Fundamentals of Nanotransistors

Unit 3: The Ballistic Nanotransistor

Lecture 3.7: Unit 3 Summary

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Our goal in Unit 3

A ballistic treatment of transport

$$I_{DS} = W\left|Q_n\left(V_{GS},V_{DS}\right)\right|\left\langle \upsilon\left(V_{GS},V_{DS}\right)\right\rangle$$

Unit 2:
MOS electrostatics

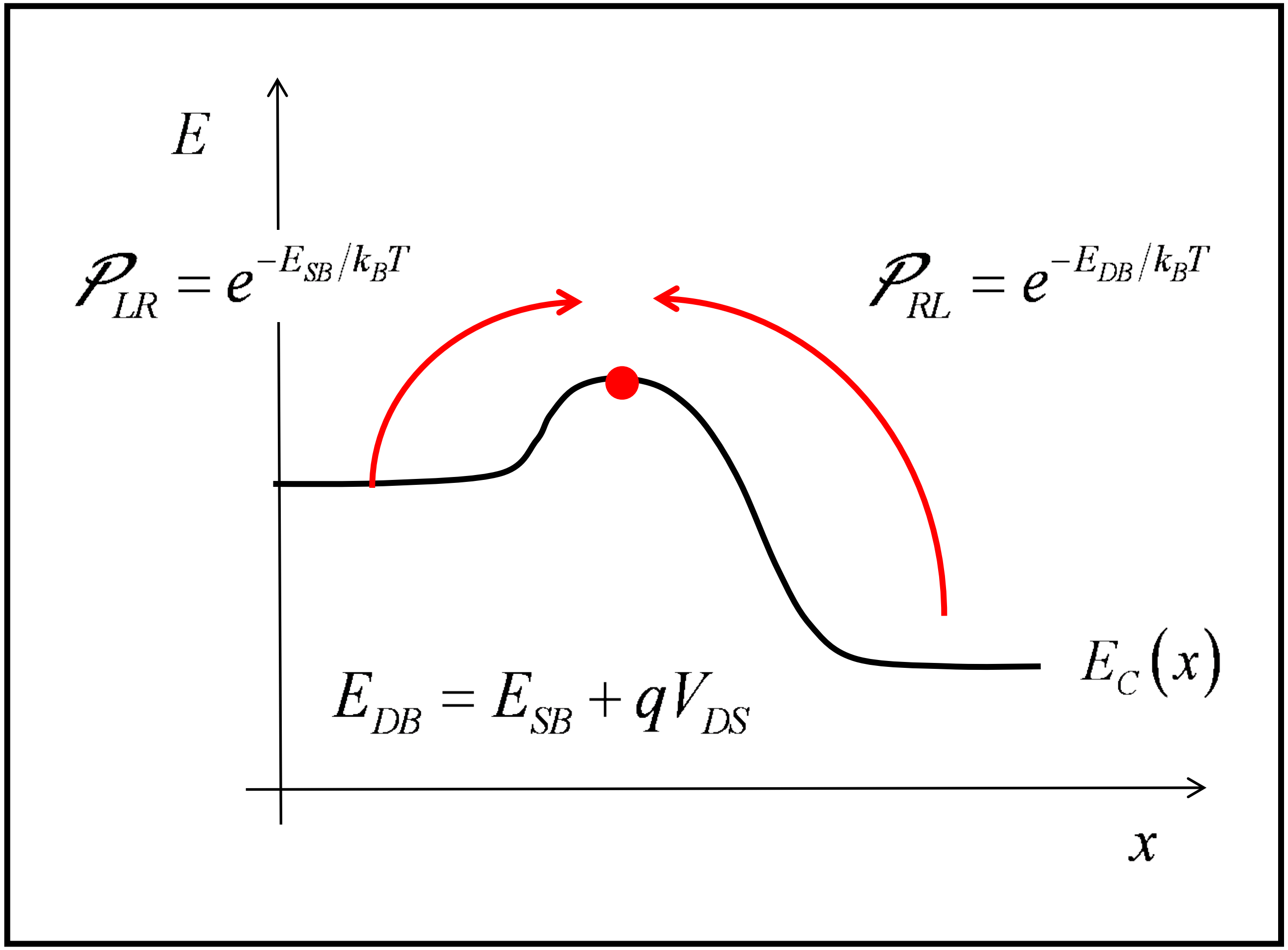
Unit 3

Lecture 3.1: Introduction

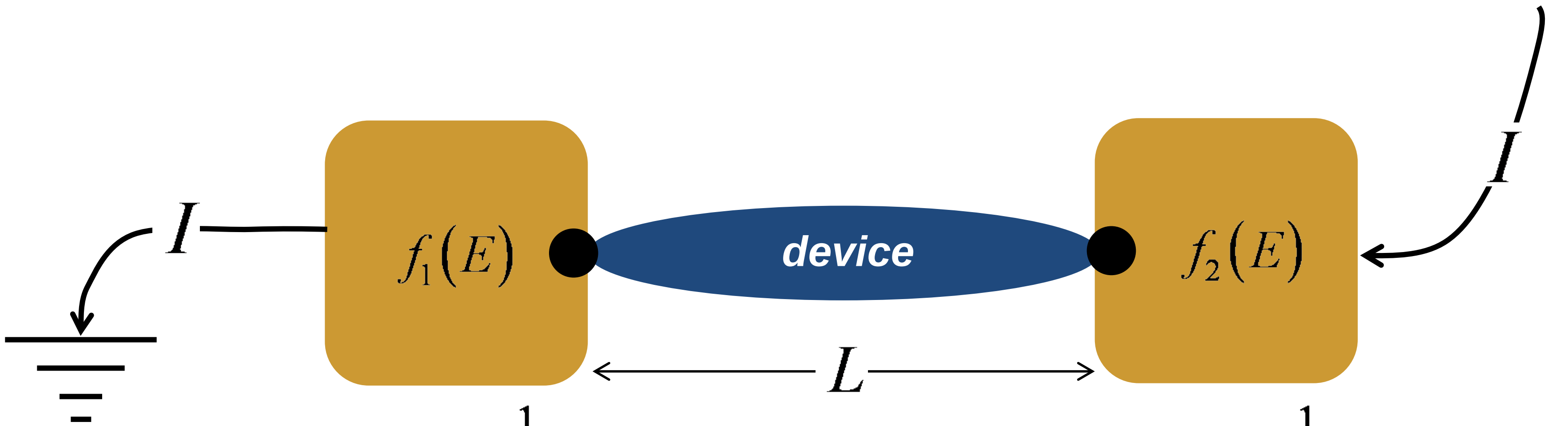
Lecture 3.2: Landauer Approach
Lecture 3.3: More on Landauer
Lecture 3.4: The Ballistic MOSFET
Lecture 3.5: The Velocity at the VS
Lecture 3.6: Revisiting the VS Model

Lecture 3.7 Summary

Lecture 3.1: thermionic emission



Lecture 3.2: Landauer Approach



$$f_1(E) = \frac{1}{1 + e^{(E - E_{F1})/k_B T_1}}$$

$$f_2(E) = \frac{1}{1 + e^{(E - E_{F2})/k_B T_2}}$$

$$E_{F2} = E_{F1} - qV$$

$$\boxed{I = \frac{2q}{h} \int \mathcal{T}(E) M(E) \big(f_1(E) - f_2(E) \big) dE}$$

$$\boxed{n_S = \int_{E_C}^{\infty} \left(\frac{D_{2D}(E)}{2} f_1(E) + \frac{D_{2D}(E)}{2} f_2(E) \right) dE \ \ \mathrm{m}^{-2}}$$

Landauer Approach

$$\boxed{I = \frac{2q}{h}\int \mathcal{T}(E)M(E)\left(f_1(E) - f_2(E)\right)dE}$$

Fundamental constants

Transmission:

$\mathcal{T}(E) = 1$ ballistic

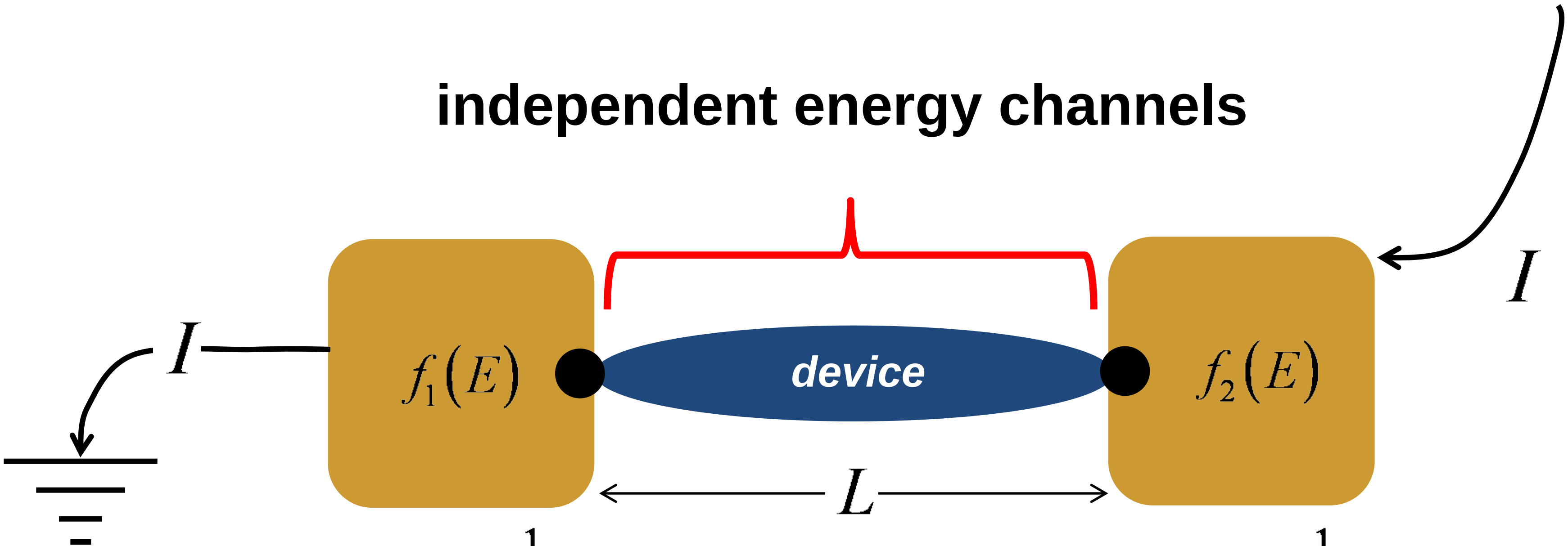
$\mathcal{T}(E) << 1$ diffusive

Channels or Modes

Ideal “Landauer” contacts

Lecture 3.2: Landauer Approach

independent energy channels



$$f_1(E) = \frac{1}{1 + e^{(E - E_{F1})/k_B T_1}}$$

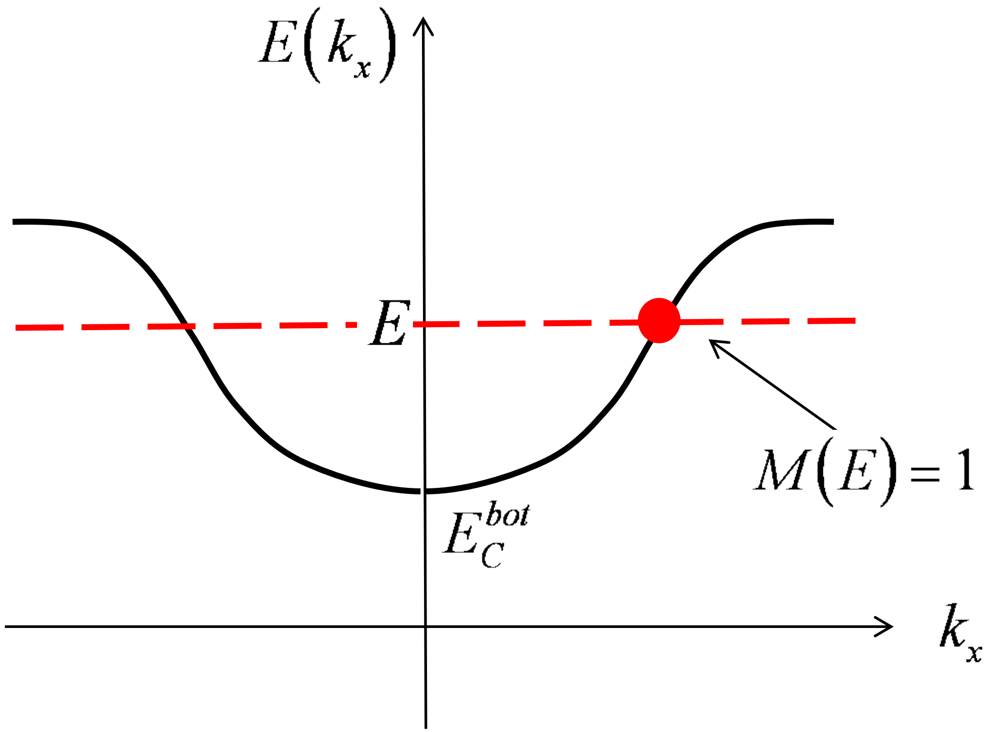
$$f_2(E) = \frac{1}{1 + e^{(E - E_{F2})/k_B T_2}}$$

$$E_{F2} = E_{F1} - qV$$

"Landauer contacts"

Channels (modes)

$$M_{1D}(E) \equiv \frac{h}{4}\left\langle \upsilon_x^+(E)\right\rangle D_{1D}(E)$$



(Easily generalized to arbitrary bandstructures in 1D, 2D, and 3D.)

Lecture 3.3: The Fermi window

$$I = \frac{2q}{h}\int \mathcal{T}(E)M(E)\left(f_1 - f_2\right)dE$$

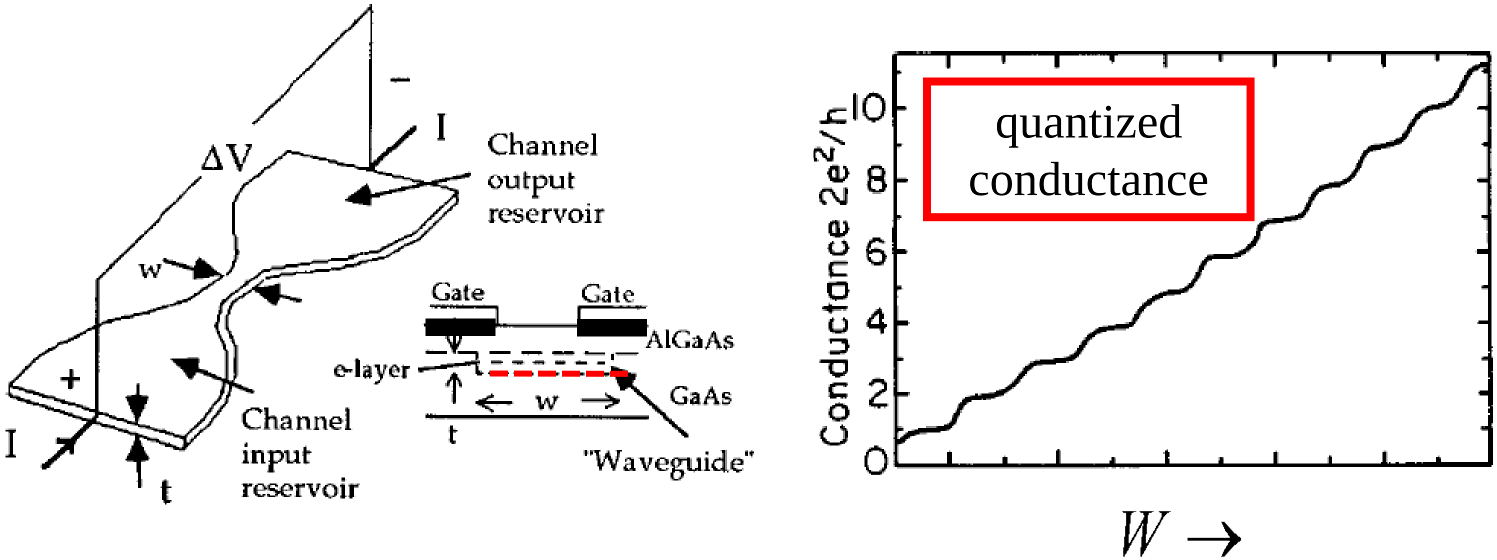
The range of energies over which $\left(f_1 - f_2\right) \neq 0$

Linear response: $f_1(E) - f_2(E) = \left(-\frac{\partial f_1}{\partial E}\right)(qV)$

$$I = GV \ \mathrm{A}$$

$$G = \frac{2q^2}{h}\int \mathcal{T}(E)M(E)\left(-\frac{\partial f_0}{\partial E}\right)dE \ \mathrm{S}$$

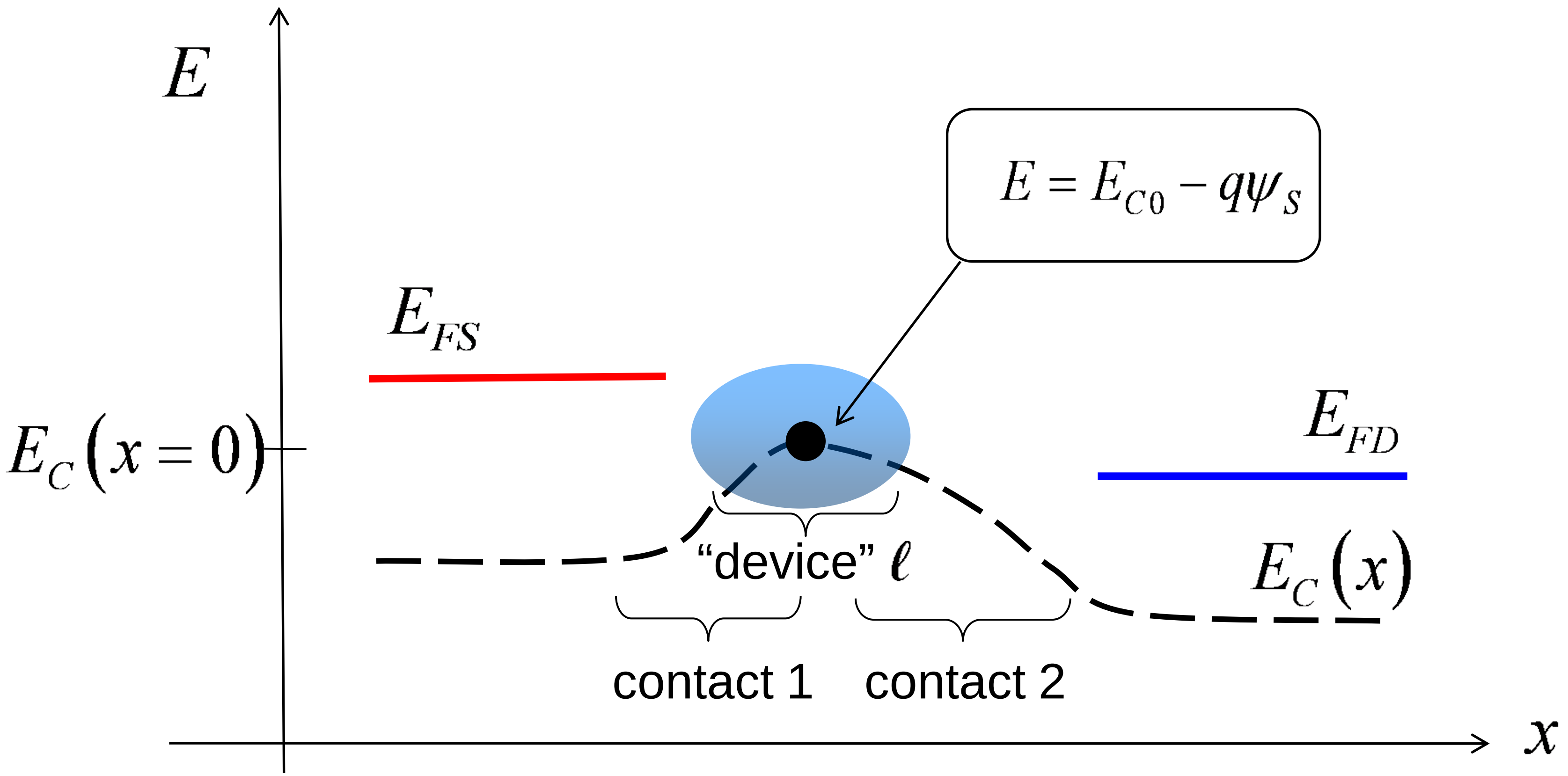
Quantized conductance



$$G(T = 0\text{ K}) = \frac{2q^2}{h}\mathcal{T}(E_F)M(E_F)$$

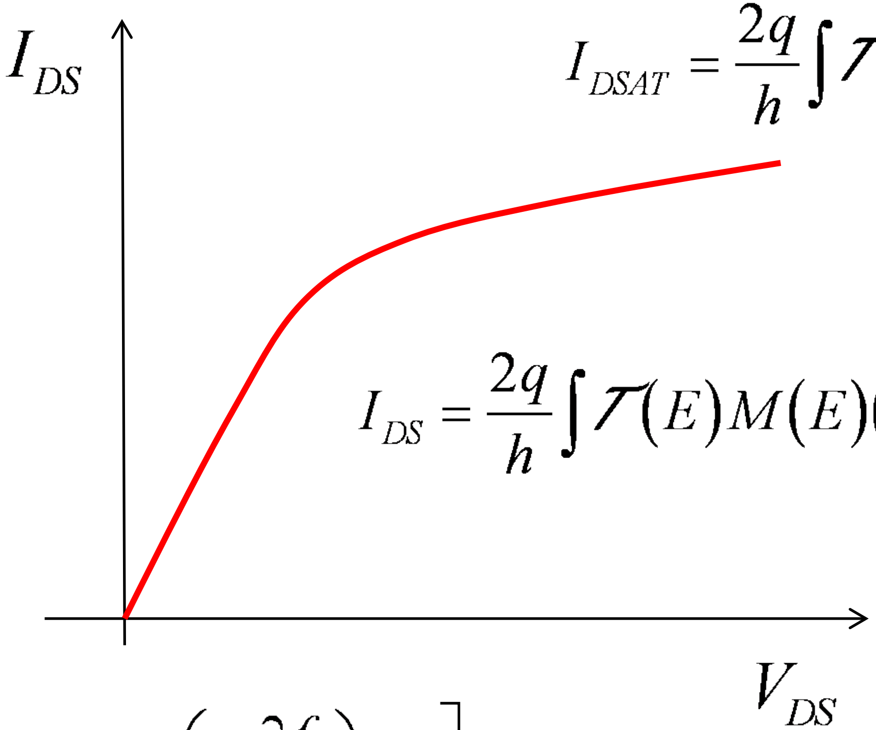
D. Holcomb, *American J. Physics*, **67**, pp. 278-297 1999.
Data from: B. J. van Wees, et al., *Phys. Rev. Lett*. **60**, 848851, 1988.

Lecture 3.4: The Ballistic MOSFET



Ballistic MOSFET

$$I_{DSAT} = \frac{2q}{h}\int \mathcal{T}(E)M(E)f_S(E)\,dE$$

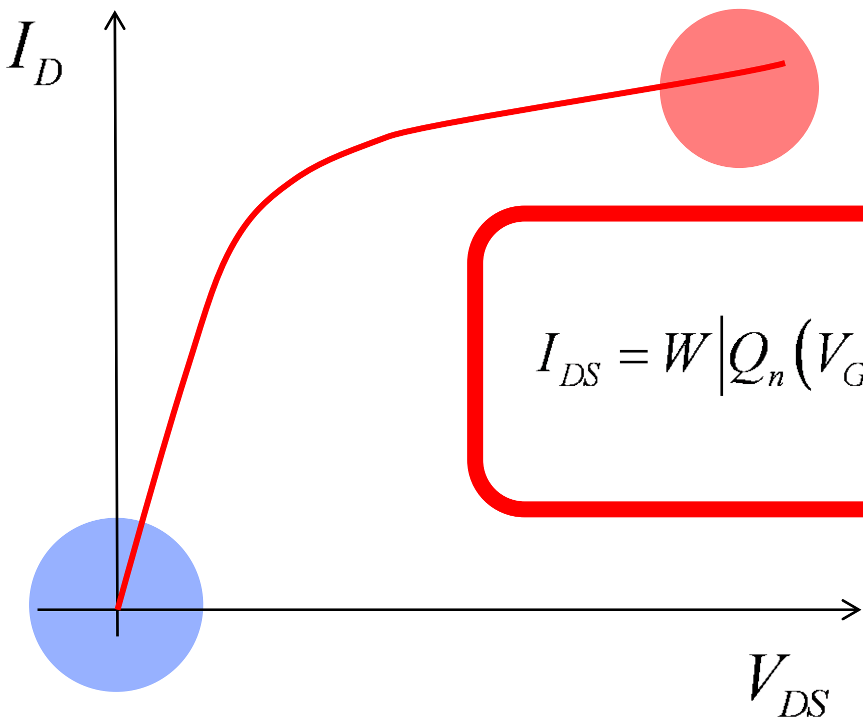


$$I_{DS} = \frac{2q}{h}\int \mathcal{T}(E)M(E)\left(f_S - f_D\right)dE$$

$$I_{DLIN} = \left[\frac{2q^2}{h}\int \mathcal{T}(E)M(E)\left(-\frac{\partial f_0}{\partial E}\right)dE\right]V_{DS}$$

Ballistic MOSFET: nondegenerate

$$I_{DSAT} = W\left|Q_n\left(V_{GS}, V_{DS}\right)\right|\upsilon_T$$

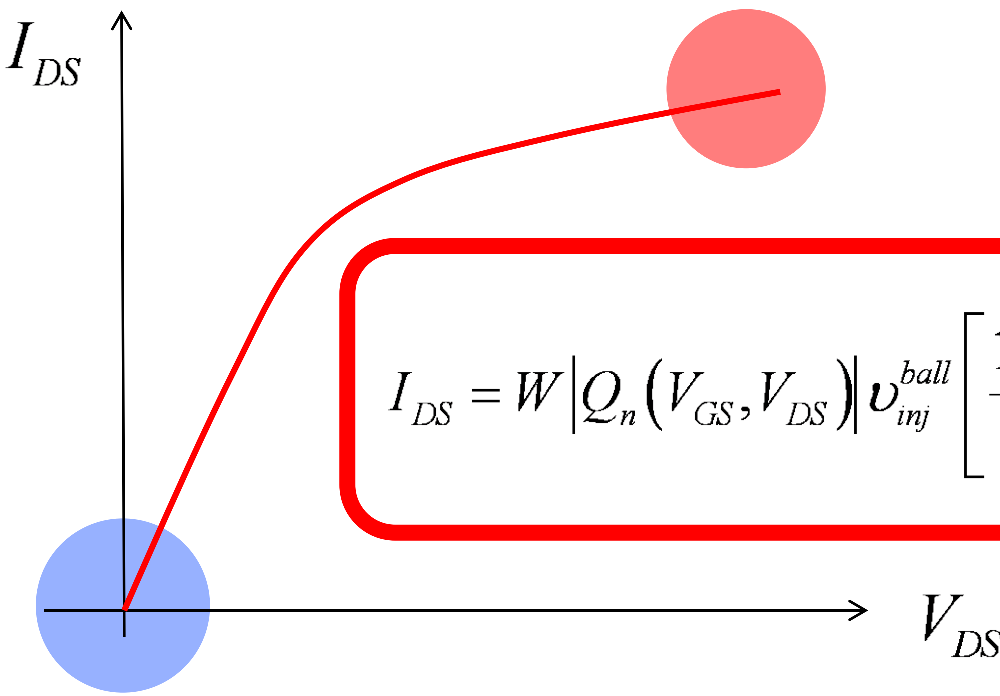


$$\boxed{I_{DS} = W\left|Q_n\left(V_{GS}, V_{DS}\right)\right|\upsilon_T\left(\frac{1-e^{-qV_{DS}/k_BT}}{1+e^{-qV_{DS}/k_BT}}\right)}$$

$$I_{DLIN} = W\frac{\upsilon_T}{2k_BT/q}\left|Q_n\left(V_{GS}, V_{DS}\right)\right|V_{DS}$$

Ballistic MOSFET: degenerate

$$I_{DSAT} = W\left|Q_n\left(V_{GS},V_{DS}\right)\right|\upsilon_{inj}^{ball}$$



$$\boxed{I_{DS} = W\left|Q_n\left(V_{GS},V_{DS}\right)\right|\upsilon_{inj}^{ball}\left[\frac{1-\mathcal{F}_{1/2}\left(\eta_{FD}\right)/\mathcal{F}_{1/2}\left(\eta_{FS}\right)}{1+\mathcal{F}_{0}\left(\eta_{FD}\right)/\mathcal{F}_{0}\left(\eta_{FS}\right)}\right]}$$

$$I_{DLIN} = W\left|Q_n\left(V_{GS},V_{DS}\right)\right|\frac{\upsilon_{inj}^{ball}}{2k_BT/q}\left\{\frac{\mathcal{F}_{-1/2}\left(\eta_{FS}\right)}{\mathcal{F}_{1/2}\left(\eta_{Fs}\right)}\right\}V_{DS}$$

(See Sec. 13.6 of Lecture Notes.)

FD statistics: Prescription

3) $$I_{DS} = W\left|Q_n\left(V_{GS},V_{DS}\right)\right|\upsilon_{inj}^{ball}\left[\frac{1-\mathcal{F}_{1/2}\left(\eta_{FD}\right)/\mathcal{F}_{1/2}\left(\eta_{FS}\right)}{1+\mathcal{F}_{0}\left(\eta_{FD}\right)/\mathcal{F}_{0}\left(\eta_{FS}\right)}\right]$$

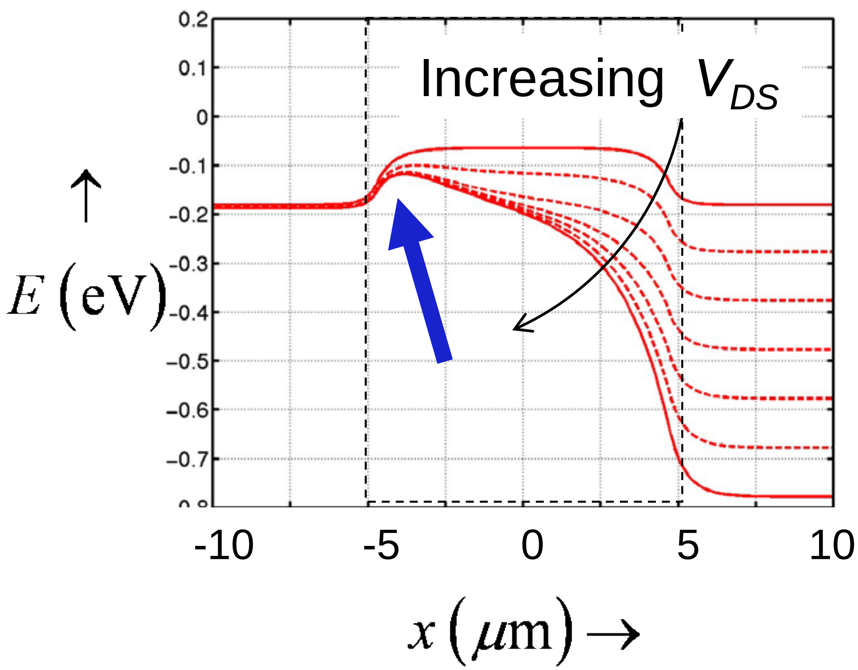
1) $$Q_n\left(V_{GS},V_{DS}\right) = -q\frac{N_{2D}}{2}\left(\mathcal{F}_0\left(\eta_{FS}\right)+\mathcal{F}_0\left(\eta_{FD}\right)\right)\mathrm{m}^{-2}$$

2) $$\upsilon_{inj}^{ball} = \left\langle\left\langle \upsilon_x^+\right\rangle\right\rangle = \upsilon_T\frac{\mathcal{F}_{1/2}\left(\eta_{FS}\right)}{\mathcal{F}_{0}\left(\eta_{FS}\right)} \qquad \upsilon_T = \sqrt{\frac{2k_BT}{\pi m^*}}$$

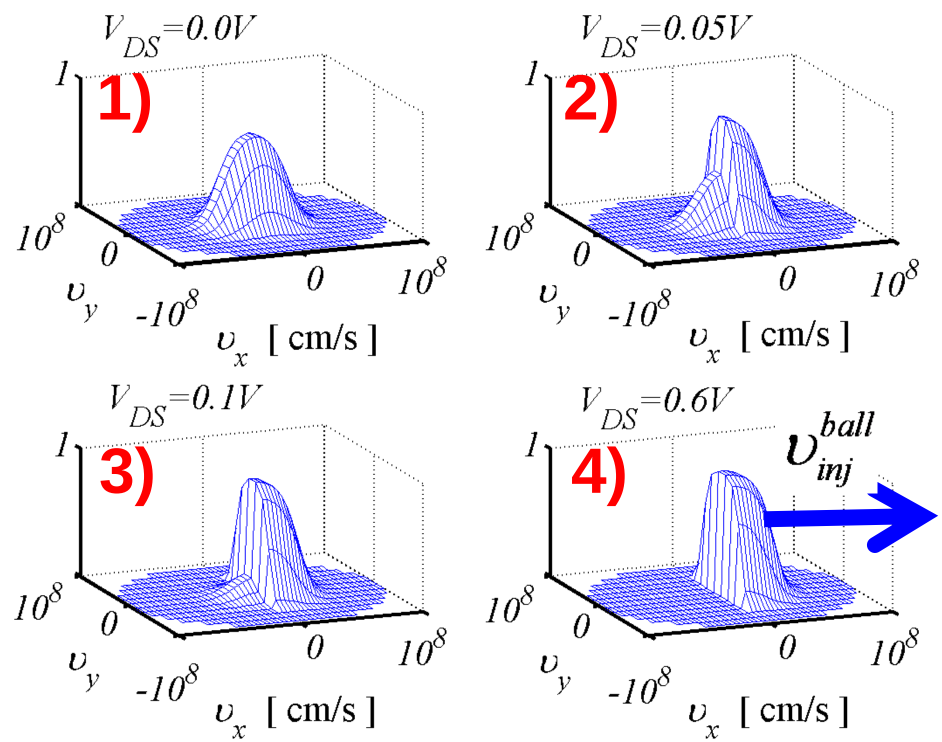
$$\eta_{FD} = \eta_{FS} - qV_{DS}/k_BT$$

Lecture 3.5: Velocity at the VS

E_X *vs*. x for V_{GS} = 0.5V

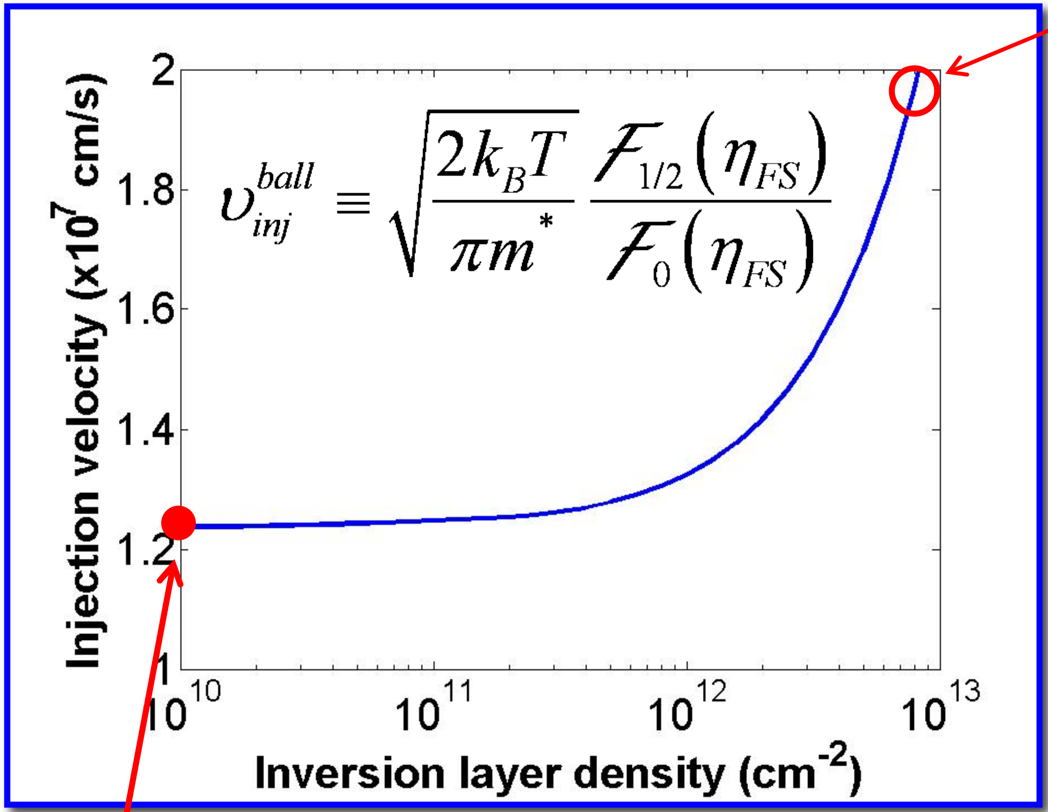


$f(\upsilon_x, \upsilon_y)$



(Numerical simulations of an L = 10 nm double gate Si MOSFET from J.-H. Rhew and M.S. Lundstrom, *Solid-State Electron.*, **46**, 1899, 2002)

Ballistic injection velocity



$$\upsilon_{inj}^{ball} \to (4/3\pi)\upsilon_F$$

$$\upsilon_F = \sqrt{\frac{2(E_F - E_C)}{m^*}}$$

fully degenerate

$$\boxed{\upsilon_T = \sqrt{2k_BT/\pi m^*}}$$

non-degenerate

Comments on FD statistics

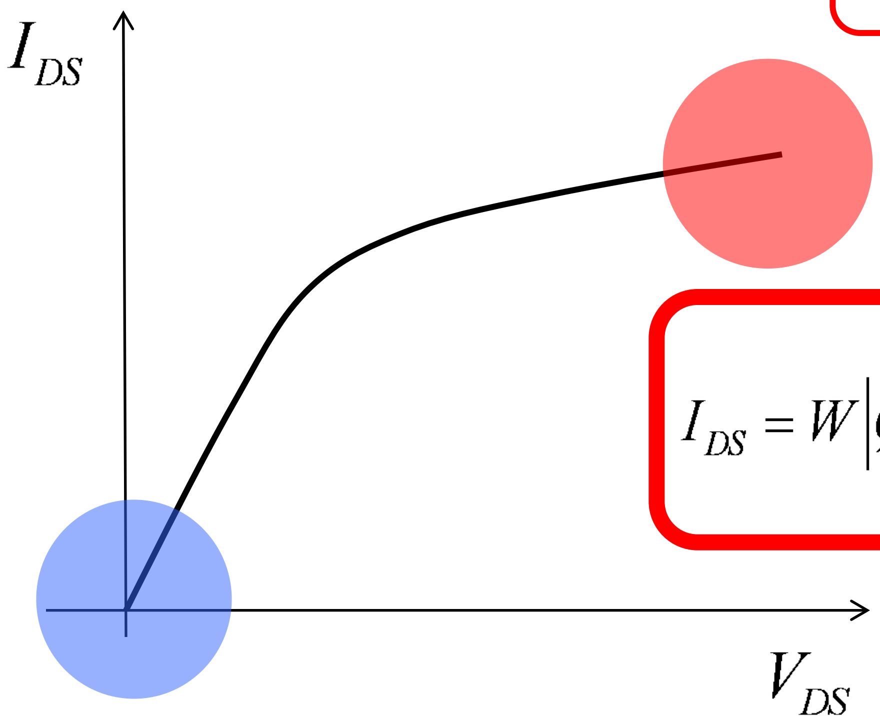
We have assumed 2D carriers, parabolic energy bands, and that a single subband at energy, E_C, is occupied.

For a 1D, nanowire MOSFET, with parabolic energy bands, the same general considerations apply, but the Fermi-Dirac integrals will be of different order.

See Lecture 13 of the Lecture Notes for more discussion.

Lecture 3.8: VS model

$$I_{DSAT} = W\left|Q_n\left(V_{GS},V_{DS}\right)\right|\upsilon_{sat}$$



$$I_{DS} = W\left|Q_n\left(V_{GS},V_{DS}\right)\right|F_{SAT}\left(V_{DS}\right)\upsilon_{sat}$$

$$I_{DLIN} = \frac{W}{L}\mu_n\left|Q_n\left(V_{GS},V_{DS}\right)\right|V_{DS}$$

Connection to ballistic model

$$\frac{1}{\mu_n} \rightarrow \frac{1}{\mu_{app}}$$ "apparent mobility"

$$\upsilon_{sat} \rightarrow \upsilon_{inj}$$ "injection velocity"

Ballistic MOSFETs:

$$\mu_{app} = \mu_B \qquad \mu_B = \frac{\upsilon_T L}{2 k_B T / q} \ \mathrm{cm^2/V\text{-}s}$$

(non-degenerate)

$$\upsilon_{inj} = \upsilon_{inj}^{ball} \qquad \upsilon_{inj}^{ball} = \upsilon_T \qquad \upsilon_T = \sqrt{\frac{2 k_B T}{\pi m^*}}$$

Our goal in Unit 3

A ballistic treatment of transport

$$I_{DS} = W\left|Q_n\left(V_{GS}, V_{DS}\right)\right|\left\langle \upsilon\left(V_{GS}, V_{DS}\right)\right\rangle$$

Unit 2:
MOS electrostatics

$$\mathcal{T}(E) = 1$$

$$\left\langle \upsilon\left(V_{DS}\right)\right\rangle = \upsilon_T\left(\frac{1 - e^{-qV_{DS}/k_BT}}{1 + e^{-qV_{DS}/k_BT}}\right) \qquad \text{(MB)}$$

$$\left\langle \upsilon\left(V_{GS}, V_{DS}\right)\right\rangle = \upsilon_{inj}^{ball}\left[\frac{1 - \mathcal{F}_{1/2}\left(\eta_{FD}\right)/\mathcal{F}_{1/2}\left(\eta_{FS}\right)}{1 + \mathcal{F}_0\left(\eta_{FD}\right)/\mathcal{F}_0\left(\eta_{FS}\right)}\right]$$

$$\upsilon_{inj}^{ball} = \left\langle\left\langle \upsilon_x^+ \right\rangle\right\rangle = \upsilon_T \frac{\mathcal{F}_{1/2}\left(\eta_{FS}\right)}{\mathcal{F}_0\left(\eta_{FS}\right)}$$

Unit 4:

$$\mathcal{T}(E) \leq 1$$