

# Fundamentals of Nanotransistors

## Unit 4: Transmission Theory of the MOSFET

### Lecture 4.6: Connection to the VS Model

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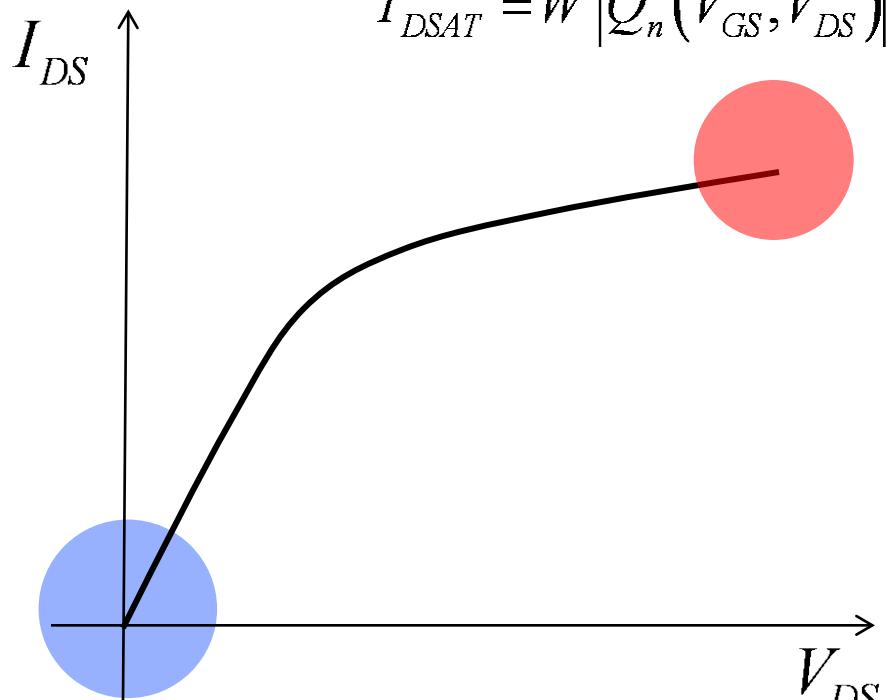
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# Transmission model

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$$I_{DSAT} = W |Q_n(V_{GS}, V_{DS})| v_{inj}$$

$$v_{inj} = \left( \frac{\mathcal{T}_{SAT}}{2 - \mathcal{T}_{SAT}} \right) v_T$$

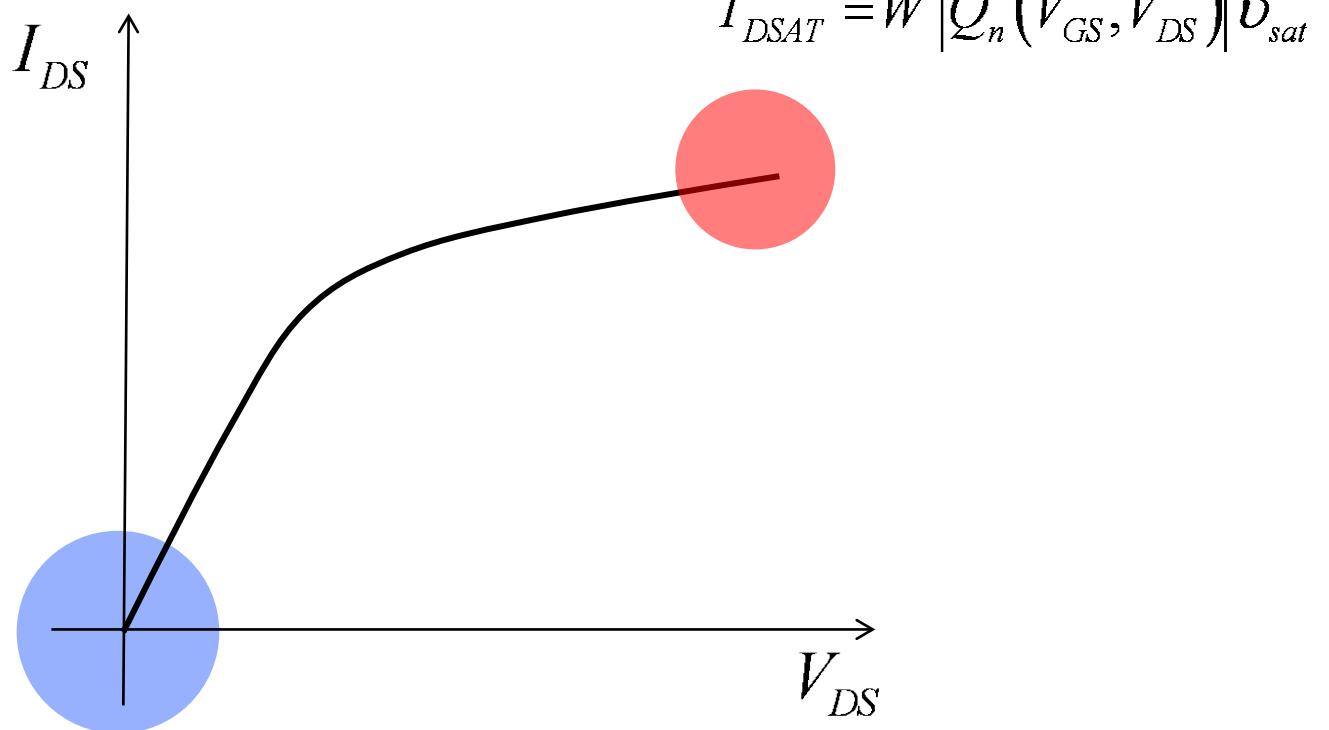
$$\mathcal{T}_{SAT} = \frac{\lambda_0}{\lambda_0 + \ell}$$

$$\ell \ll L$$

$$I_{DLIN} = W |Q_n(V_{GS}, V_{DS})| \left( \mathcal{T}_{LIN} \frac{v_T}{2k_B T/q} \right) V_{DS} \quad \mathcal{T}_{LIN} = \frac{\lambda_0}{\lambda_0 + L}$$

# Traditional (diffusive) model

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$$I_{DLIN} = \frac{W}{L} \mu_n |Q_n(V_{GS}, V_{DS})| V_{DS}$$

# Connection: Linear region

Traditional:

$$I_{DS} = \frac{W}{L} \mu_n C_{inv} (V_{GS} - V_T) V_{DS} \quad \text{above threshold}$$

Transmission:

$$I_{DS} = \mathcal{T}_{LIN} \left( WC_{inv} (V_{GS} - V_T) \frac{v_T}{(2k_B T / q)} \right) V_{DS} \quad \mathcal{T}_{LIN} = \frac{\lambda_0}{\lambda_0 + L}$$

$$I_{DS} = \frac{W}{L + \lambda_0} \mu_n C_{inv} (V_{GS} - V_T) V_{DS}$$

$$I_{DS} = \frac{W}{L} \mu_{app} C_{inv} (V_{GS} - V_T) V_{DS}$$

$$1/\mu_{app} = 1/\mu_n + 1/\mu_B$$

# Apparent mobility

$$I_{DS} = \frac{W}{L} \mu_{app} C_{inv} (V_{GS} - V_T) V_{DS}$$
$$\frac{1}{\mu_{app}} = \frac{1}{\mu_n} + \frac{1}{\mu_B}$$

“Mathiessen’s Rule”

$$\mu_n = \frac{v_T \lambda_0}{2 k_B T / q} \quad \text{due to scattering in the channel}$$

$$\mu_B = \frac{v_T L}{2 k_B T / q} \quad \text{ballistic mobility (due to scattering in the source and drain)}$$

# Example

Estimate the apparent mobility for a 22 nm N-MOSFET.

$$\mu_n \approx 200 \text{ cm}^2/\text{V-s}$$

$$\mu_B = \frac{v_T L}{2 k_B T / q}$$

$$v_T = \sqrt{\frac{2k_B T}{\pi m_t^*}} = 1.2 \times 10^7 \text{ cm/s}$$

$$\mu_B \approx 500 \text{ cm}^2/\text{V-s}$$

$$\frac{1}{\mu_{app}} = \frac{1}{\mu_n} + \frac{1}{\mu_B} = \frac{1}{200} + \frac{1}{500}$$

$$\mu_{app} \approx 140 \frac{\text{cm}^2}{\text{V-s}}$$

This device operates in the quasi-ballistic regime.

(Assumes confinement in the <100> direction.)

# Connection: Saturation region

Traditional:

$$I_{DSAT} = WC_{inv}(V_{GS} - V_T)v_{sat} \quad \text{above threshold}$$

Transmission:

$$I_{DSAT} = WC_{inv}(V_{GS} - V_T) \left( \frac{\mathcal{T}_{SAT}}{2 - \mathcal{T}_{SAT}} \right) v_T \quad \mathcal{T}_{SAT} = \frac{\lambda_0}{\lambda_0 + \ell}$$

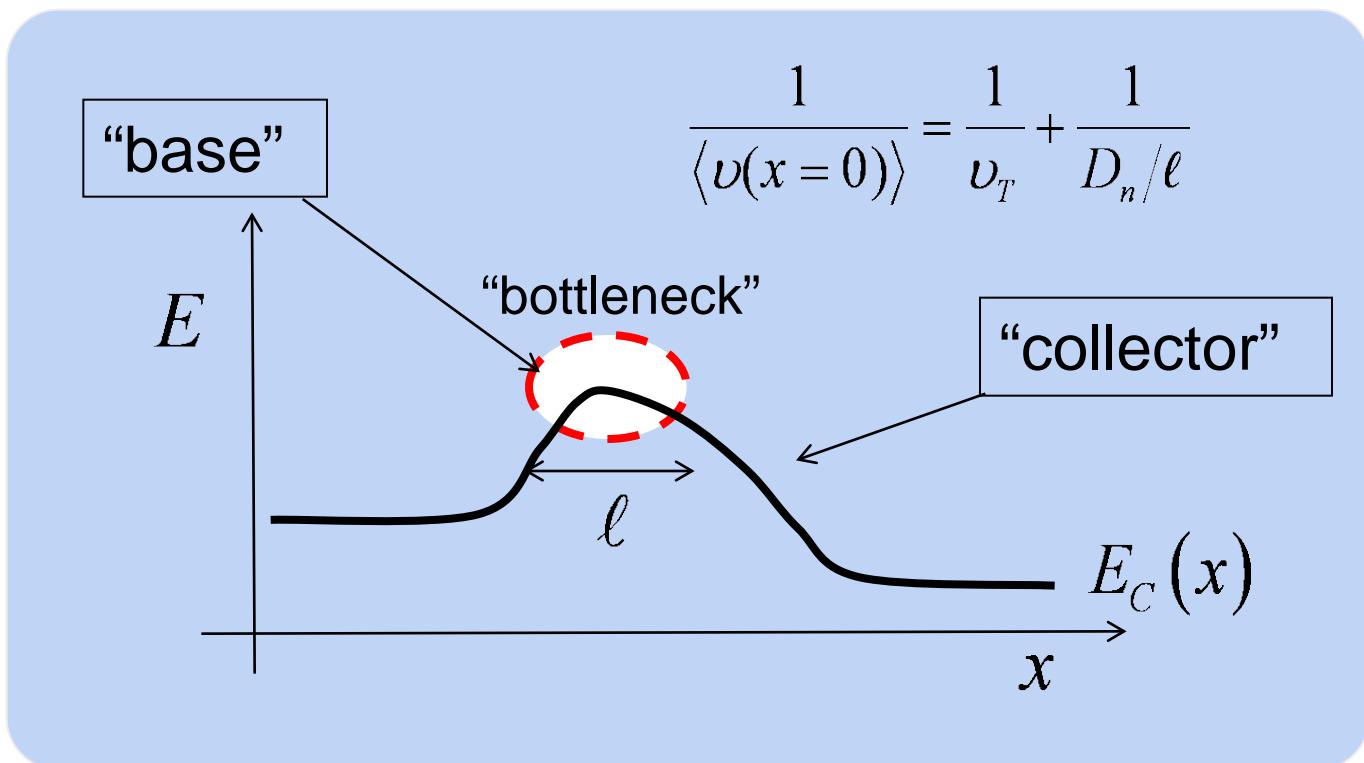
$$I_{DSAT} = WC_{inv}(V_{GS} - V_T) \left[ \frac{1}{v_T} + \frac{1}{(D_n/\ell)} \right]^{-1}$$

$$D_n = \frac{v_T \lambda_0}{2}$$

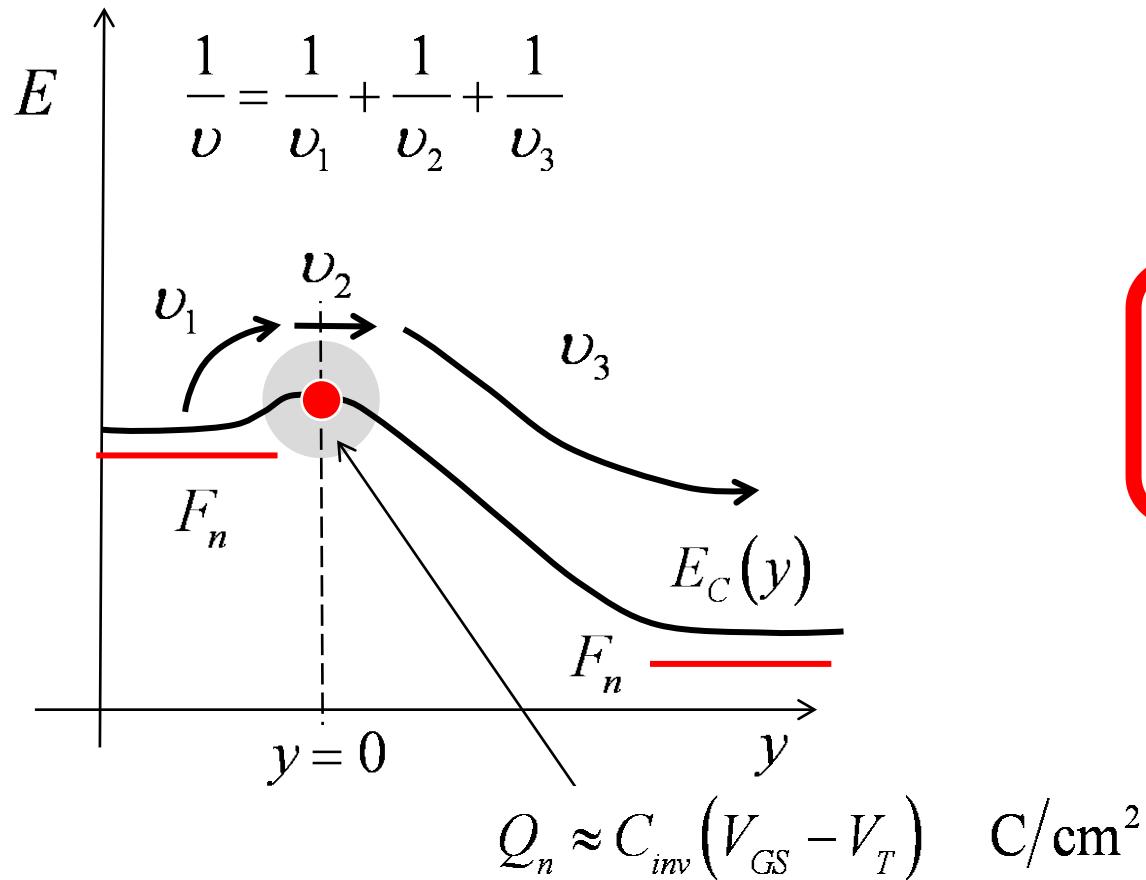
*How do we interpret this result?*

# Saturation current in a nanoscale MOSFET

$$I_{DS} = WC_{inv} \left( V_{GS} - V_T \right) \langle v(x=0) \rangle$$



# Injection velocity



$$\frac{1}{v} \approx \frac{1}{v_1} + \frac{1}{v_2}$$

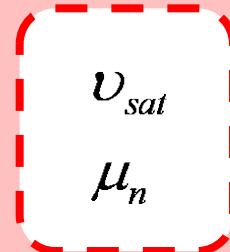
$$\frac{1}{v_{inj}} = \frac{1}{v_T} + \frac{1}{D_n/\ell}$$

# VS model for the MOSFET

$$I_{DS} = W Q_n(V_{GS}, V_{DS}) \langle v(0) \rangle$$

$$\langle v(0) \rangle = \left\{ \frac{V_D / V_{DSAT}}{\left[ 1 + (V_D / V_{DSAT})^\beta \right]^{1/\beta}} \right\} v_{sat} = F_{SAT}(V_{DS}) v_{sat}$$

$$V_{DSAT} = \frac{v_{sat} L}{\mu_n}$$



# a more physical VS model for a MOSFET

$$1) \quad \mu_n \rightarrow \mu_{app} \quad \frac{1}{\mu_{app}} = \frac{1}{\mu_n} + \frac{1}{\mu_B}$$

$$2) \quad v_{sat} \rightarrow v_{inj} = \left( \frac{\mathcal{T}_{SAT}}{2 - \mathcal{T}_{SAT}} \right) v_T = \left[ \frac{1}{v_T} + \frac{1}{D_n/\ell} \right]^{-1}$$

$$v_{inj}^{ball} = v_T \quad (\text{non-degenerate carrier statistics})$$

# Summary

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## A Simple Semiempirical Short-Channel MOSFET Current–Voltage Model Continuous Across All Regions of Operation and Employing Only Physical Parameters

Ali Khakifirooz, *Member, IEEE*, Osama M. Nayfeh, *Member, IEEE*, and Dimitri Antoniadis, *Fellow, IEEE*

$$\frac{1}{\mu_n} \rightarrow \frac{1}{\mu_{app}} \quad \text{"apparent mobility"}$$

$$v_{sat} \rightarrow v_{inj} \quad \text{"injection velocity"}$$

# Next lecture

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Using the VS model to characterize the electrical performance of nanotransistors.