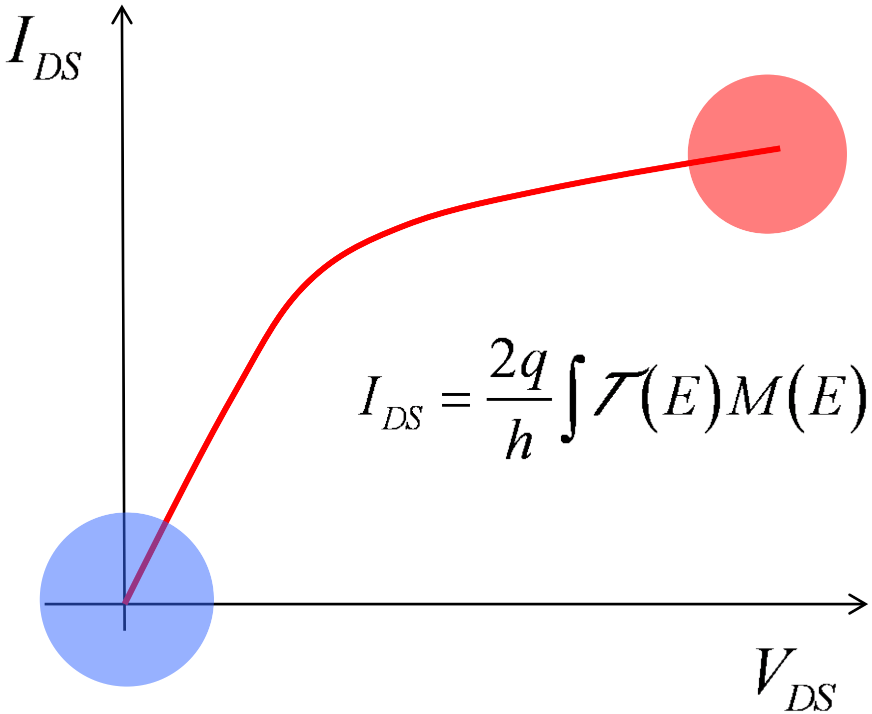
Fundamentals of Nanotransistors

Unit 4: Transmission Theory of the MOSFET

Lecture 4.4: Transmission Theory of the MOSFET: I

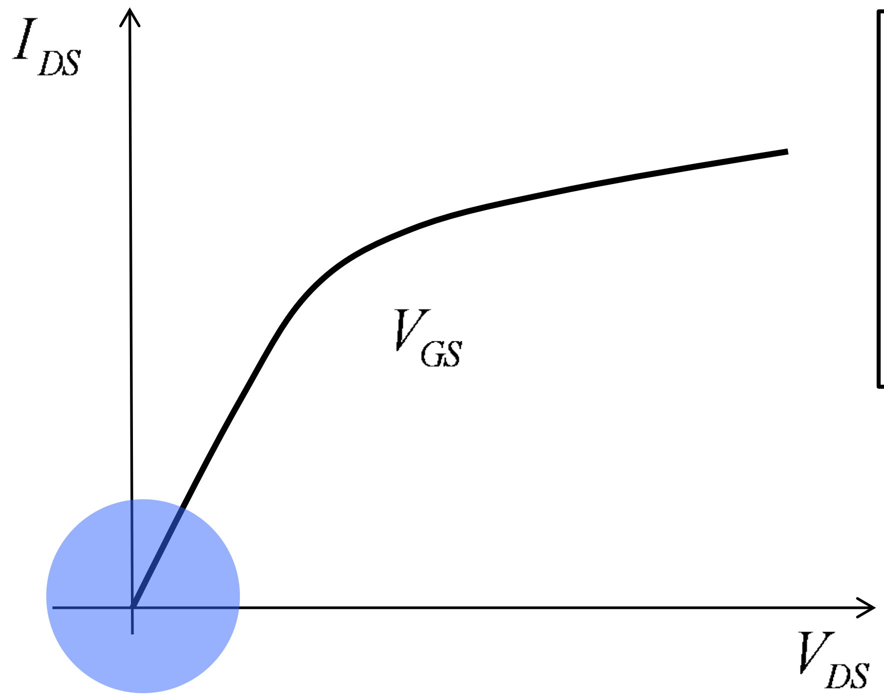
Mark Lundstrom
lundstro@purdue.edu
Electrical and Computer Engineering
Birck Nanotechnology Center
Purdue University, West Lafayette, Indiana USA

Transmission theory of the MOSFET



$$I_{DS} = \frac{2q}{h}\int \mathcal{T}(E)M(E)\left(f_S - f_D\right)dE$$

Linear region with scattering



assume: $\lambda(E) = \lambda_0$

$$\mathcal{T}(E) = \mathcal{T}_0 = \frac{\lambda_0}{\lambda_0 + L}$$

$$I_{DLIN} = W\left|Q_n\left(V_{GS}, V_{DS}\right)\right| \frac{\upsilon_T}{2k_BT/q} V_{DS} \rightarrow I_{DLIN} = \mathcal{T}_0 W\left|Q_n\left(V_{GS}, V_{DS}\right)\right| \frac{\upsilon_T}{2k_BT/q} V_{DS}$$

Linear region in the diffusive limit

$$I_{DLIN} = \mathcal{T}_0 W \left|Q_n(V_{GS}, V_{DS})\right| \frac{\upsilon_T}{2k_BT/q} V_{DS}$$

$$\mathcal{T}(E) = \mathcal{T}_0 = \frac{\lambda_0}{\lambda_0 + L} \rightarrow \frac{\lambda_0}{L}$$

$$I_{DLIN} = \frac{\lambda_0}{L} W \left|Q_n(V_{GS}, V_{DS})\right| \frac{\upsilon_T}{2k_BT/q} V_{DS}$$

$$I_{DLIN} = \frac{W}{L}\left(\frac{\upsilon_T \lambda_0}{2k_BT/q}\right) \left|Q_n(V_{GS}, V_{DS})\right| V_{DS}$$

$$\left(\frac{\upsilon_T \lambda_0}{2k_BT/q}\right) = \left(\frac{\upsilon_T \lambda_0}{2}\right)\left(\frac{1}{k_BT/q}\right)$$

$$= \left(\frac{D_n}{k_BT/q}\right)$$

$$= \mu_n$$

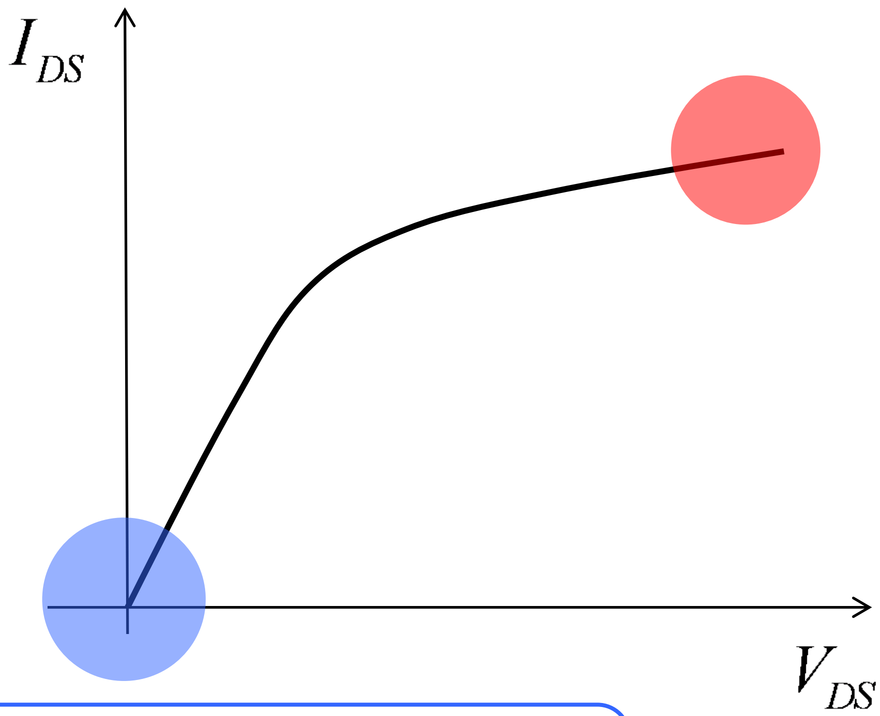
$$I_{DLIN} = \frac{W}{L} \mu_n \left|Q_n(V_{GS}, V_{DS})\right| V_{DS}$$

$$\mu_n = \frac{\upsilon_T \lambda_0}{2k_BT/q}$$

✓

Saturation region with scattering

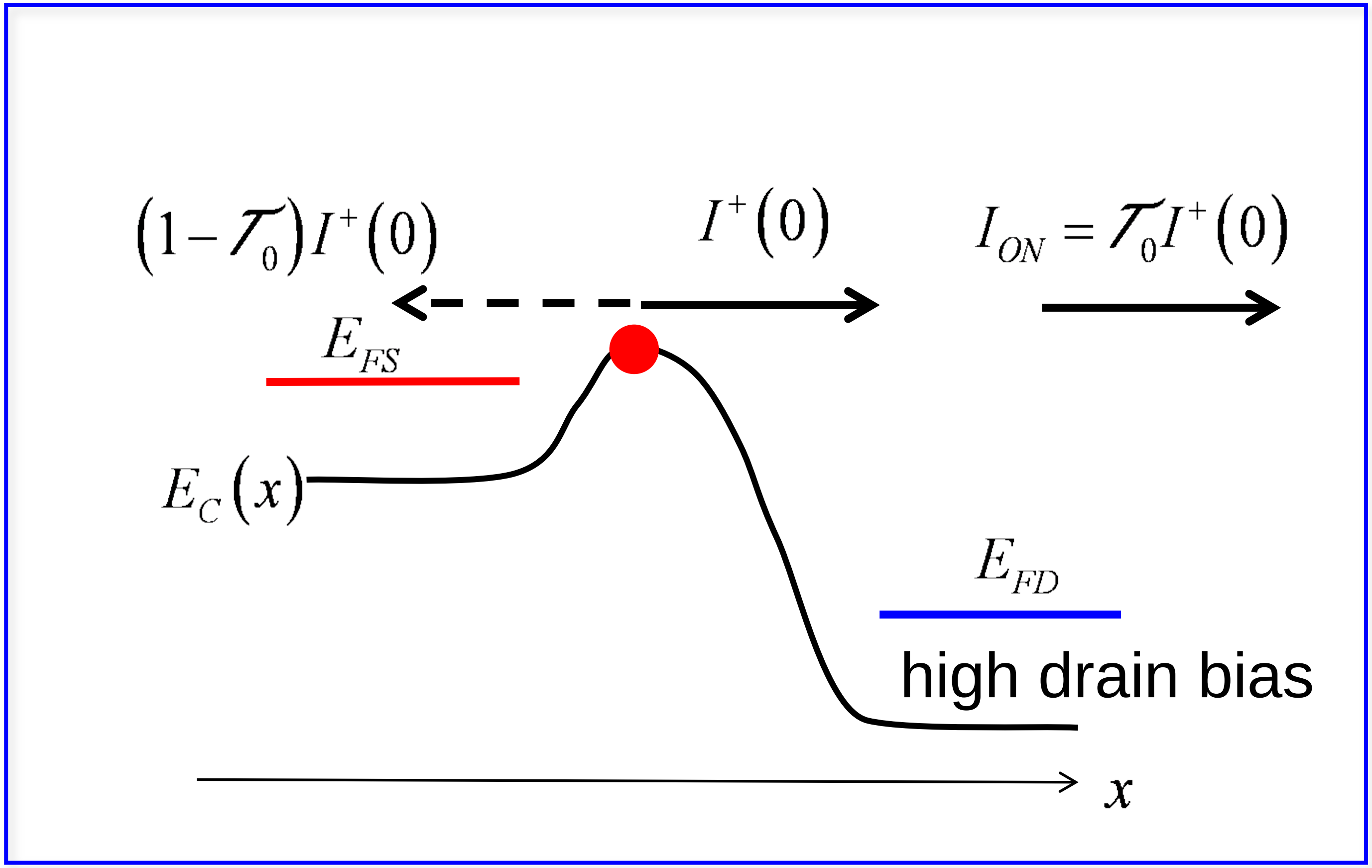
$$I_{DSAT} = \mathcal{T}_0 W \left| Q_n \left(V_{GS}, V_{DS} \right) \right| \upsilon_T$$



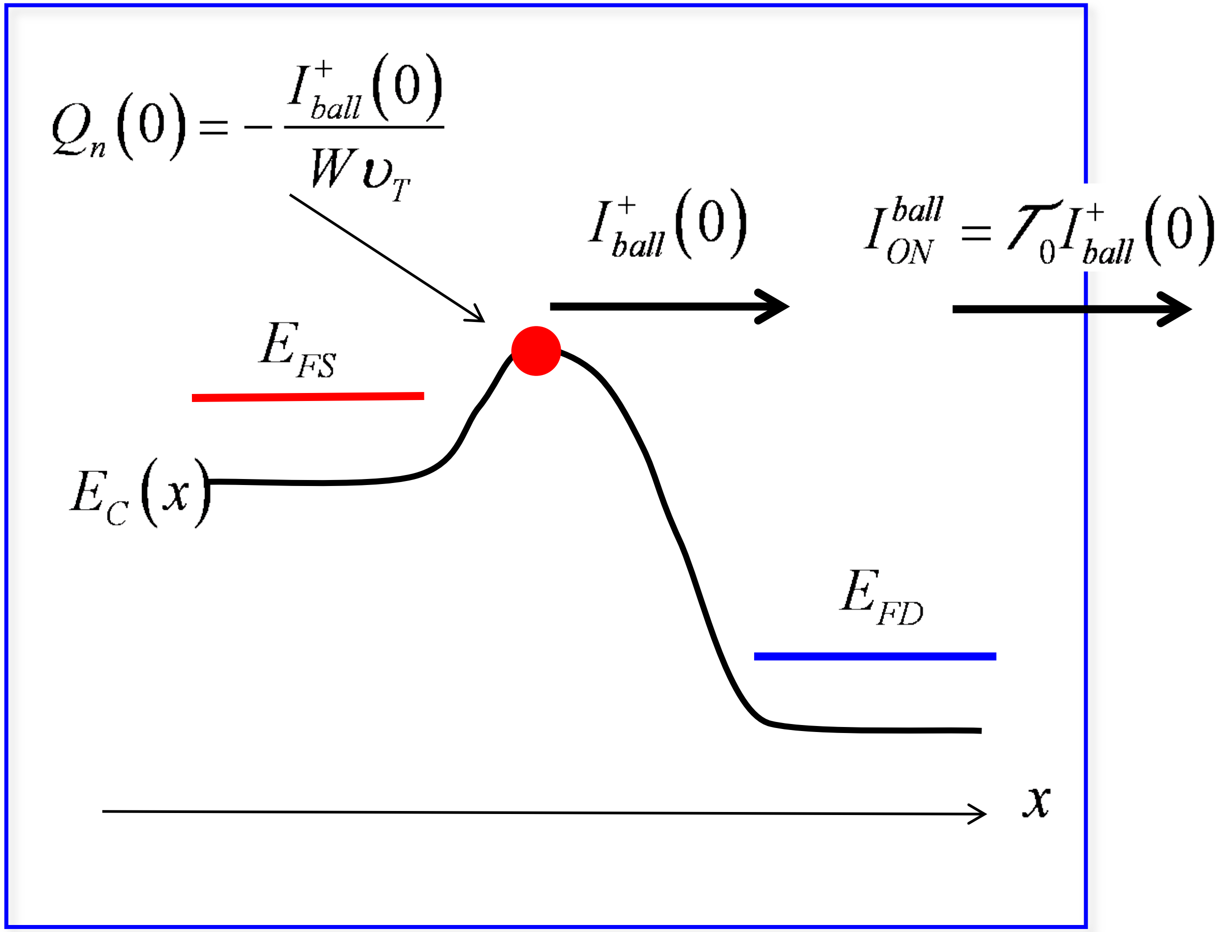
X

$$I_{DLIN} = \mathcal{T}_0 W \left| Q_n \left(V_{GS}, V_{DS} \right) \right| \frac{\upsilon_T}{2k_BT/q} V_{DS}$$

On-current and transmission



Consider ballistic case first



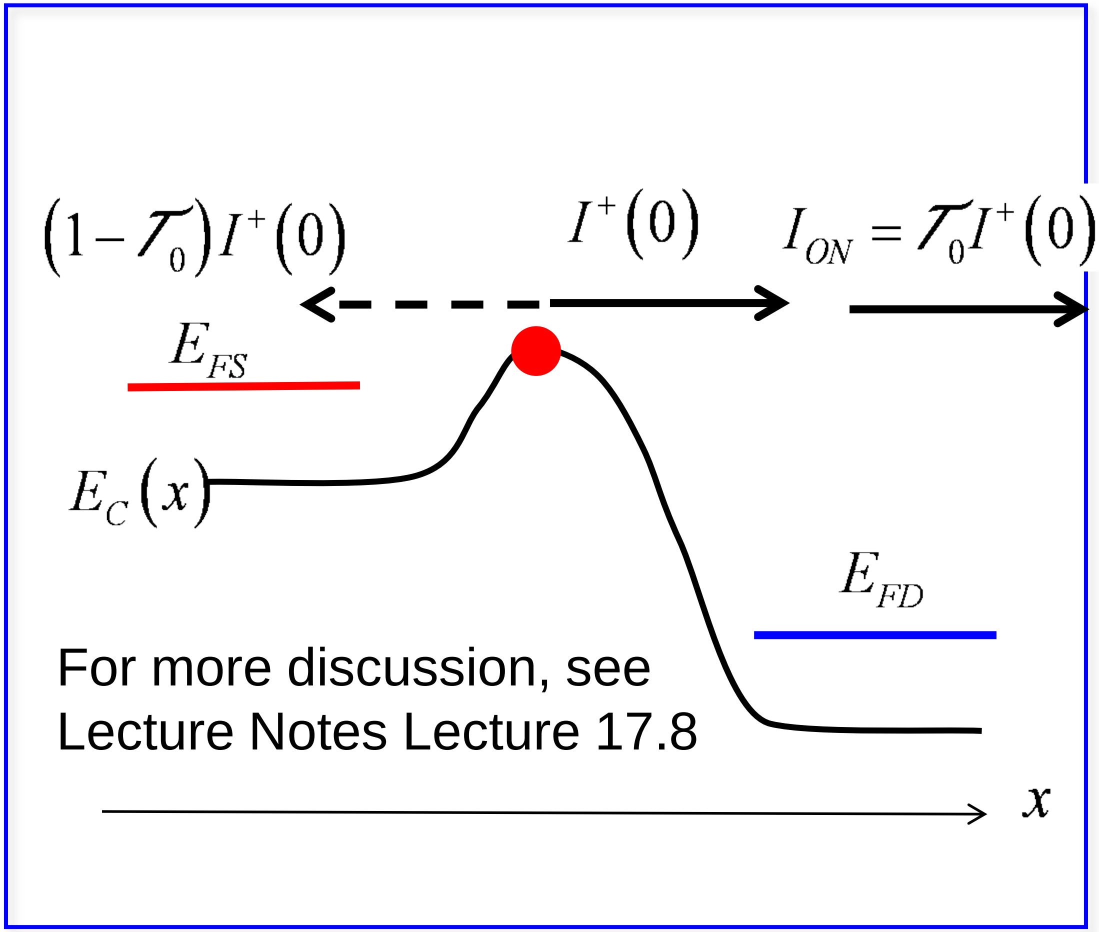
On-current and transmission

$$Q_n(0) = -\frac{I^+_{ball}(0)}{W\upsilon_T}$$

$$Q_n(0) = -\frac{I^+(0) + (1-\mathcal{T}_0)I^+(0)}{W\upsilon_T}$$

$$I^+(0) = \frac{I^+_{ball}(0)}{(2-\mathcal{T}_0)}$$

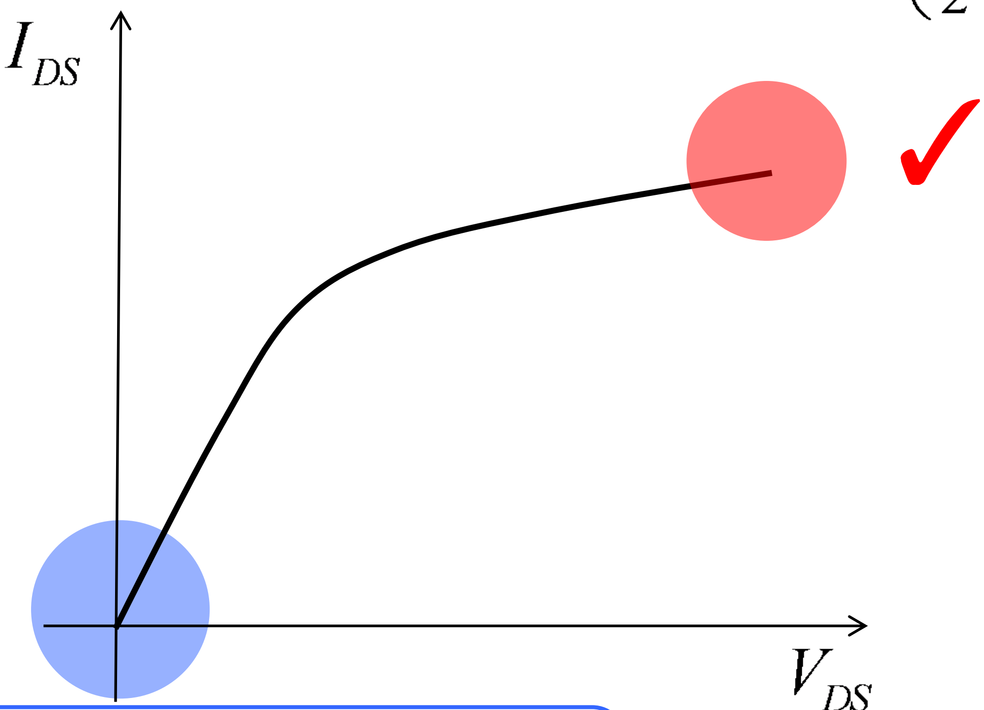
$$I_{ON} = \left(\frac{\mathcal{T}_0}{2-\mathcal{T}_0}\right) I^+_{ball}(0)$$



For more discussion, see Lecture Notes Lecture 17.8

Saturation current

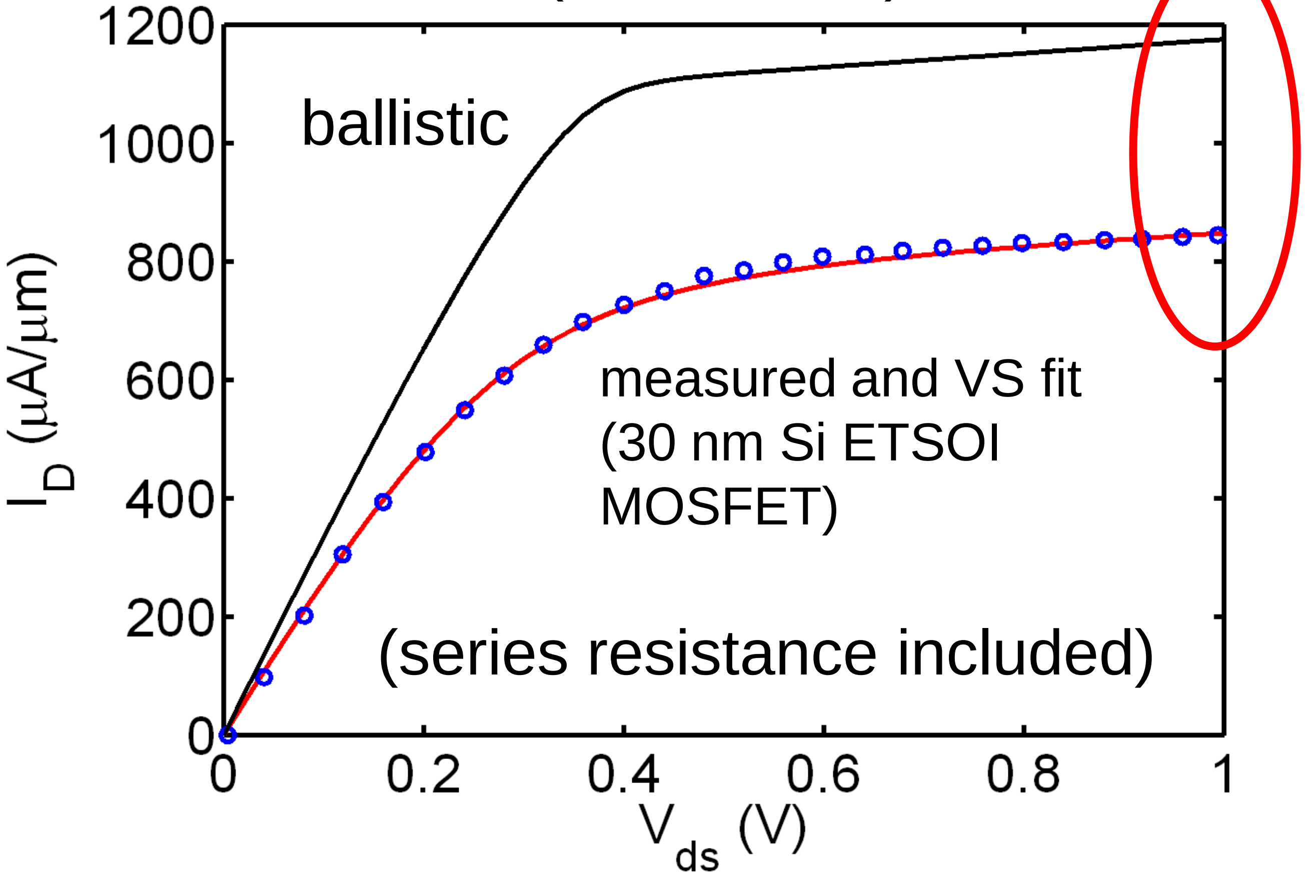
$$I_{DSAT} = \left(\frac{\mathcal{T}_0}{2-\mathcal{T}_0}\right) W \left| Q_n\left(V_{GS}, V_{DS}\right) \right| \upsilon_T$$



$$I_{DLIN} = \mathcal{T}_0 W \left| Q_n\left(V_{GS}, V_{DS}\right) \right| \frac{\upsilon_T}{2k_B T/q} V_{DS}$$

Transmission in saturation

(Lecture 3.6)



$$\frac{I_{ON}}{I_{ON}^{ball}} = 0.70$$

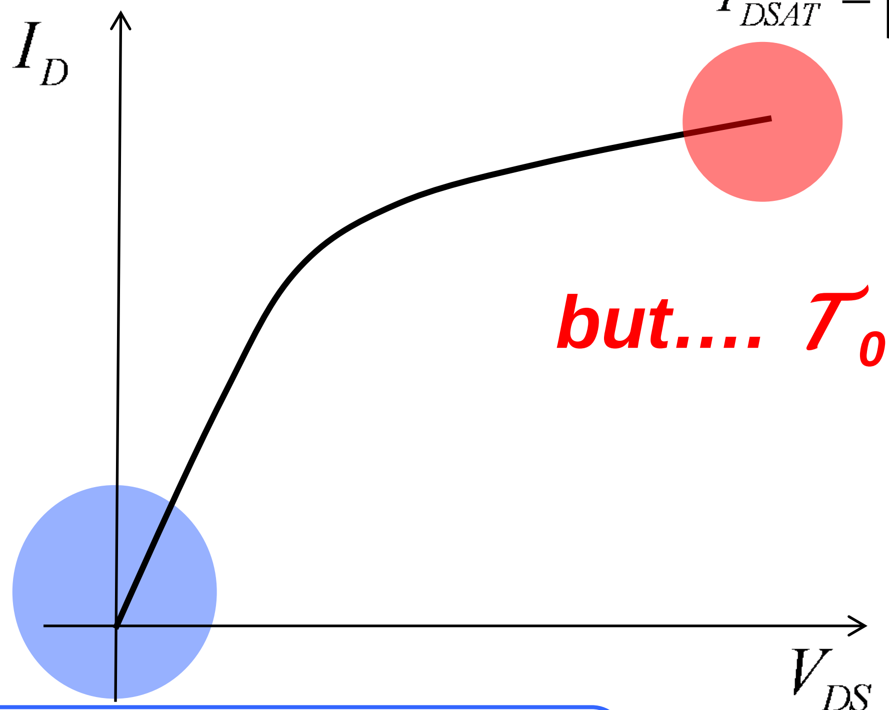
$$\frac{I_{DSAT}}{I_{DSAT}^{ball}} = \left(\frac{\mathcal{T}_0}{2 - \mathcal{T}_0}\right)$$

$$\mathcal{T}_0 = 0.82$$

Fig. 15.2 *Fundamentals of Nanotransistors*, World Scientific Lecture Notes, 2015.

Saturation current

$$I_{DSAT} = \left(\frac{\mathcal{T}_{SAT}}{2-\mathcal{T}_{SAT}}\right) W \left|Q_n\left(V_{GS}, V_{DS}\right)\right| \upsilon_T$$



but…. $\mathcal{T}_0$ depends on V_{DS}!

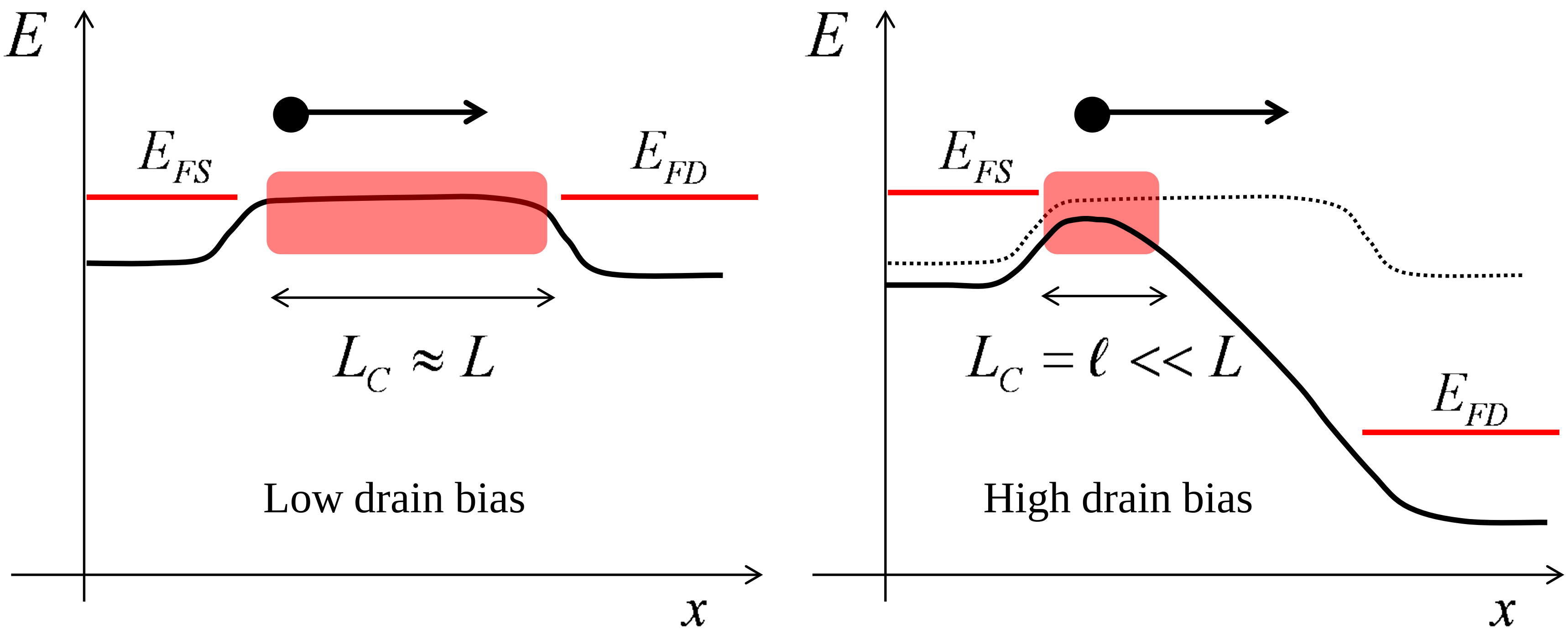
$$\boxed{\mathcal{T}_{LIN} < \mathcal{T}_{SAT}}$$

$$\boxed{I_{DS} = \mathcal{T}_{LIN} W \left|Q_n\left(V_{GS}, V_{DS}\right)\right| \frac{\upsilon_T}{2k_BT/q} V_{DS}}$$

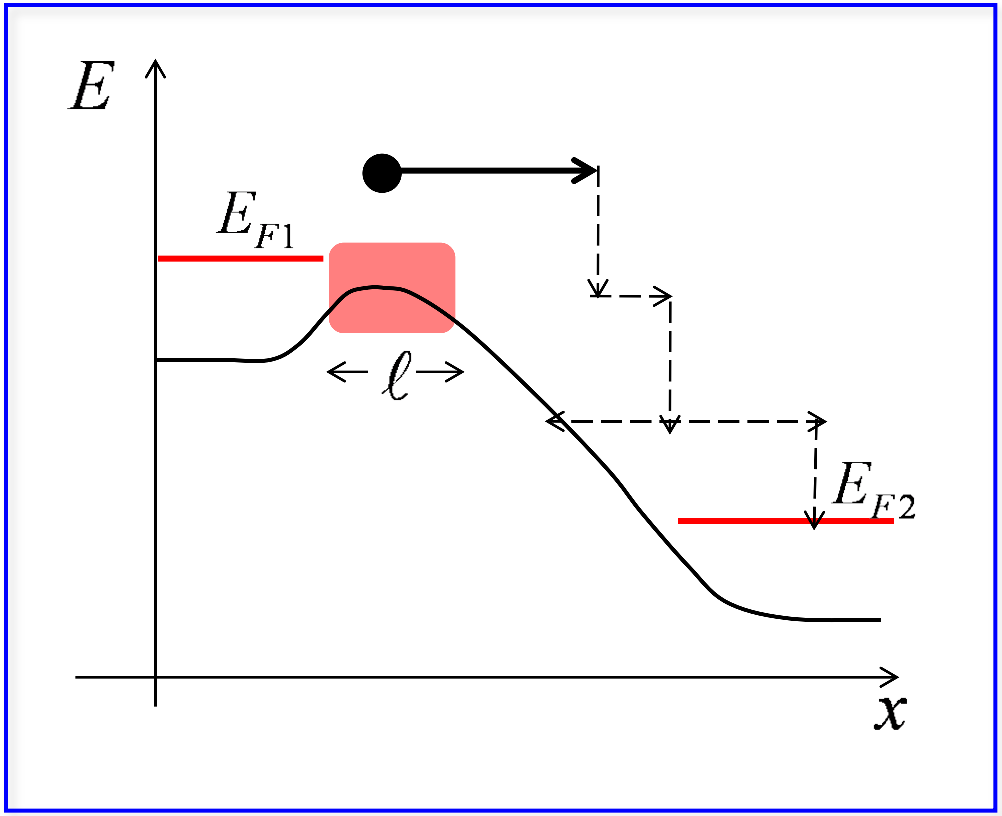
(Lecture 4.6 Characterization)

Scattering under low and high V_{DS}

$$\mathcal{T}_{LIN} = \frac{\lambda_0}{\lambda_0 + L} \qquad \mathcal{T}(V_{DS}) = \frac{\lambda_0}{\lambda_0 + L_C(V_{DS})} \qquad \mathcal{T}_{SAT} = \frac{\lambda_0}{\lambda_0 + \ell}$$



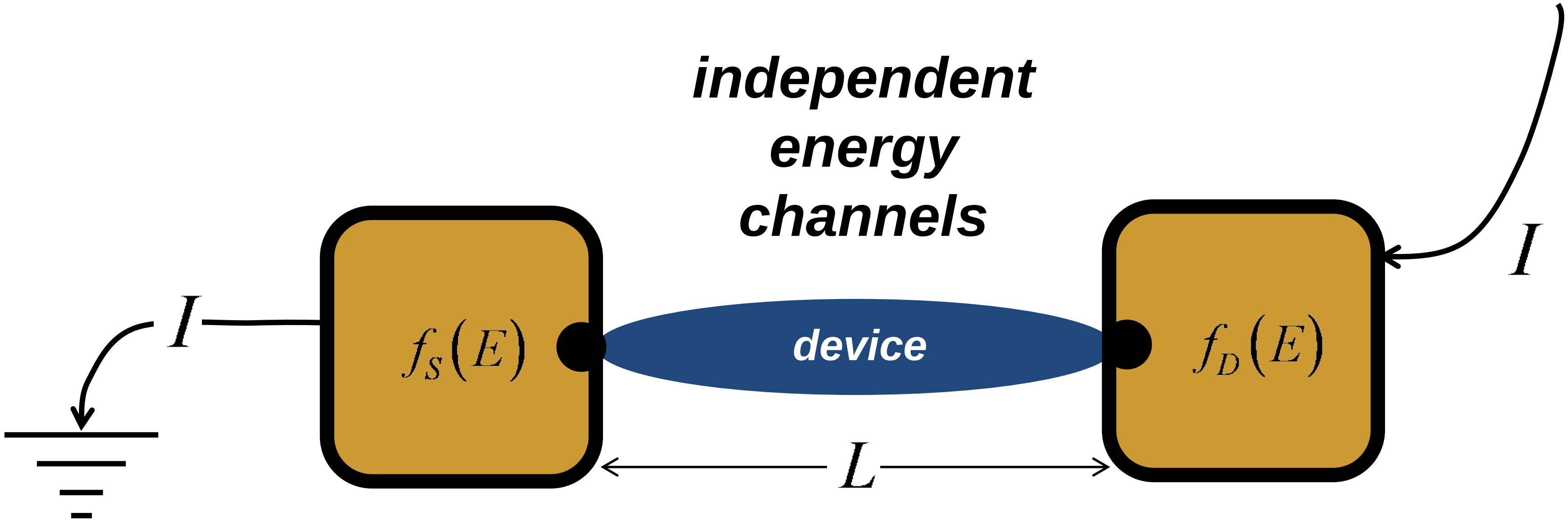
Operation near the "ballistic limit"



$$\mathcal{T}_{SAT} = \frac{\lambda_0}{\lambda_0 + \ell}$$

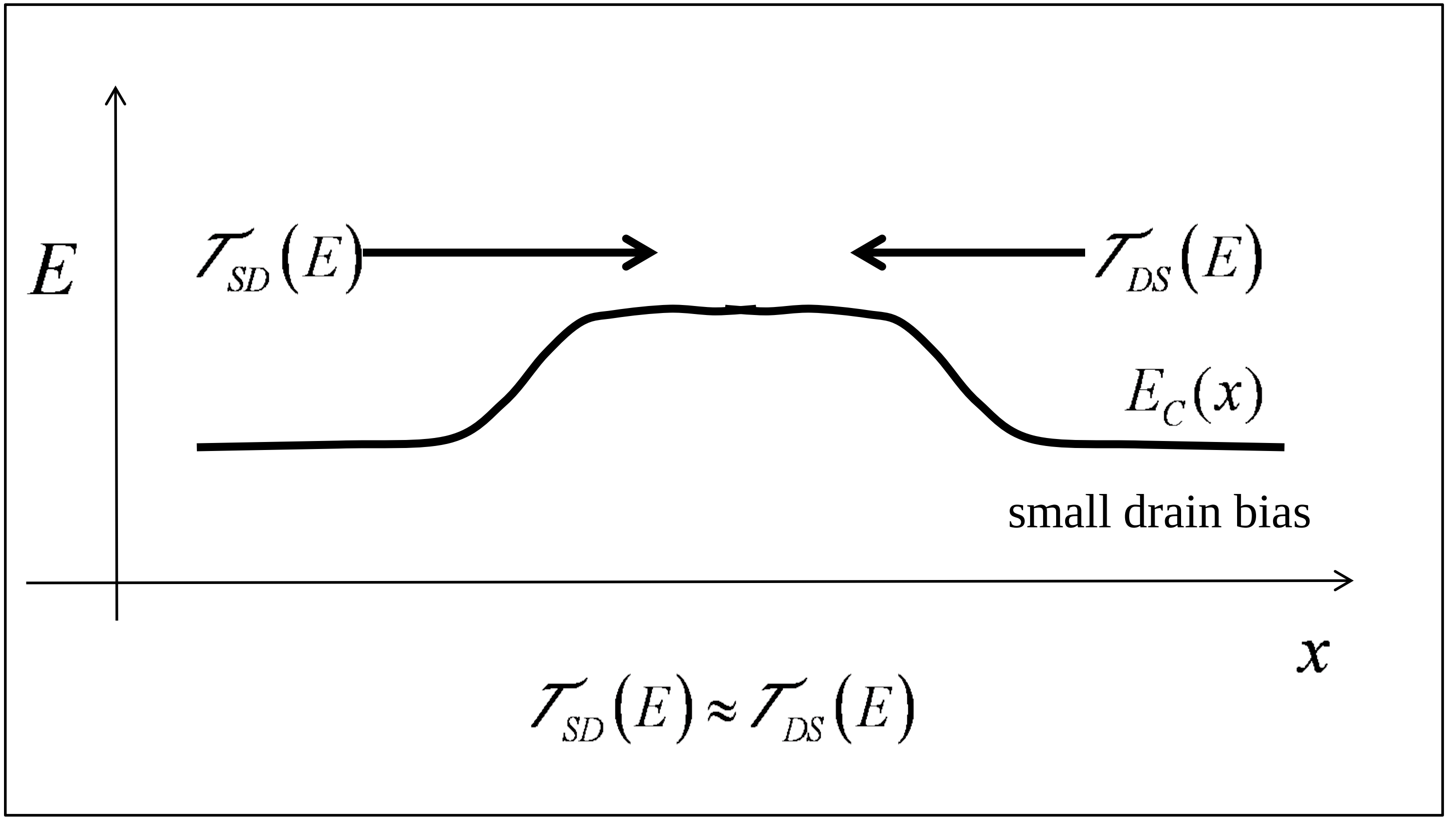
Operation near the ballistic limit current just means that $\mathcal{T}_{SAT} \rightarrow 1$, **it does not imply** that there is little scattering.

Landauer Approach

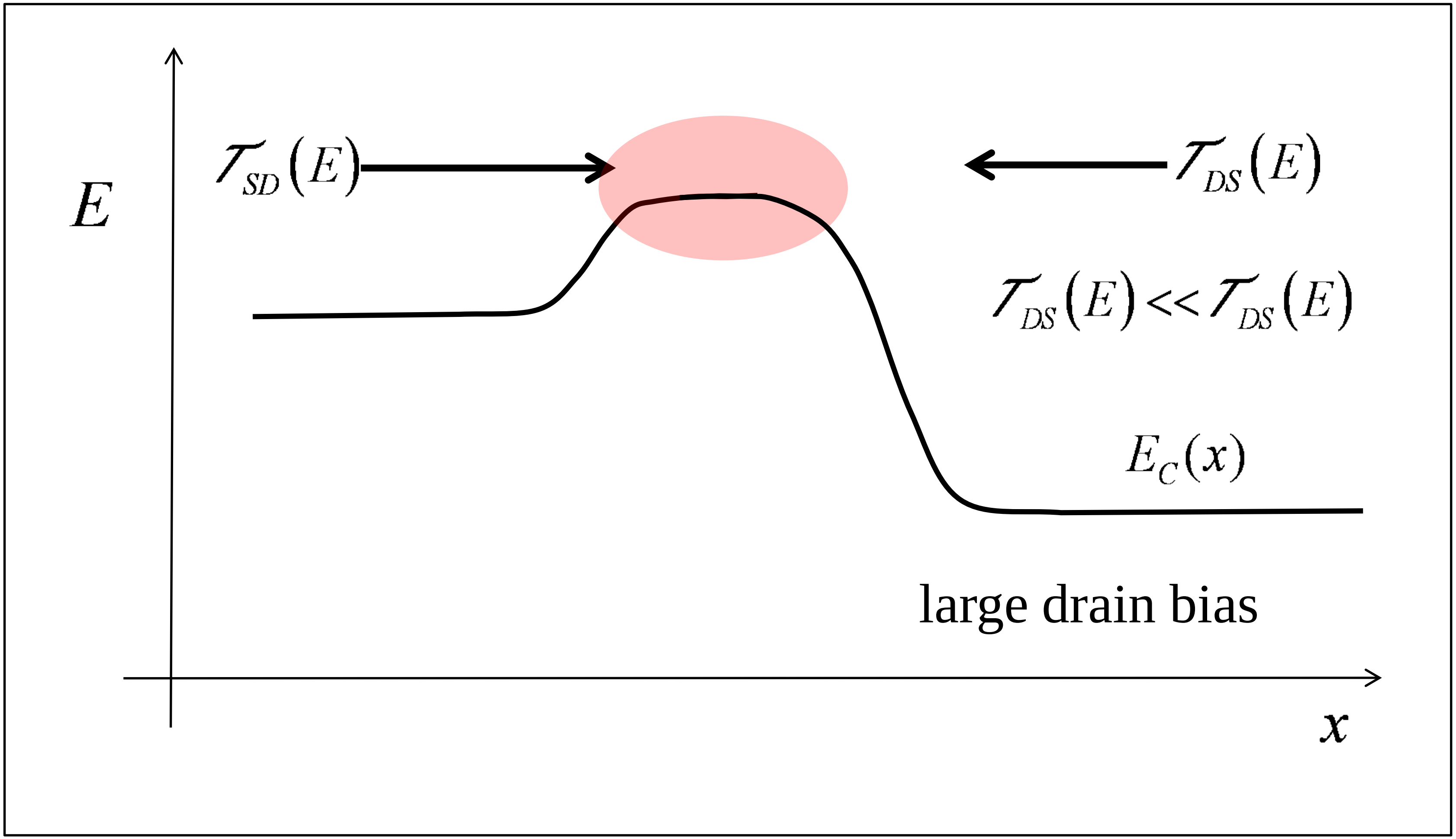


$$\mathcal{T}_{SD}(E) \approx \mathcal{T}_{DS}(E) \quad ?$$

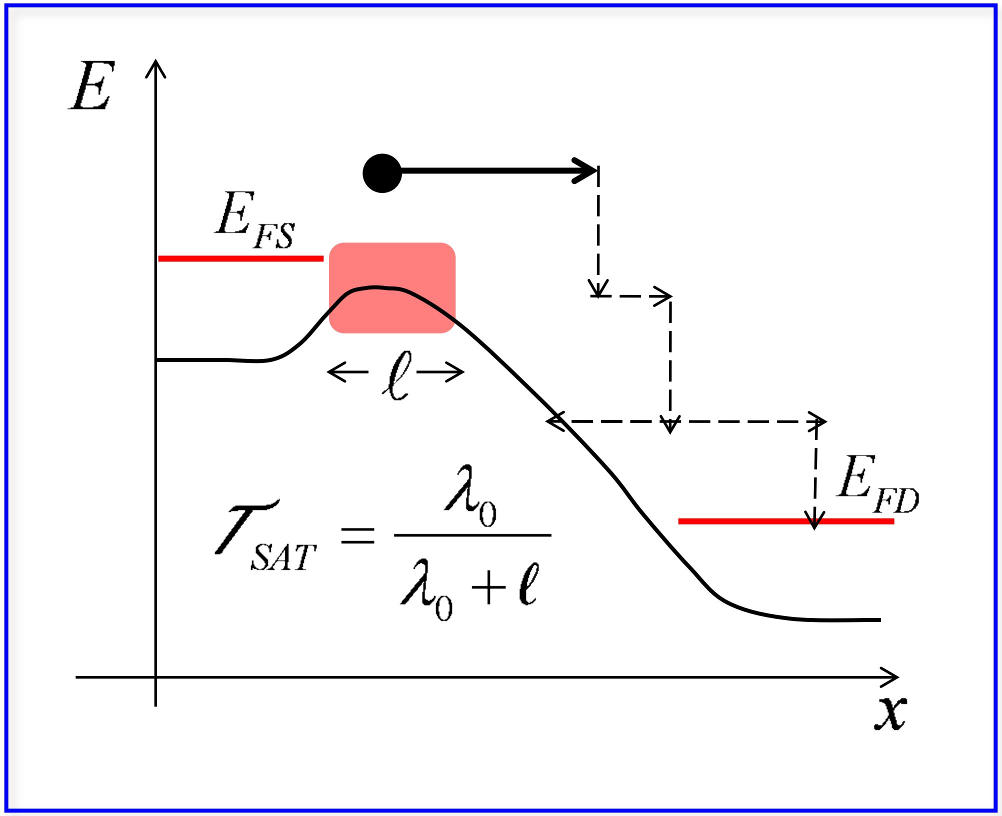
Low drain to source bias



Large drain to source bias



Is mobility relevant at the nanoscale?



- mobility is related to the near-eq. MFP
- backscattering in the critical region is also controlled by the near-eq. MFP.
- mobility determines the on-current
- but the MFP near the drain is very short.

Summary

- Scattering lowers the drain current

- Transmission is **higher** under high drain bias than under low drain bias.

- Under low drain bias, transmission is determined by backscattering in the **entire channel**.

- Under high drain bias, transmission is determined by backscattering in a **short, “bottleneck region”** near the top of the barrier.