HW 1 Solution

1.1. (a) $\dfrac{q^2}{h} = \dfrac{(1.6\times10^{-19}\ \text{coul})^2}{6.626\times10^{-34}\ \text{J-sec.}}$

$= \dfrac{1.6^2}{6.626}\times10^{-4}\ S = 38.6\ \mu S$

(b) $\dfrac{h}{q^2} = 25.9\ K\Omega$

(c) $kT = 1.38\times10^{-23}\times300\ \text{J}$

$= \dfrac{4.14\times10^{-21}}{1.6\times10^{-19}}\ eV \approx 25\ meV$

(d) $\dfrac{1}{2}\,9.1\times10^{-31}\ kg\times v^2 = 10\times10^{-3}\times1.6\times10^{-19}\ \text{J}$

$v^2 = \dfrac{3.2\times10^{-21}}{9.1\times10^{-31}} = 35.16\times10^{8}\ \dfrac{m^2}{s^2}$

$v = 5.93\times10^4\ m/s$

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1.2. $f_1(E) - f_0(E) = f(E,\mu_1) - f(E,\mu_0)$

$$f(E,\mu) = \frac{1}{\exp\left(\frac{E-\mu}{kT}\right)+1}$$

$$\frac{\partial f}{\partial \mu} \approx \frac{f(E,\mu_1) - f(E,\mu_0)}{\mu_1 - \mu_0}$$

$$f_1(E) - f_0(E) \approx \left(\frac{\partial f}{\partial \mu}\right)_{\mu=\mu_0} (\mu_1 - \mu_0)$$

$$= \left(\frac{-\partial f}{\partial E}\right)_{\mu=\mu_0} (\mu_1 - \mu_0)$$

$$\frac{\partial f}{\partial \mu} = \frac{df}{dx} \cdot \frac{\partial x}{\partial \mu} \rightarrow -1/kT \qquad x \equiv \frac{E-\mu}{kT}$$

$$\frac{\partial f}{\partial E} = \frac{df}{dx} \frac{\partial x}{\partial E} \rightarrow +1/kT$$

$$f_1(E) - f_0(E) \approx \left(-\frac{\partial f_0}{\partial E}\right)(\mu_1 - \mu_0)$$

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1.3. Suppose $G(E) = \underbrace{q I_0}_{\text{constant with dimensions of conductance * energy = charge * current}} \delta(E-\varepsilon)$

constant with dimensions of conductance * energy
= charge * current

$$G = \int dE \left(-\frac{\partial f_0}{\partial E}\right) G(E)$$

$$= q I_0 \int dE \left(-\frac{\partial f_0}{\partial E}\right) \delta(E-\varepsilon)$$

$$= q I_0 \left(-\frac{\partial f_0}{\partial E}\right)_{E=\varepsilon} \equiv G_0$$

$$G_S = q I_0 \int dE \left(-\frac{\partial f_0}{\partial E}\right) \frac{E-\mu_0}{qT} \delta(E-\varepsilon)$$

$$= q I_0 \left(-\frac{\partial f_0}{\partial E}\right)_{E=\varepsilon} \frac{\varepsilon-\mu_0}{qT}$$

$$= G_0 \cdot (\varepsilon-\mu_0)/qT$$

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$$G_P = q\,I_0 \int dE \left(-\frac{\partial f_0}{\partial E}\right) \frac{E-\mu_0}{q}\,\delta(E-\varepsilon)$$

$$= q I_0 \left(-\frac{\partial f_0}{\partial E}\right)_{E=\varepsilon} \frac{\varepsilon-\mu_0}{q}$$

$$= G_0\,(\varepsilon-\mu_0)/q$$

$$G_Q = q\,I_0 \int dE \left(-\frac{\partial f_0}{\partial E}\right) \frac{(E-\mu_0)^2}{q^2 T}\,\delta(E-\varepsilon)$$

$$= q I_0 \left(-\frac{\partial f_0}{\partial E}\right)_{E=\varepsilon} \frac{(\varepsilon-\mu_0)^2}{q^2 T}$$

$$= G_0\,(\varepsilon-\mu_0)^2 / q^2 T$$

$$|S| = G_S/G = (\varepsilon-\mu_0)/qT$$

$$|\Pi| = G_P/G = (\varepsilon-\mu_0)/q$$

$$G_K = G_Q - G_P G_S/G$$

$$\frac{G_K}{G} = \underbrace{G_Q/G}_{\frac{(\varepsilon-\mu_0)^2}{q^2 T}} - \underbrace{G_P/G}_{\frac{\varepsilon-\mu_0}{q}} \cdot \underbrace{G_S/G}_{\frac{\varepsilon-\mu_0}{qT}} = 0$$

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1.4.

$$G = \int dE \left(-\frac{\partial f_0}{\partial E}\right) \cdot G(E)$$

$$\frac{e^{-(E-\mu_0)/kT}}{kT} \qquad \frac{q^2}{h}\left(E/E_0\right)^n$$

$$= \frac{q^2}{h} e^{\mu_0/kT} \int \frac{dE}{kT} e^{-E/kT} \left(E/E_0\right)^n$$

$$= \frac{q^2}{h} \left(\frac{kT}{E_0}\right)^n e^{\mu_0/kT} \underbrace{\int dx\, e^{-x} x^n}_{n!}$$

$$G_S = \int dE \frac{e^{-(E-\mu_0)/kT}}{kT} \frac{q^2}{h}\left(E/E_0\right)^n \frac{E-\mu_0}{qT}$$

$$= \frac{q^2}{h}\left(\frac{kT}{E_0}\right)^n e^{\mu_0/kT} \frac{k}{q} \underbrace{\int dx\, e^{-x} x^n \left(x - \frac{\mu_0}{kT}\right)}_{(n+1)! - \frac{\mu_0}{kT} n!}$$

$$|S| = \frac{G_S}{G} = \frac{k}{q} \frac{(n+1)! - \mu_0/kT \; n!}{n!}$$

$$= \frac{k}{q}\left(n + 1 - \frac{\mu_0}{kT}\right)$$

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