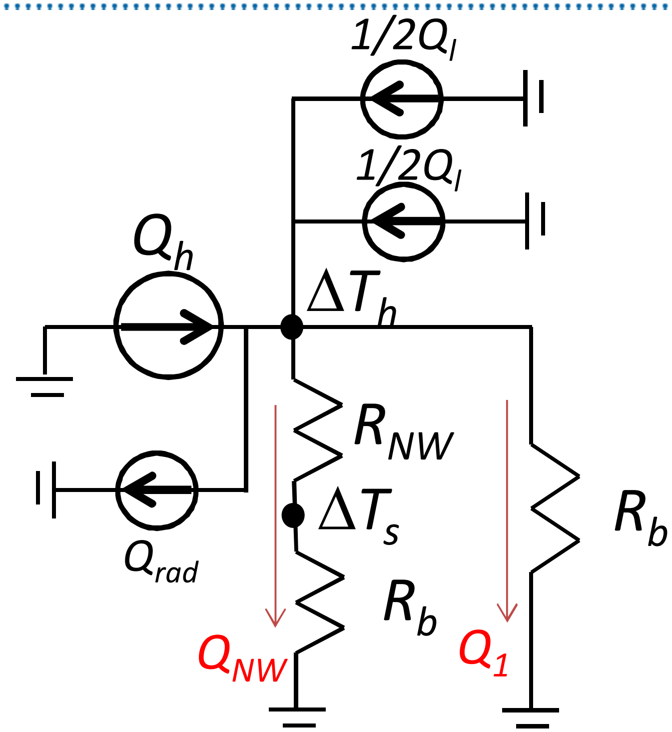
Prob. 3. Individual nanowire characterization

3-1.





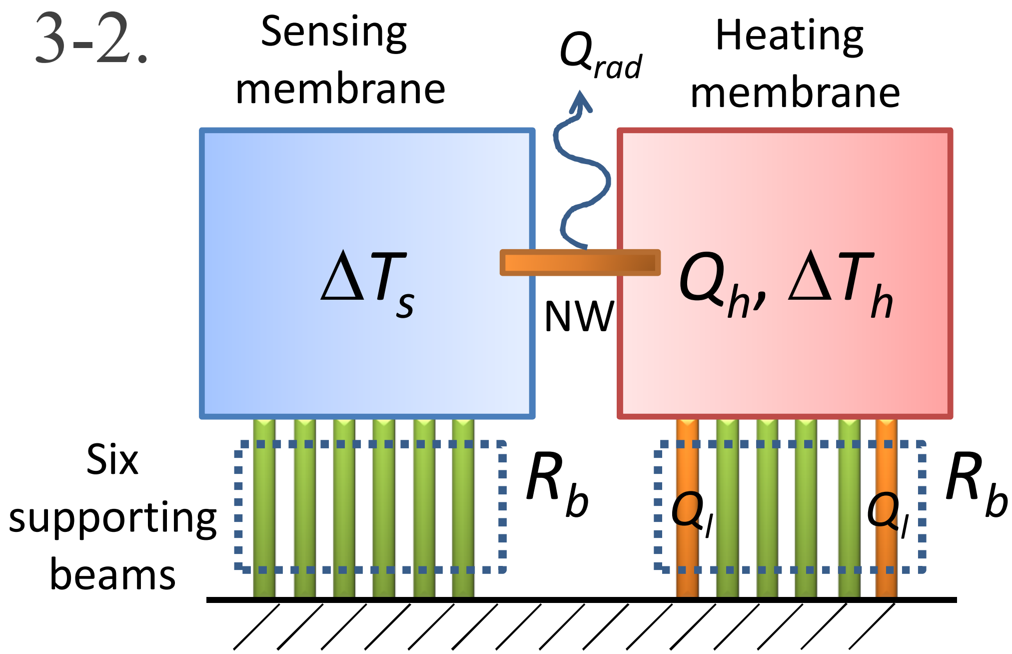
$$Q_{NW} = \frac{\Delta T_h - \Delta T_s}{R_{NW}} = \frac{\Delta T_s}{R_b} \qquad Q_1 = \frac{\Delta T_h}{R_b}$$

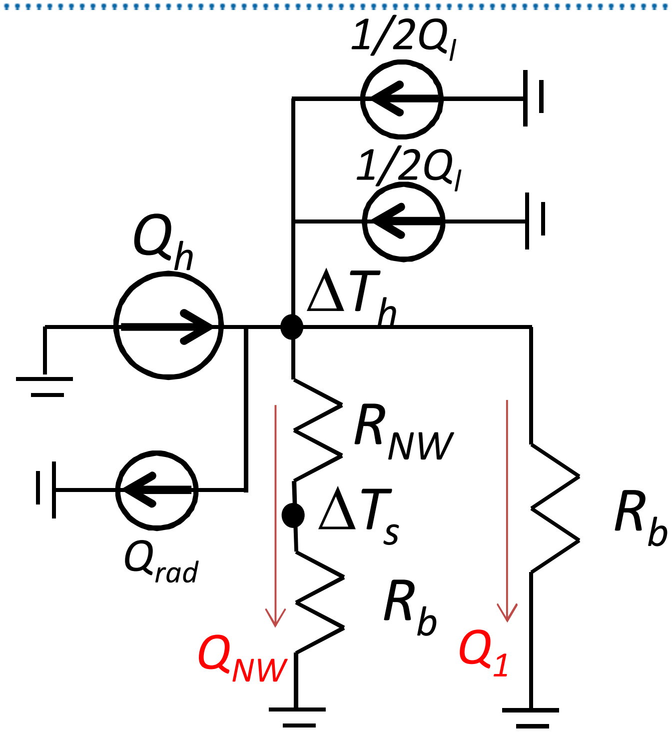
Heat energy conservation at the heating membrane:

$$Q_h + Q_l - Q_{rad} = Q_{NW} + Q_1 = \frac{\Delta T_s}{R_b} + \frac{\Delta T_h}{R_b} \Rightarrow \therefore R_b = \frac{\Delta T_h + \Delta T_s}{Q_h + Q_l - Q_{rad}}$$

Prob. 3. Individual nanowire characterization

3-2.





Including the contact resistances

$$Q_{NW} = \frac{\Delta T_h - \Delta T_s}{R_{NW} + R_{c1} + R_{c2}} = \frac{\Delta T_s}{R_b}$$

$$\therefore R_{NW} = R_b \frac{\Delta T_h - \Delta T_s}{\Delta T_s} - R_{c1} - R_{c2}$$

Prob. 3. Individual nanowire characterization

3-3.

a) Thermal resistance of nanowire is underestimated if radiation loss from the nanowire surface is not taken into account.

$$R_{NW} = R_b \frac{\Delta T_h - \Delta T_s}{\Delta T_s} - R_{c1} - R_{c2} \qquad R_b = \frac{\Delta T_h + \Delta T_s}{Q_h + Q_l - Q_{rad}}$$

b) Radiation loss can be negligibly small if the nanowire is sufficiently thick and short.

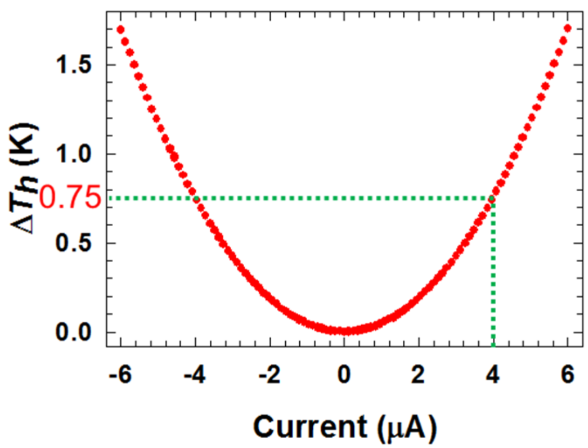
c) Thermal contact resistances between nanowire and membranes reduce the actual temperature gradient across the nanowire, and create non-uniform temperature on the portion of the nanowire that is in contact with the membrane surface.

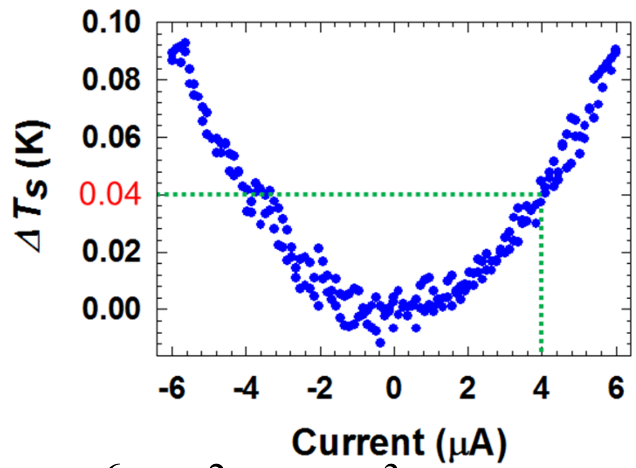
d) The nanowire thermal resistance is overestimated unless the thermal contact resistances are accurately quantified and added.

e) All of the above

Prob. 3. Individual nanowire characterization

3-4.





$$\Delta T_h = 0.75\ \mathrm{K} \qquad Q_h = (4\times10^{-6}\ \mathrm{A})^2(2\times10^3\ \Omega) = 3.2\times10^{-8}\ \mathrm{W}$$

$$\Delta T_s = 0.04\ \mathrm{K} \qquad Q_l = (4\times10^{-6}\ \mathrm{A})^2(0.47\times10^3\ \Omega) = 7.52\times10^{-9}\ \mathrm{W}$$

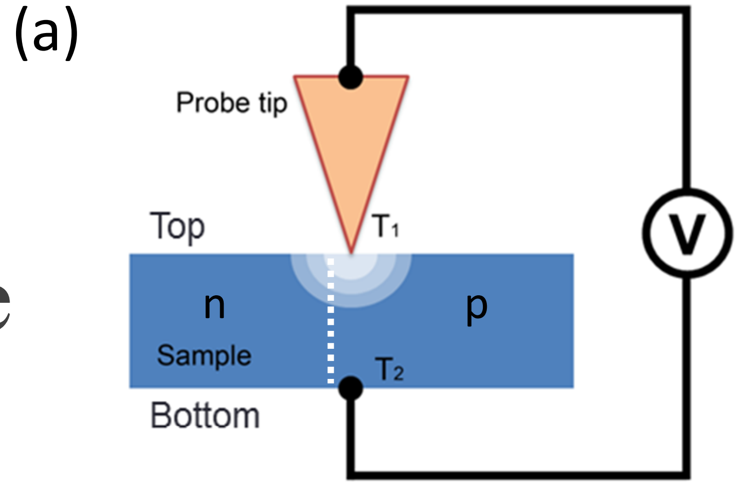
$$\therefore R_b = \frac{\Delta T_h + \Delta T_s}{Q_h + Q_l} = \frac{0.75 + 0.04\ \mathrm{K}}{3.2\times10^{-8} + 7.52\times10^{-9}\ \mathrm{W}} = 2.0\times10^7\ \mathrm{K/W}$$

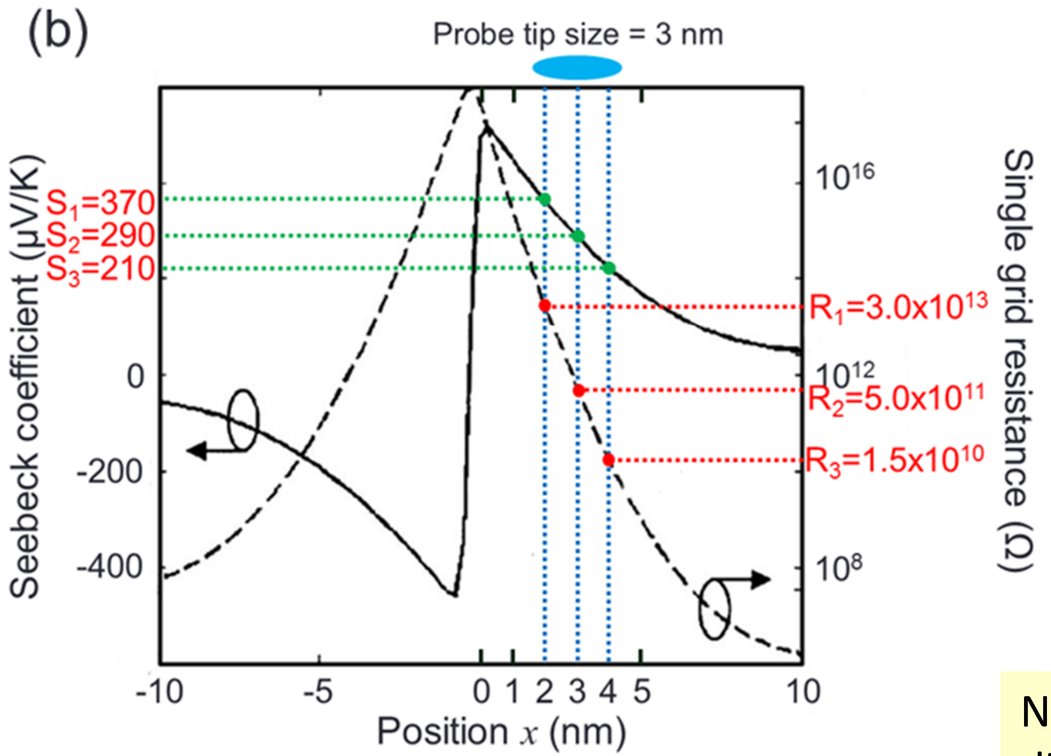
$$\therefore \kappa_{NW} = \frac{1}{R_{NW}}\frac{l_{NW}}{A_{NW}} = \frac{1}{R_b}\frac{\Delta T_s}{\Delta T_h - \Delta T_s}\frac{l_{NW}}{A_{NW}}$$

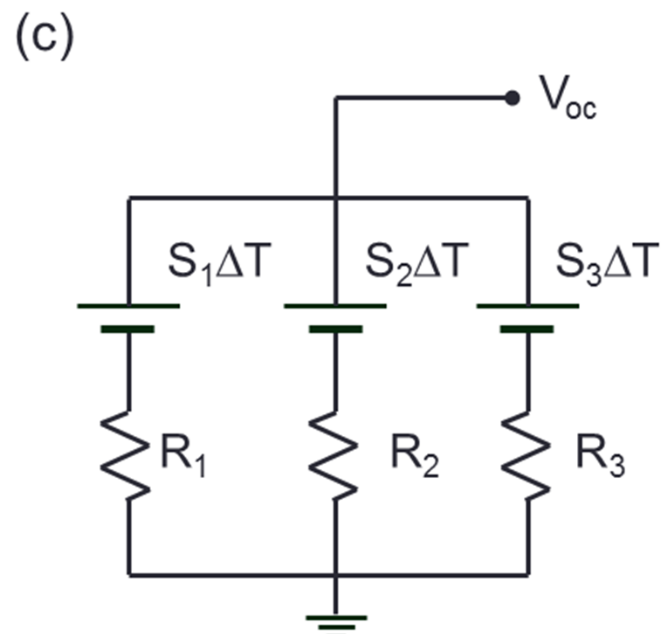
$$= \frac{1}{2\times10^7\ \mathrm{K/W}}\frac{0.04}{0.75-0.04}\frac{3\times10^{-6}\ \mathrm{m}}{\pi(148/2\times10^{-9}\ \mathrm{m})^2} = 0.49\ \mathrm{W/mK}$$

Includes contact thermal resistance

Prob. 4. Scanning thermal probe method



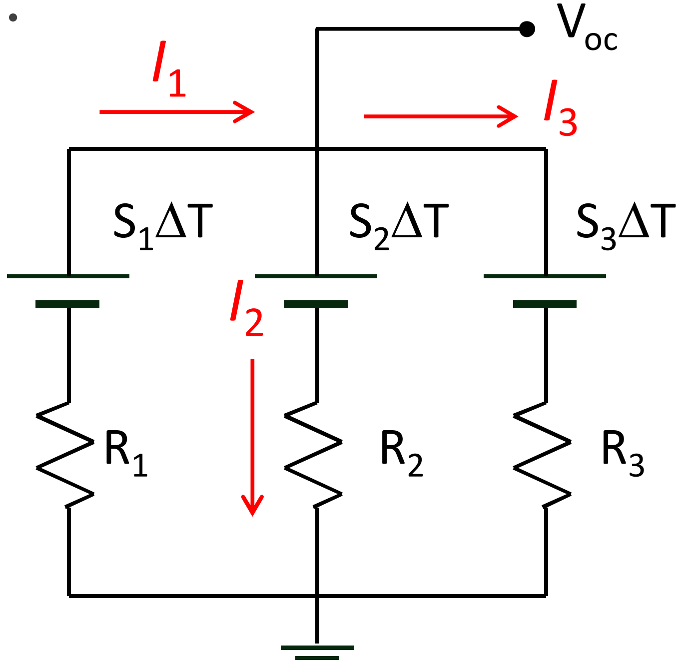




Neglect temperature distribution around the lead

Prob. 4. Scanning thermal probe method

4-1.



Kirchoff's current law:

$$I_1 = I_2 + I_3$$

Kirchoff's voltage law:

$$\begin{aligned} V_{oc} &= S_1\Delta T - I_1 R_1 \\ &= S_2\Delta T + I_2 R_2 \\ &= S_3\Delta T + I_3 R_3 \end{aligned}$$

"Effective" Seebeck coefficient

$$\therefore S = \frac{V_{oc}}{\Delta T} = \frac{\dfrac{S_1}{R_1} + \dfrac{S_2}{R_2} + \dfrac{S_3}{R_3}}{\dfrac{1}{R_1} + \dfrac{1}{R_2} + \dfrac{1}{R_3}} = 212.4\ \mu\mathrm{V/K}$$

Prob. 4. Scanning thermal probe method

4-2.

Which one is closest to the measured Seebeck coefficient value, and why?

a) S_1, because R_1 is much larger than the other two resistances.

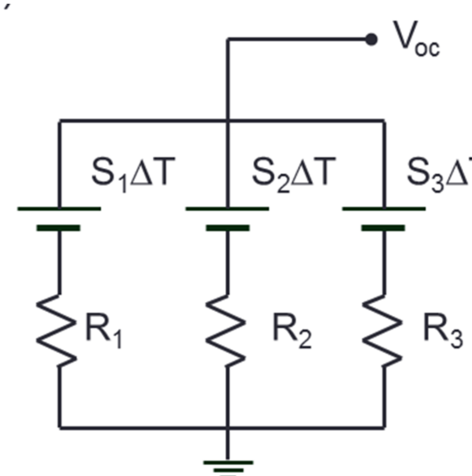
b) $(S_1+S_2)/2$, because R_3 is much smaller than the other two resistances.

c) S_2, because the probe tip is centered at the position of R_2.

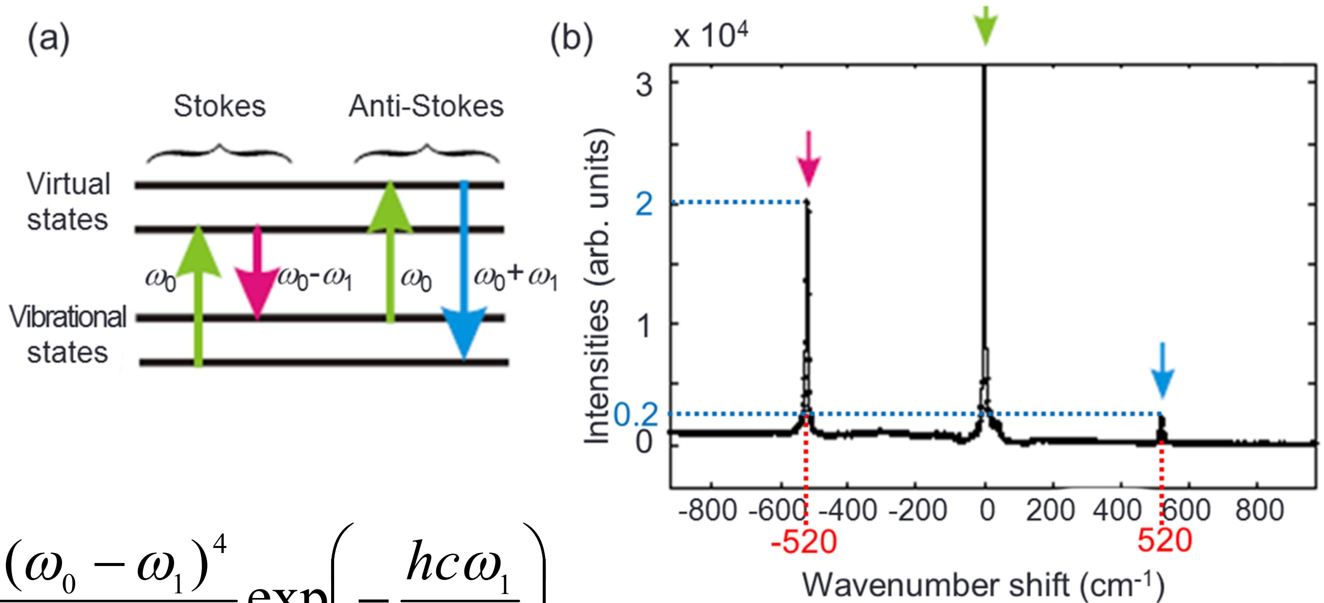
d) $(S_2+S_3)/2$, because R_1 is much larger than the other two resistances.

e) S_3, because R_3 is much smaller than the other two resistances.

$$S = \frac{V_{oc}}{\Delta T} = \frac{\frac{S_1}{R_1} + \frac{S_2}{R_2} + \frac{S_3}{R_3}}{\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}}$$



Prob. 5. Raman spectroscopy



$$\frac{I_{AS}}{I_S} = \frac{(\omega_0-\omega_1)^4}{(\omega_0+\omega_1)^4}\exp\left(-\frac{hc\omega_1}{k_B T}\right)$$

$$\therefore T = \frac{hc\omega_1}{k_B}\frac{1}{\ln\left[\frac{I_S}{I_{AS}}\frac{(\omega_0-\omega_1)^4}{(\omega_0+\omega_1)^4}\right]} = \frac{(6.626\times10^{-34}\ \mathrm{m^2kg/s})(3\times10^8\ \mathrm{m/s})(520\times10^2\,\mathrm{m^{-1}})}{(1.38\times10^{-23}\ \mathrm{m^2kg/(s^2K)})\ln\left[\left(\frac{2}{0.2}\right)\left(\frac{1/(632.8\times10^{-7})-520}{1/(632.8\times10^{-7})+520}\right)^4\right]}$$

$$= 367.3\ \mathrm{K}$$

Prob. 6. Ballistic and diffusive thermal transport

Diffusive: $t_d = \dfrac{l^2}{D}$

Ballistic: $t_b = \dfrac{l}{v_s}$

$$\frac{l_c}{v_s} = \frac{l_c^{\ 2}}{D}$$

$$\therefore\ l_c = \frac{D}{v_s} = \frac{8.8\times10^{-5}\ \text{m}^2/\text{s}}{8.433\times10^3\ \text{m/s}} = 10.4\ \text{nm}$$ for Silicon

$$\therefore\ t_0 = \frac{l_c}{v_s} = \frac{10.4\ \text{nm}}{8.433\times10^3\ \text{m/s}} = 1.24\ \text{ps}$$