Unit 4: Transmission Theory of the MOSFET

Lecture 4.2: Transmission

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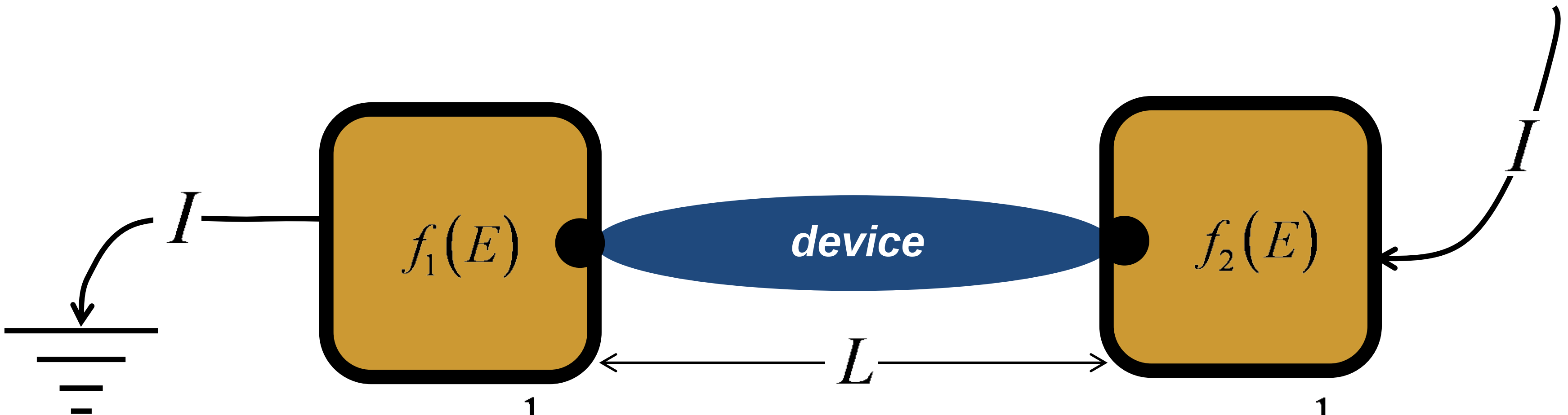
Landauer Approach



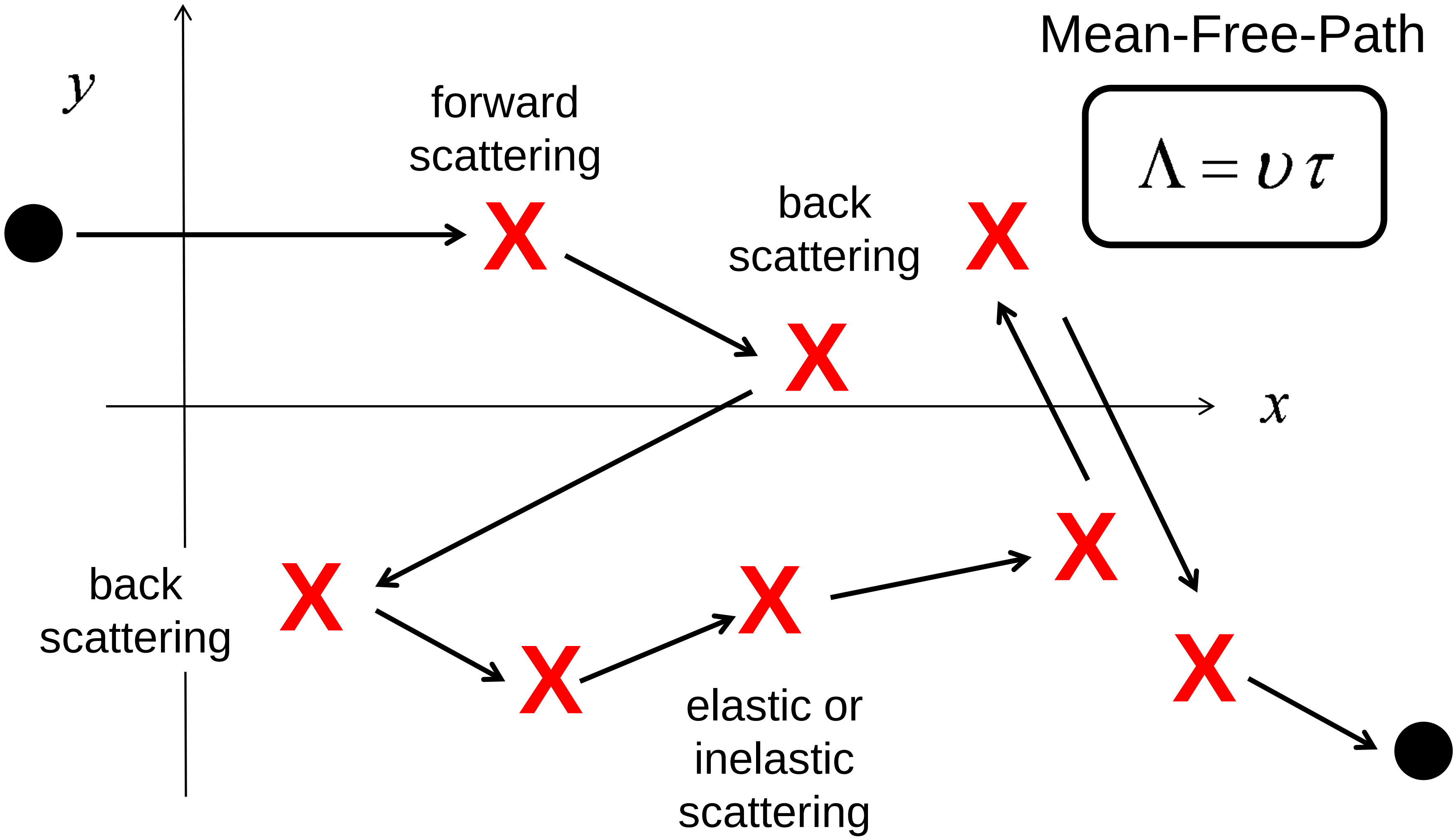
$$f_1(E) = \frac{1}{1 + e^{(E - E_{F1})/k_B T}}$$

$$f_2(E) = \frac{1}{1 + e^{(E - E_{F2})/k_B T}}$$

$$E_{F2} = E_{F1} - qV$$

$$I = \frac{2q}{h} \int \mathcal{T}(E) M(E) \left(f_1(E) - f_2(E) \right) dE$$

Transmission and MFP



Mean-Free-Path

$$\boxed{\Lambda = \upsilon \tau}$$

Scattering time / rate

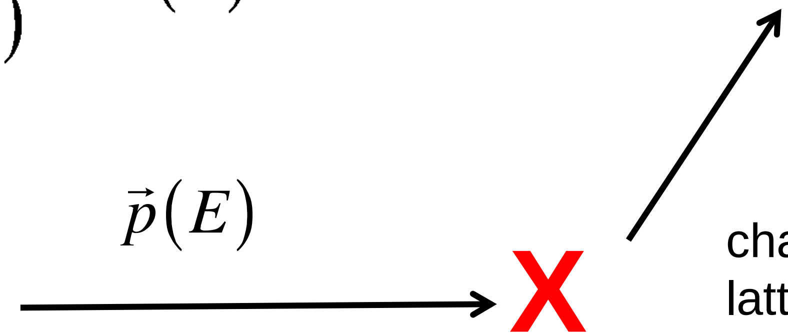
$$\frac{1}{\tau(E)} \propto D(E)$$

$\vec{p}\,'(E')$

elastic
inelastic

isotropic
anisotropic

$\vec{p}(E)$



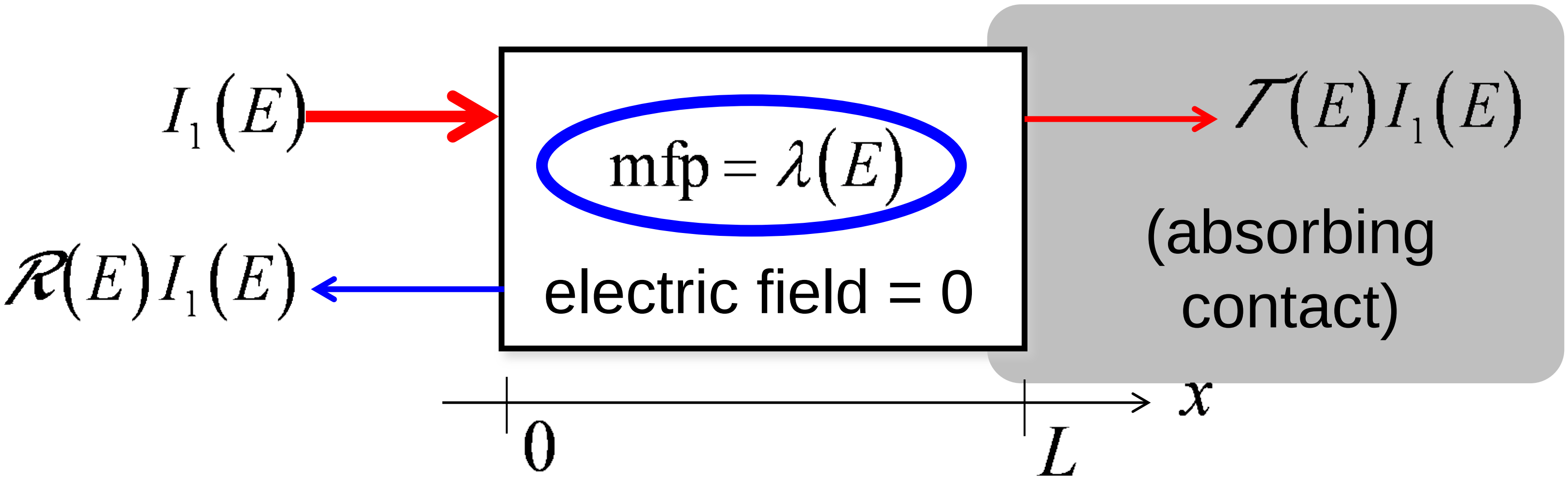
charged impurities
lattice vibrations (phonons)
surface roughness,
other electrons
holes.
etc.

$$\Lambda(E) = \upsilon(E)\tau(E)$$

$\tau(E)$: scattering time - average time between scattering events

$1/\tau(E)$: scattering rate - probability per sec of scattering

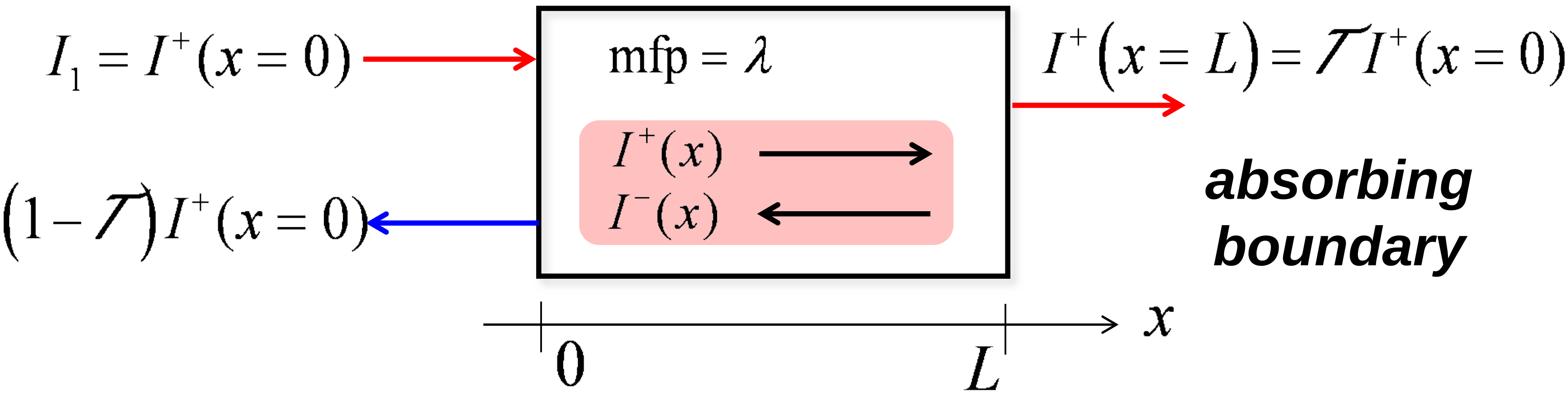
Transmission across a field-free slab



Consider a flux of carriers injected from the left into a field-free slab of length, *L*. The flux that emerges at $x = L$ is $\mathcal{T}$ times the incident flux, where $0 < \mathcal{T} < 1$. The flux that emerges from $x = 0$ is $\mathcal{R}$ times the incident flux, where $\mathcal{T} + \mathcal{R} = 1$, assuming no carrier recombination-generation.

How is $\mathcal{T}$ related to the mean-free-path **for backscattering within the slab?**

Internal fluxes (i)



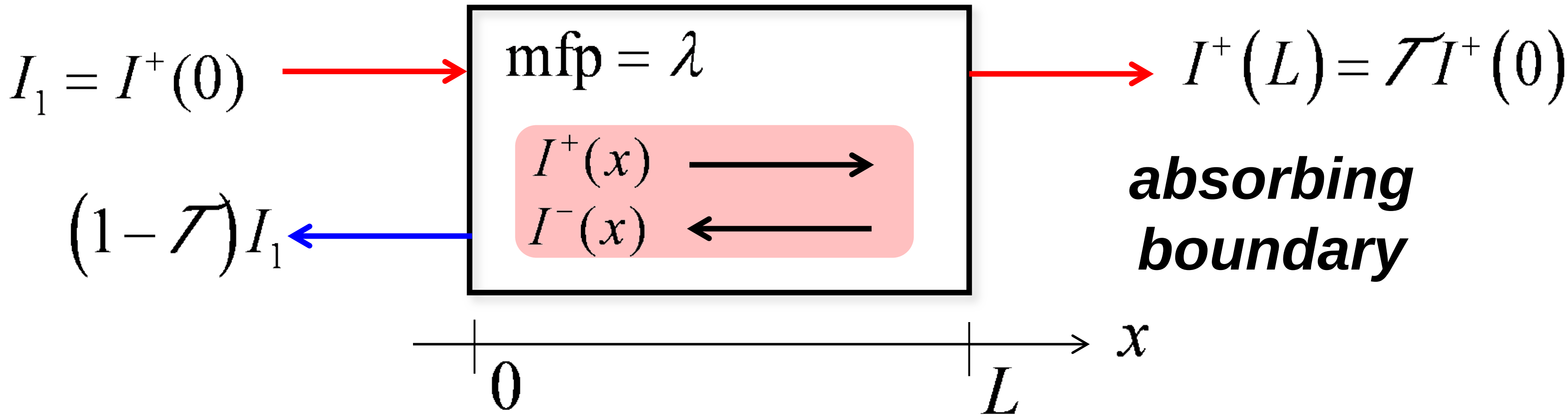
$$\frac{dI^+(x)}{dx} = -\frac{I^+(x)}{\lambda} + \frac{I^-(x)}{\lambda}$$

$$I = I^+(x) - I^-(x) \quad \text{(constant)}$$

$$\boxed{\frac{dI^+(x)}{dx} = -\frac{I}{\lambda}}$$

$$\boxed{I^+(x) = I^+(0) - I\frac{x}{\lambda}}$$

Exiting flux



$$\boxed{I^+(x) = I^+(0) - I\frac{x}{\lambda}}$$

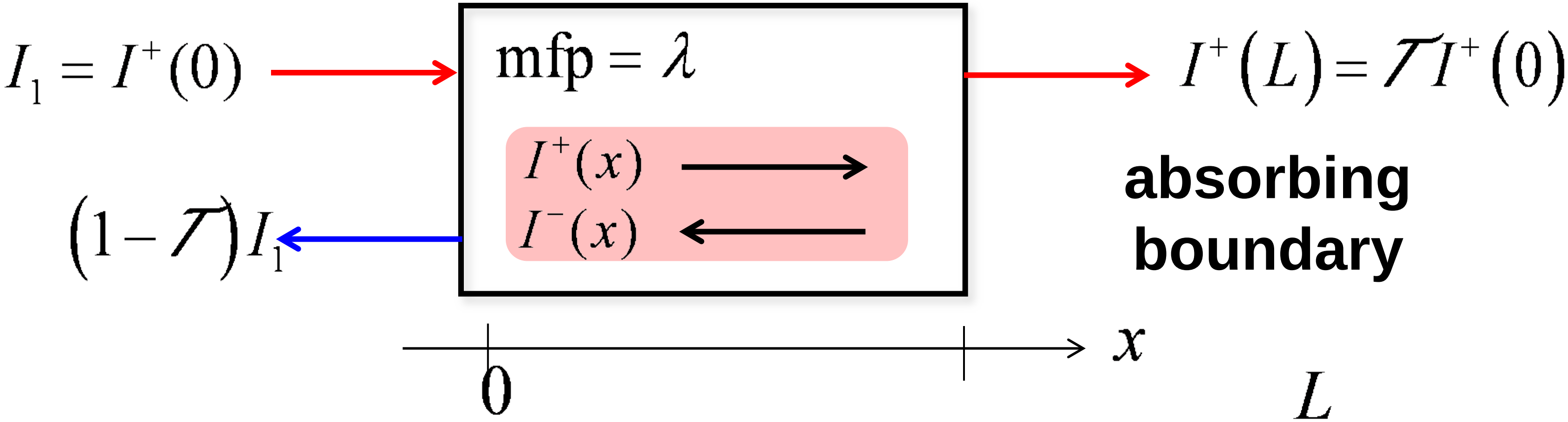
$$I^+(x) = I^+(0) - \left(I^+(x) - I^-(x)\right)\frac{x}{\lambda}$$

$$I^+(L) = I^+(0) - \left(I^+(L) - I^-(L)\right)\frac{L}{\lambda}$$

$I^-(L) = 0$ (boundary condition)

$$I^+(L) = I^+(0) - I^+(L)\frac{L}{\lambda}$$

Transmission



$$\boxed{I^+(L) = I^+(0) - I^+(L)\frac{L}{\lambda}}$$

$$I^+(L) = \frac{I^+(0)}{1 + L/\lambda}$$

$$\frac{I^+(L)}{I^+(0)} = \mathcal{T} = \frac{\lambda}{\lambda + L}$$

$$\mathcal{T}(E) = \frac{\lambda(E)}{\lambda(E) + L}$$

$$\mathcal{T} \to 0 \quad L >> \lambda$$

$$\mathcal{T} \to 1 \quad L << \lambda$$

MFP and transmission

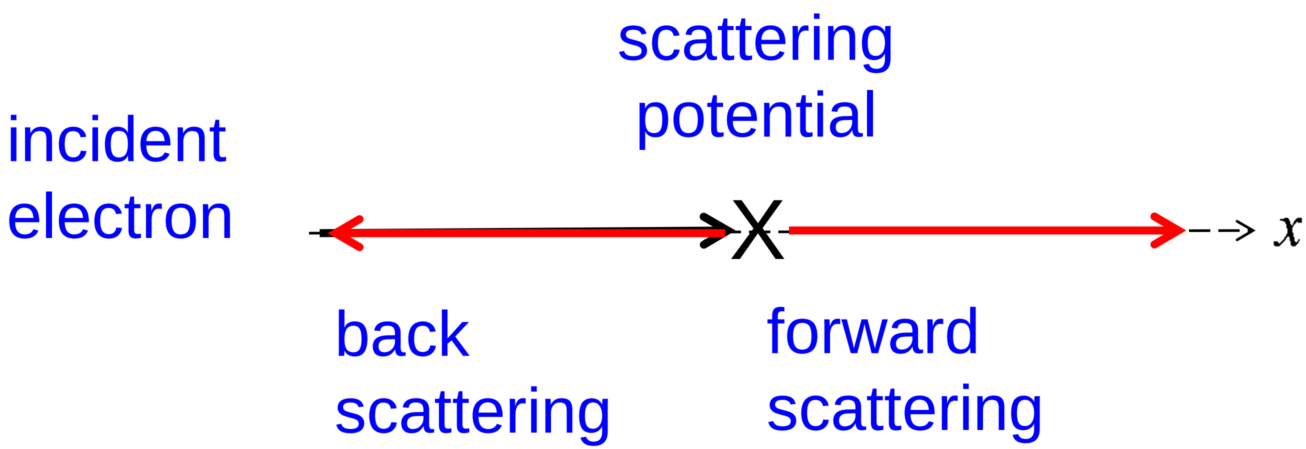
$$\Lambda(E) = \upsilon(E)\tau(E)$$

$$T\left(\Lambda(E)\right)$$

$$\mathcal{T}\left(\lambda(E)\right) = \frac{\lambda(E)}{\lambda(E) + L}$$

$$\Lambda(E) \text{ vs. } \lambda(E)\,?$$

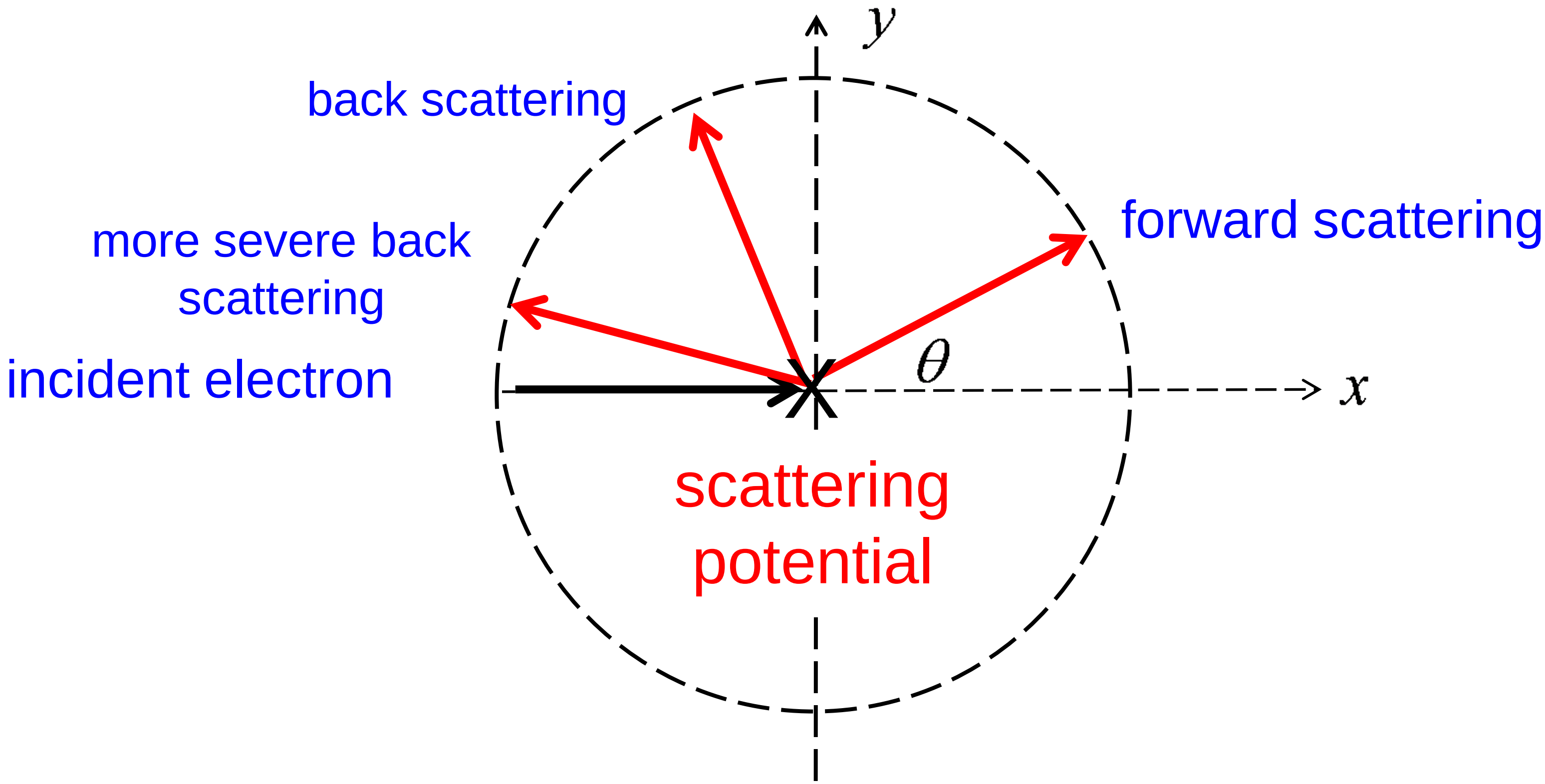
Backscattering in 1D



If we assume that the scattering is ***isotropic*** (equal probability of scattering forward or back) then average time between backscattering events is 2τ.

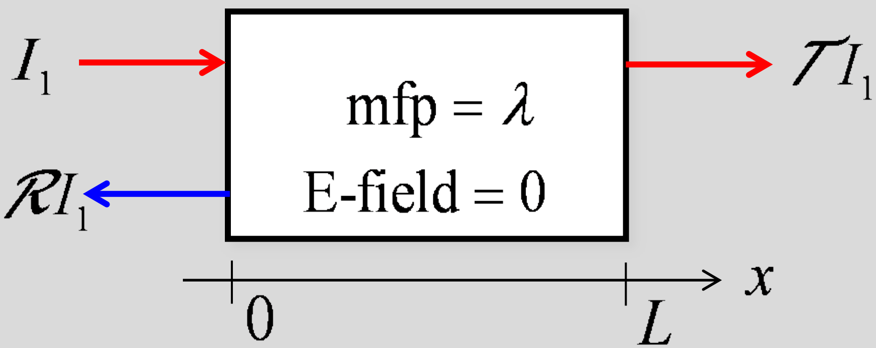
$$\lambda(E) = 2\upsilon(E)\tau(E) = 2\Lambda(E)$$

Backscattering in 2D

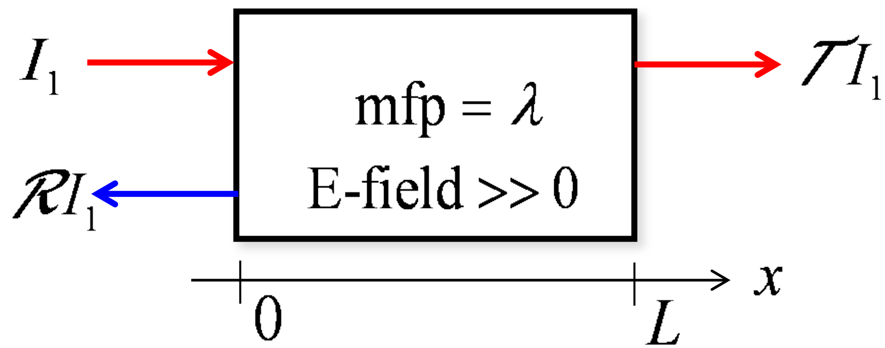


$$\lambda(E) = \frac{\pi}{2} \upsilon(E)\tau(E)$$

Transmission across a slab with an electric field



$$\mathcal{T}(E) = \frac{\lambda(E)}{\lambda(E) + L}$$

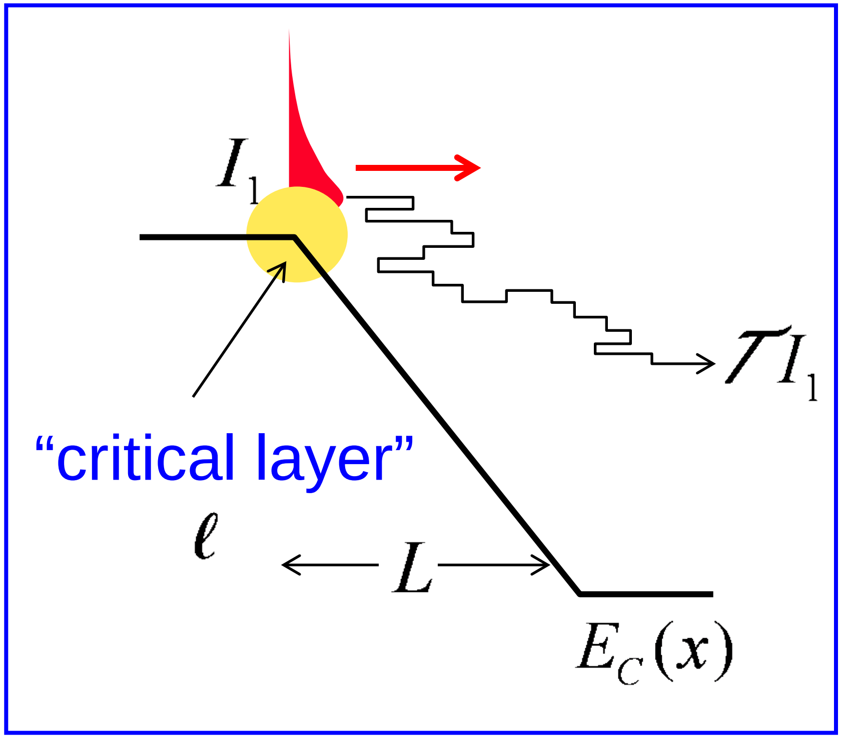


$$\mathcal{T}(E) = ?$$

This turns out to be a **difficult** problem.

How can we understand the essential physics?

Transport “downhill”



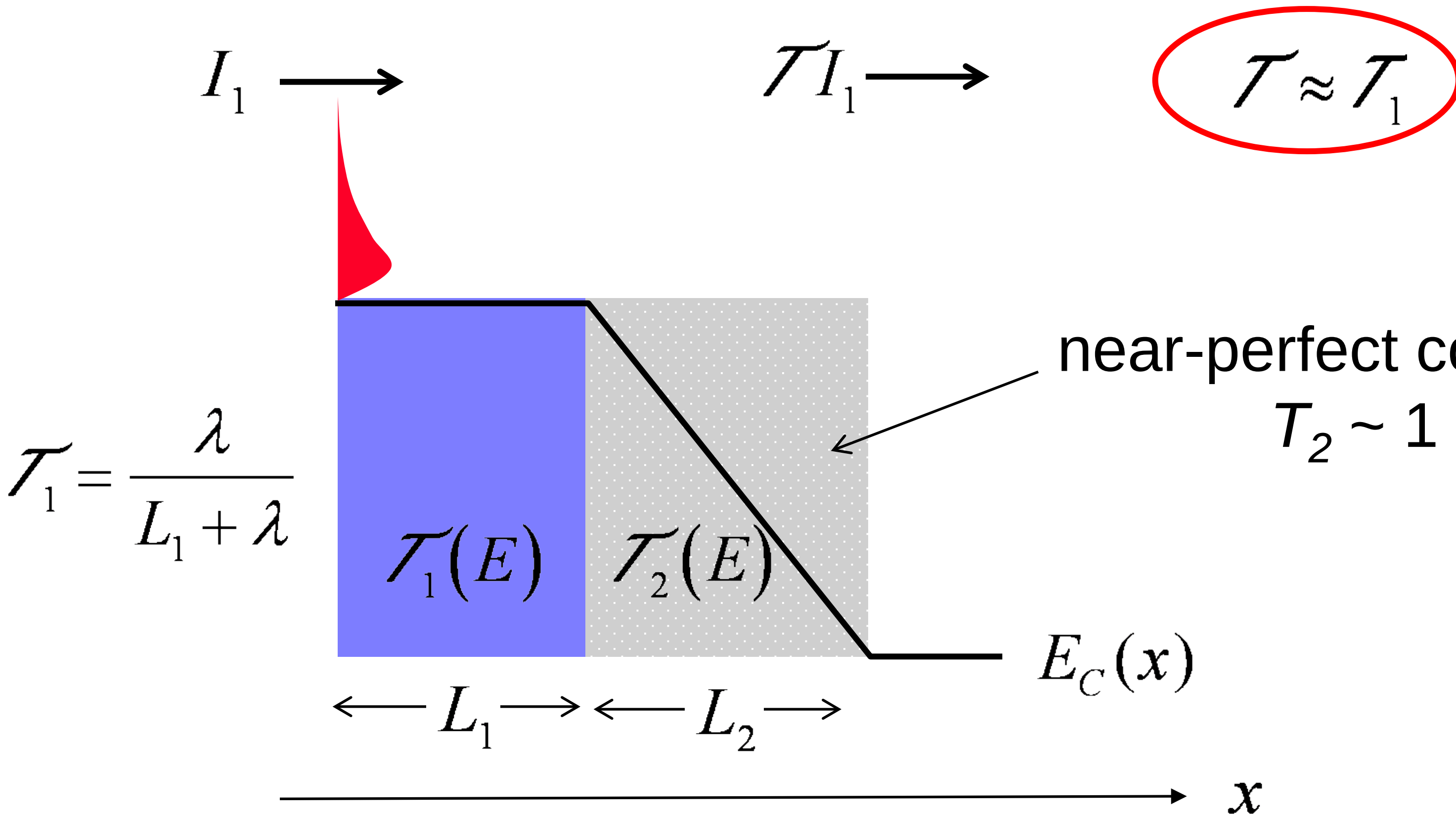
$\ell << L$

$\mathcal{T} \approx 1$:

High field regions are good carrier collectors.

Peter J. Price, “Monte Carlo calculation of electron transport in solids,” *Semiconductors and Semimetals*, **14**, pp. 249-334, 1979

field-free region followed by high-field region



Transmission is controlled by the low-field region.

Summary

1) Transmission is related to the MFP ***for backscattering***

$$\mathcal{T}(E) = \frac{\lambda(E)}{L + \lambda(E)}$$ (no electric field)

2) Ballistic transport: $L << \lambda \quad \mathcal{T} \rightarrow 1$

3) Diffusive transport: $L >> \lambda \quad \mathcal{T} \rightarrow \dfrac{\lambda}{L} << 1$

4) High-field regions are good collectors $\mathcal{T} \approx 1$

5) Transmission is controlled by the low field part of a region