Fundamentals of Nanotransistors

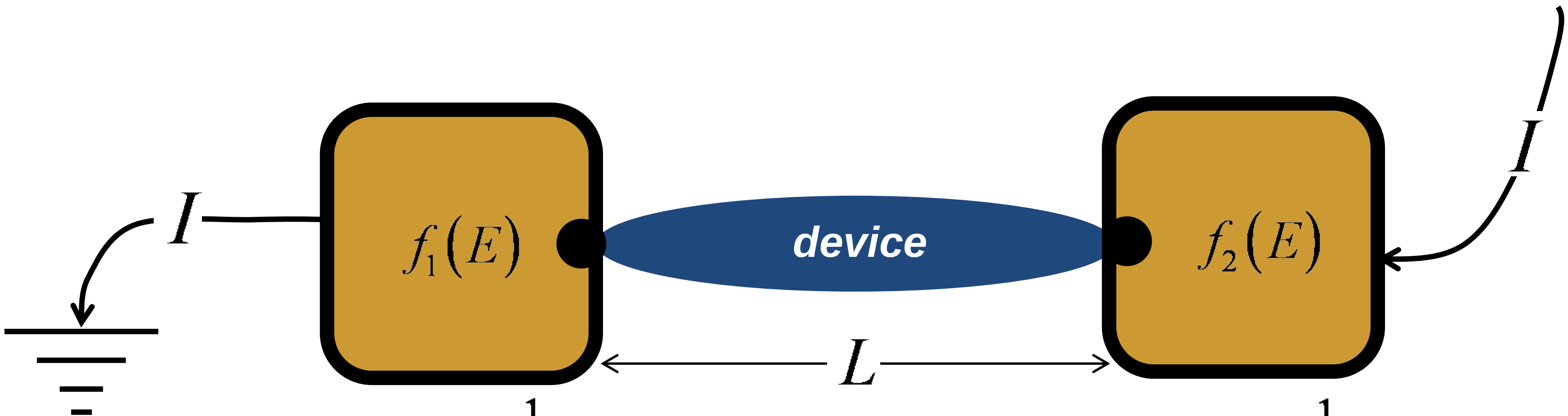
Unit 4: Transmission Theory of the MOSFET

Lecture 4.3: MFP and Diffusion Coefficient

Mark Lundstrom

lundstro@purdue.edu
Electrical and Computer Engineering
Birck Nanotechnology Center
Purdue University, West Lafayette, Indiana USA

Landauer Approach



$$f_1(E) = \frac{1}{1 + e^{(E - E_{F1})/k_B T}}$$

$$f_2(E) = \frac{1}{1 + e^{(E - E_{F2})/k_B T}}$$

$$E_{F2} = E_{F1} - qV$$

$$I = \frac{2q}{h} \int \mathcal{T}(E) M(E) \left(f_1(E) - f_2(E) \right) dE$$

$$\mathcal{T}(E) = \frac{\lambda(E)}{L + \lambda(E)}$$

Measuring the MFP for backscattering

Question: How do we measure the MFP for backscattering?

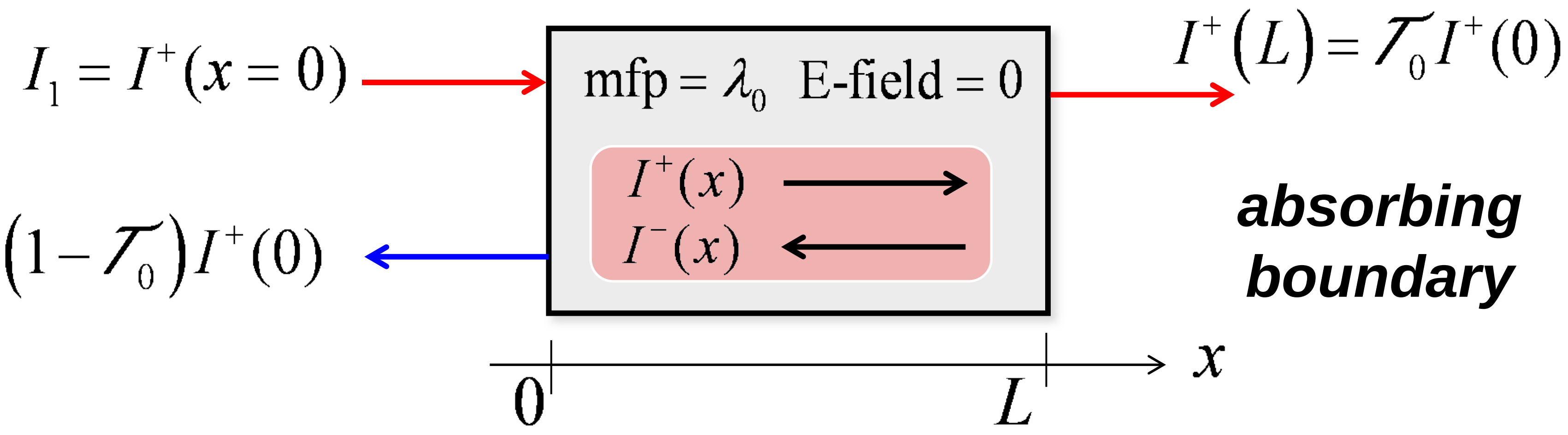
It is "easy" to measure the mobility.

Mobility and diffusion coefficient are related (near equilibrium and for non-degenerate carrier statistics) by the Einstein relation:

$$\frac{D_n}{\mu_n} = \frac{k_B T}{q}$$

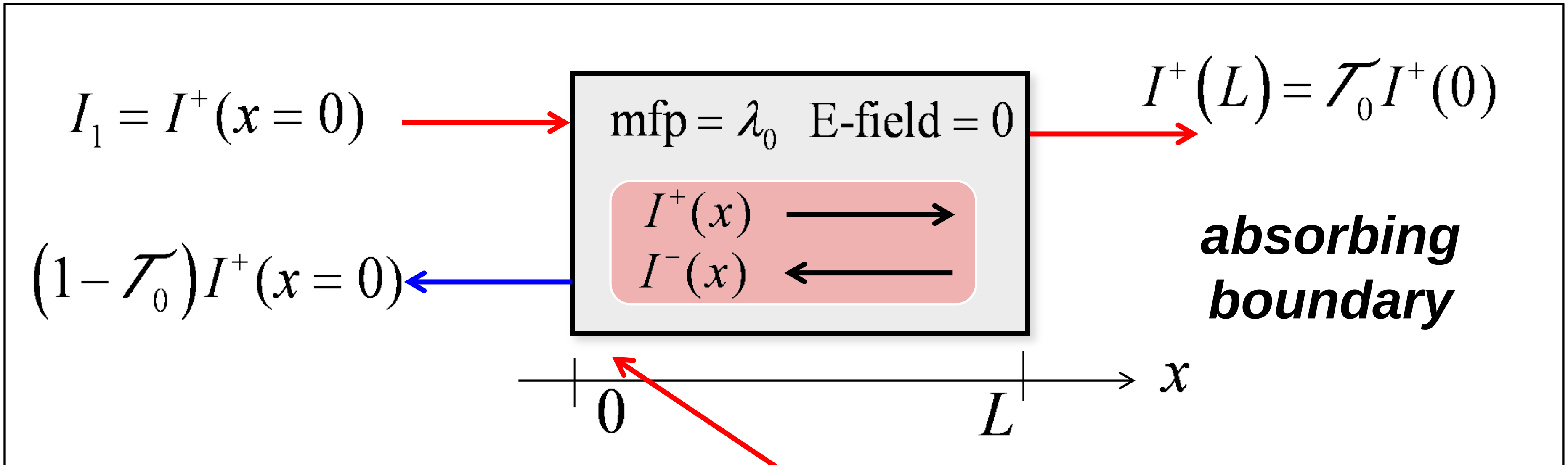
As we will show next, there is a simple relation between the MFP for backscattering and the diffusion coefficient.

The problem



We will not resolve the separate energy channels. For example, I^+ refers to the total flux integrated over all energy channels. The average MFP is λ_0, and the average transmission is $\mathcal{T}_0$.

Carrier density at $x = 0$



$$\langle\langle \upsilon_x^+ \rangle\rangle = \upsilon_T = \sqrt{\frac{2k_BT}{\pi m^*}}$$

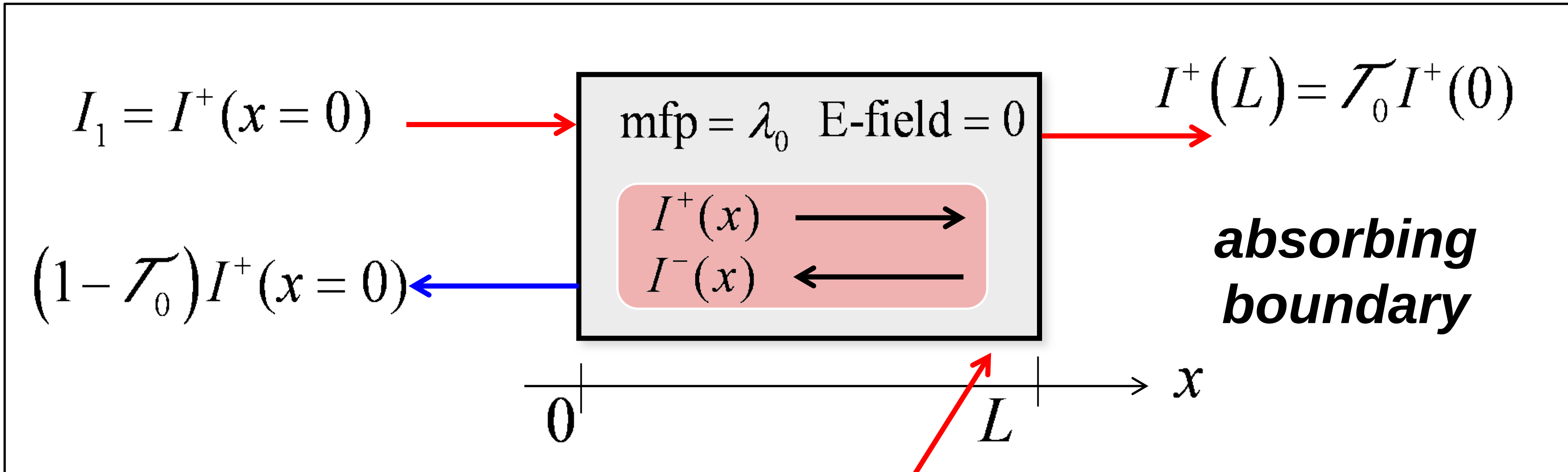
(nondegenerate)

$$n^+(0) = I^+(0) \big/ \langle\langle \upsilon_x^+ \rangle\rangle$$

$$n^-(0) = I^-(0) \big/ \langle\langle \upsilon_x^+ \rangle\rangle$$

$$n(0) = (2-\mathcal{T}_0) I^+(0) \big/ \langle\langle \upsilon_x^+ \rangle\rangle$$

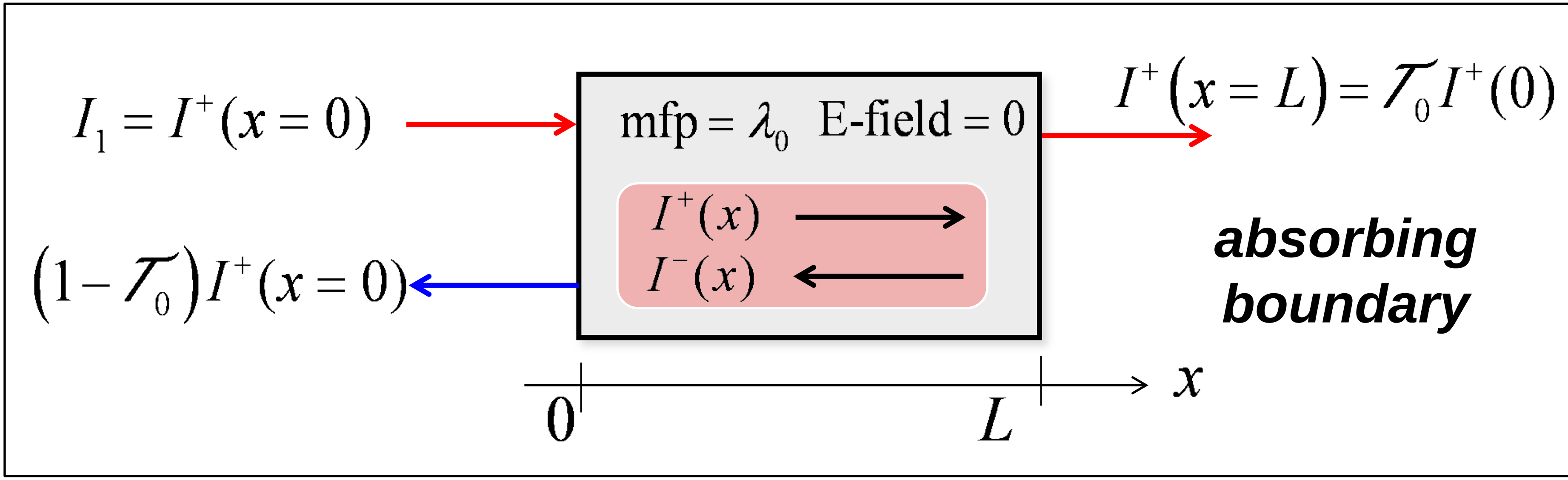
Carrier density at $x = L$

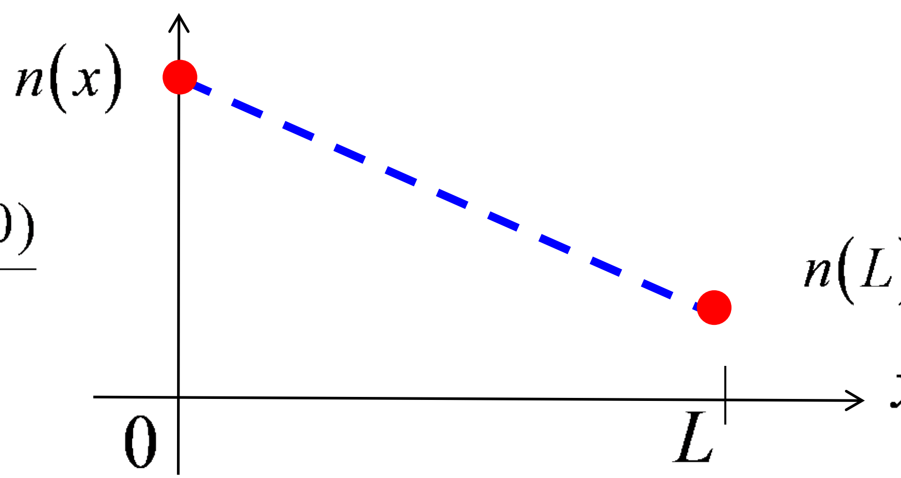


$$n^+(L) = I^+(L)/\upsilon_T \qquad n(L) = I^+(L)/\upsilon_T$$

$$n^-(L) = 0 \qquad n(L) = \mathcal{T}_0 I^+(0)/\upsilon_T$$

Carrier density profile





$$n(0) = \frac{(2-\mathcal{T}_0)I^+(0)}{\upsilon_T}$$

$$n(L) = \frac{\mathcal{T}_0 I^+(0)}{\upsilon_T}$$

Carrier density and net flux

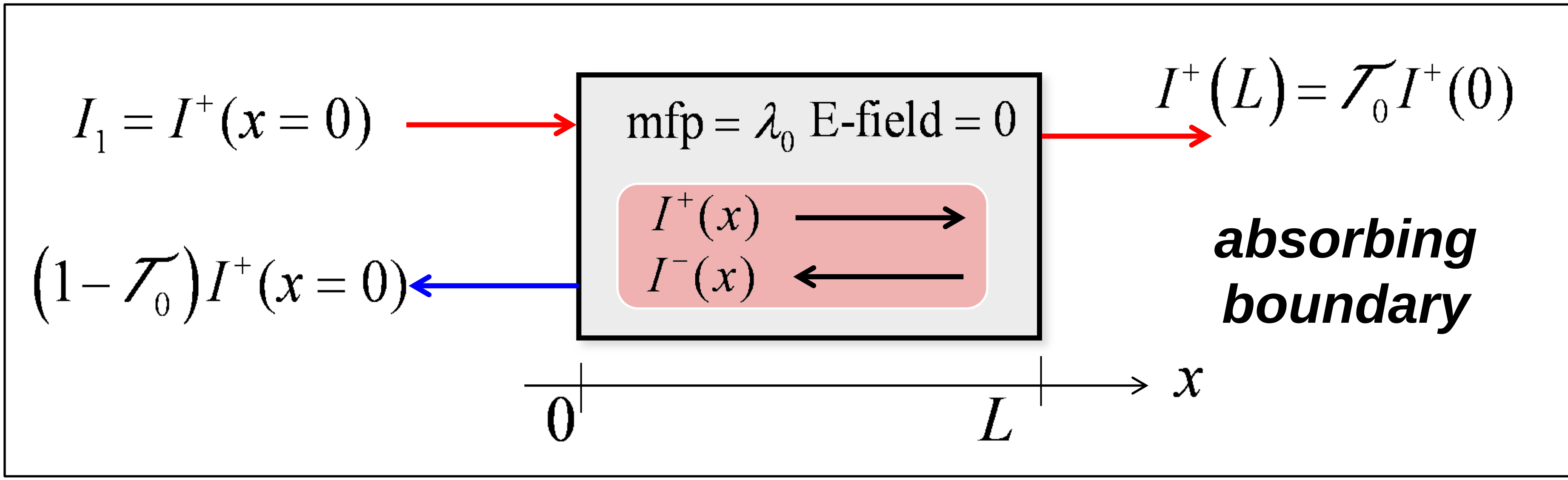
$$n(0) = \frac{\left(2 - \mathcal{T}_0\right) I^+(0)}{\upsilon_T} \qquad n(L) = \frac{\mathcal{T}_0 I^+(0)}{\upsilon_T} \qquad I = \mathcal{T}_0 I^+(0)$$

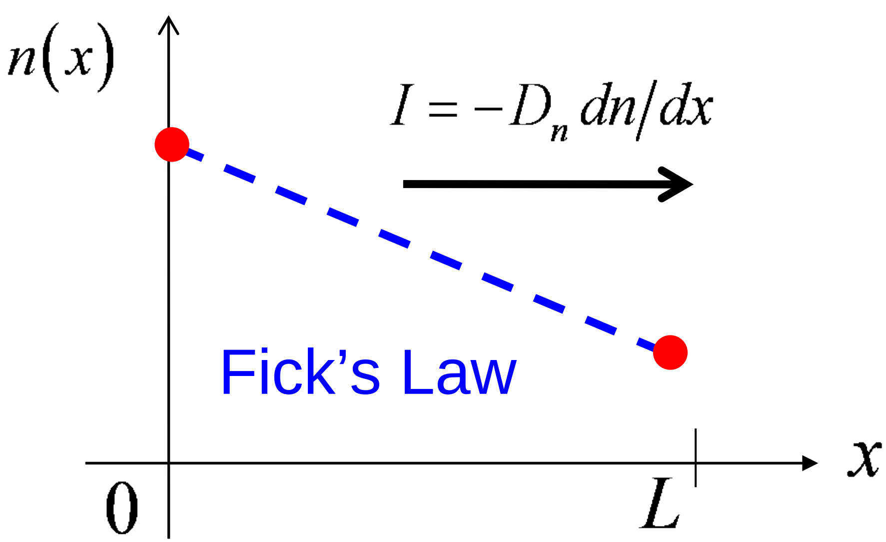
$$n(0) - n(L) = \frac{I^+(0)}{\upsilon_T} 2\left(1 - \mathcal{T}_0\right)$$

$$I = \frac{\upsilon_T}{2} \frac{\mathcal{T}_0}{1 - \mathcal{T}_0} \times \left[n(0) - n(L) \right]$$

$$I = \underbrace{\left\{ \frac{\upsilon_T}{2} \frac{\mathcal{T}_0 L}{1 - \mathcal{T}_0} \right\}}_{D_n} \times \underbrace{\left[\frac{n(0) - n(L)}{L} \right]}_{-dn/dx}$$

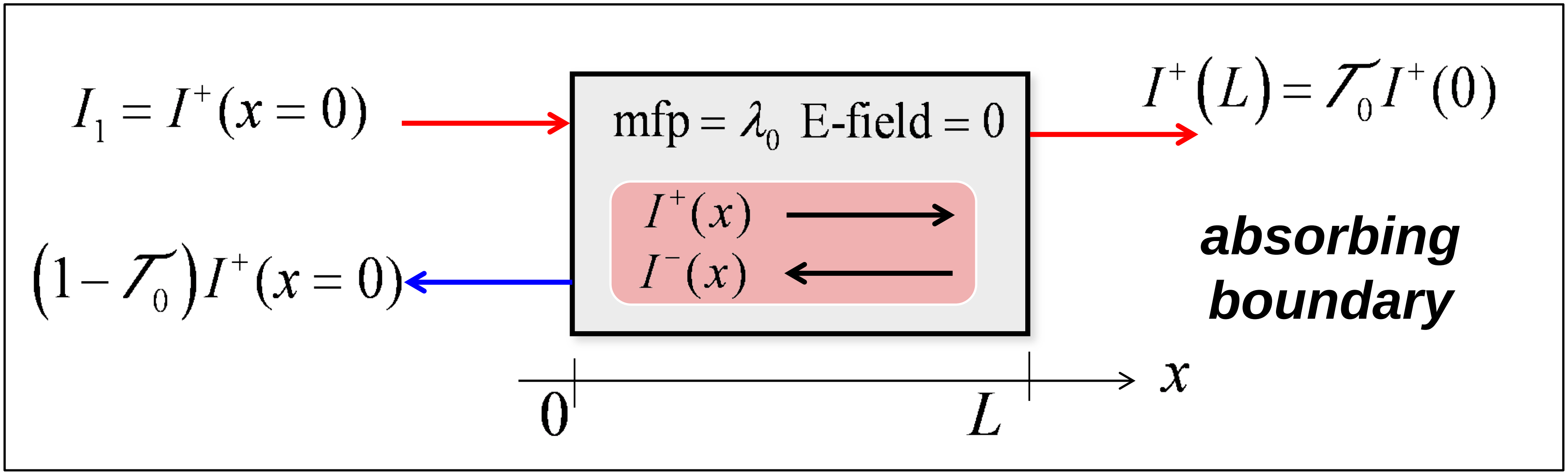
Fick's Law in a nanostructure





$$D_n = \frac{\upsilon_T}{2}\frac{\mathcal{T}_0 L}{1-\mathcal{T}_0}$$

Fick’s Law in general



$$I = -D_n \, dn/dx$$

$$\mathcal{T}_0 = \frac{\lambda_0}{\lambda_0 + L}$$

$$D_n = \frac{\upsilon_T}{2} \frac{\mathcal{T}_0 L}{1-\mathcal{T}_0}$$

$$D_n = \frac{\upsilon_T \lambda_0}{2}$$

$$\boxed{D_n = \frac{\upsilon_T \lambda_0}{2}}$$

Mobility to MFP for backscattering

1) Given the mobility: $\mu_n \ \dfrac{\text{cm}^2}{\text{V-s}}$

2) Use the Einstein Relation: $\dfrac{D_n}{\mu_n} = \dfrac{k_B T}{q}$

3) Find diffusion coefficient: $D_n = \dfrac{k_B T}{q} \mu_n \ \dfrac{\text{m}^2}{\text{s}}$

4) Extract the MFP for backscattering: $\lambda_0 = \dfrac{2D_n}{\upsilon_T}$ m

Example

Consider a Si MOSFET at T = 300 K.

$$\mu_n = 250 \ \frac{\mathrm{cm}^2}{\mathrm{V\text{-}s}} \qquad m_n^* = 0.19\, m_0 \qquad \upsilon_T = \sqrt{\frac{2k_BT}{\pi m_n^*}} = 1.2 \times 10^7 \ \mathrm{cm/s}$$

What is the MFP for backscattering?
(Assuming nondegenerate carrier statistics).

$$D_n = \frac{k_BT}{q}\mu_n = 0.026 \times 250 = 6.5 \ \frac{\mathrm{cm}^2}{\mathrm{s}}$$

$$\lambda_0 = \frac{2D_n}{\upsilon_T} = \frac{2 \times 6.5}{1.2 \times 10^7} = 10.8 \ \mathrm{nm} \quad \checkmark$$

Example 2

Consider a III-V HEMT at T = 300 K.

$$\mu_n = 12{,}500 \ \frac{\text{cm}^2}{\text{V-s}} \qquad m_n^* = 0.022 m_0 \qquad \upsilon_T = \sqrt{\frac{2k_BT}{\pi m_n^*}} = 3.6\times 10^7 \ \text{cm/s}$$

What is the MFP for backscattering?
(Assuming MB statistics).

$$D_n = \frac{k_BT}{q}\mu_n = 0.026\times 12{,}500 = 325 \ \frac{\text{cm}^2}{\text{s}}$$

$$\lambda_0 = \frac{2D_n}{\upsilon_T} = \frac{2\times 325}{3.6\times 10^7} = 153 \ \text{nm} \quad \checkmark$$

Key point of Lecture 4.3

The diffusion coefficient is directly related to the MFP for backscattering.

$$\boxed{D_n = \frac{\upsilon_T \lambda_0}{2}}$$

Non-degenerate carrier statistics.