Fundamentals of Nanotransistors

Unit 3: The Ballistic Nanotransistor

Lecture 3.5: The Velocity at the VS

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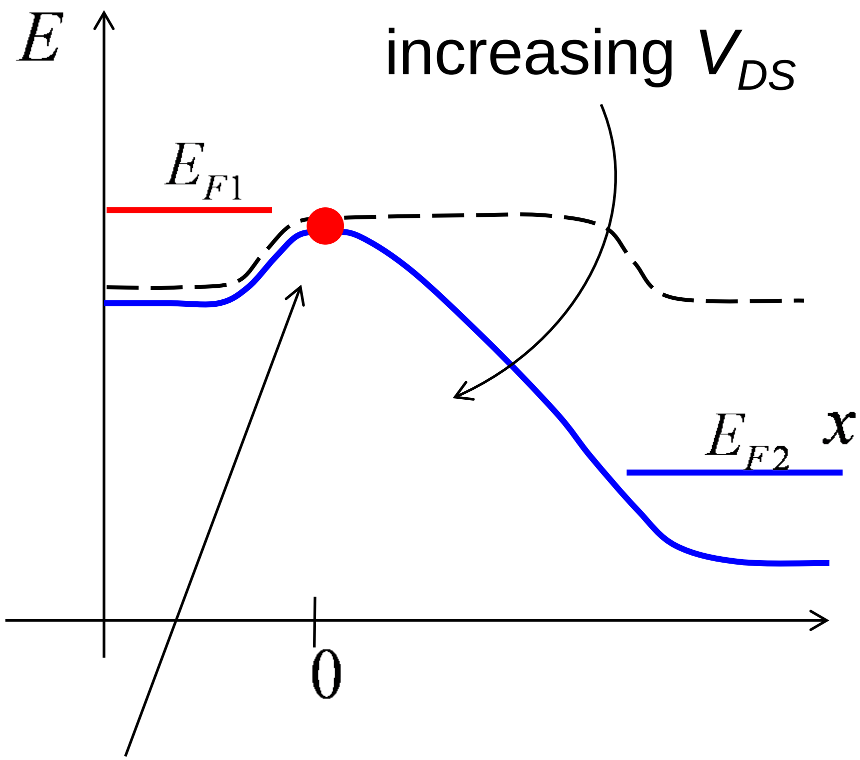
Current is velocity times charge

$$I_{DS} = W\left|Q_n\left(V_{GS}, V_{DS}\right)\right|\left\langle \upsilon\left(V_{GS}, V_{DS}\right)\right\rangle$$

Average velocity at the top of the barrier



velocity at the top of the barrier?

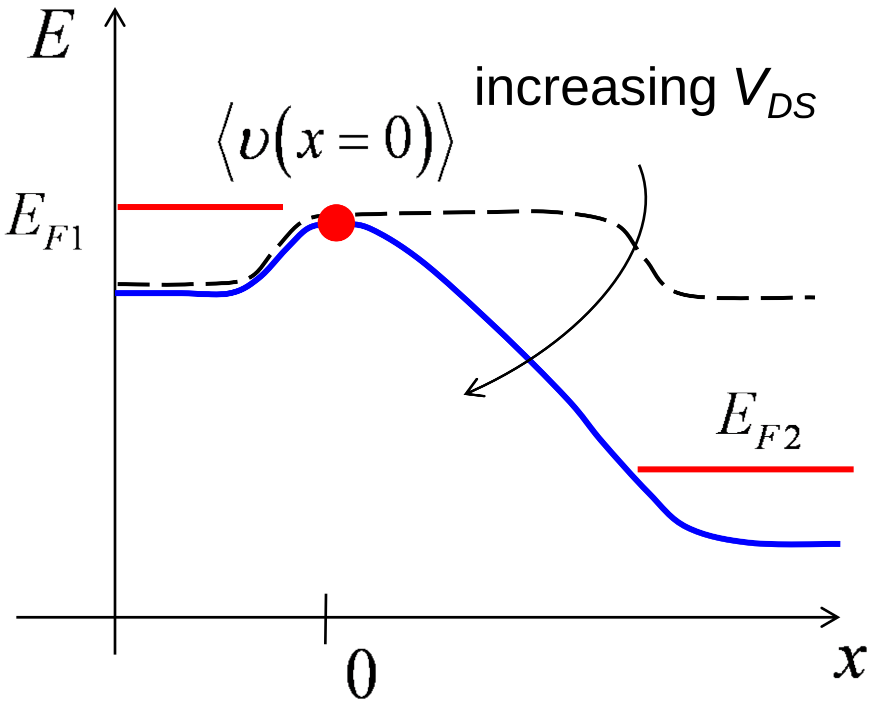
$$I_{DS} = W\left|Q_n(0)\right|\upsilon_T \frac{\left(1-e^{-qV_{DS}/k_BT}\right)}{\left(1+e^{-qV_{DS}/k_BT}\right)}$$

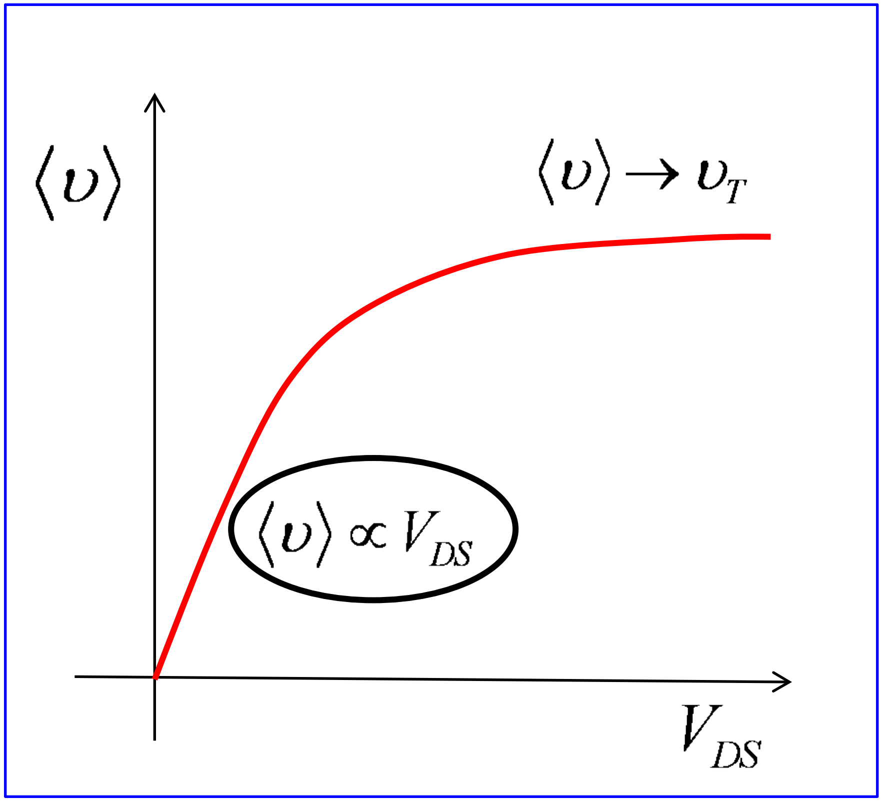
$$I_{DS} = W\left|Q_n(0)\right|\left\langle \upsilon_x(x=0)\right\rangle$$

$$\boxed{\left\langle \upsilon_x(x=0)\right\rangle = \upsilon_T \frac{\left(1-e^{-qV_{DS}/k_BT}\right)}{\left(1+e^{-qV_{DS}/k_BT}\right)}}$$

(nondegenerate carrier statistics)

Velocity vs. V_{DS}





$$\langle \upsilon_x(x=0) \rangle = \upsilon_T \frac{\left(1 - e^{-qV_{DS}/k_BT}\right)}{\left(1 + e^{-qV_{DS}/k_BT}\right)}$$

Velocity for small V_{DS}

$$\langle \upsilon(x=0) \rangle = \upsilon_T \frac{\left(1 - e^{-qV_{DS}/k_B T}\right)}{\left(1 + e^{-qV_{DS}/k_B T}\right)}$$

$$V_{DS} << k_B T/q$$

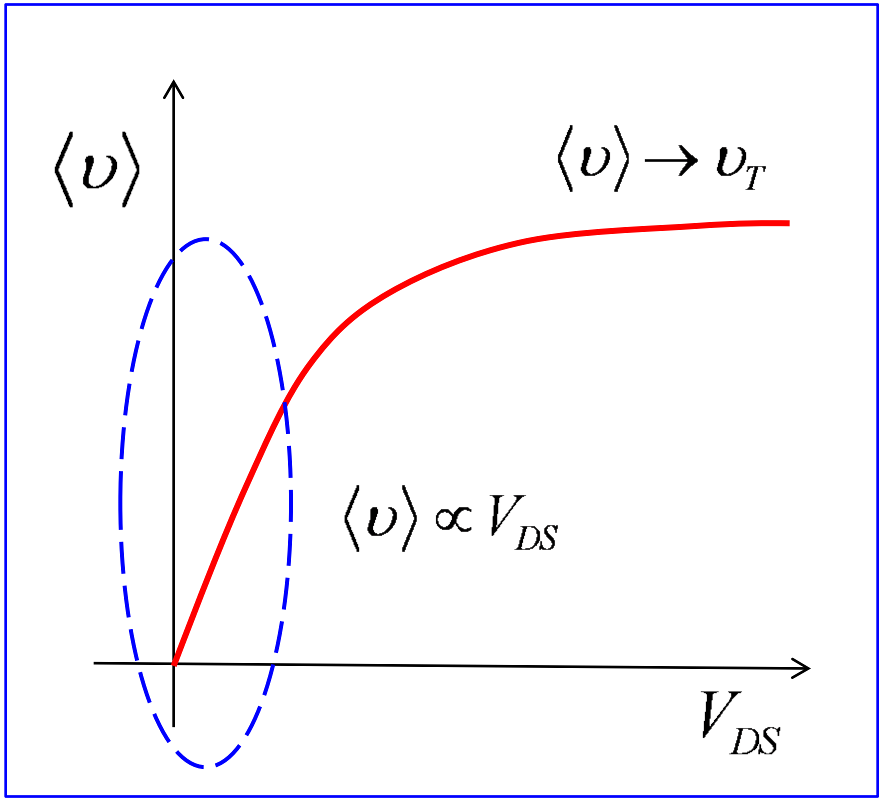
$$\langle \upsilon(x=0) \rangle = \frac{\upsilon_T}{2(k_B T/q)} V_{DS}$$

$$\langle \upsilon(x=0) \rangle = \frac{\upsilon_T L}{2(k_B T/q)} \frac{V_{DS}}{L}$$

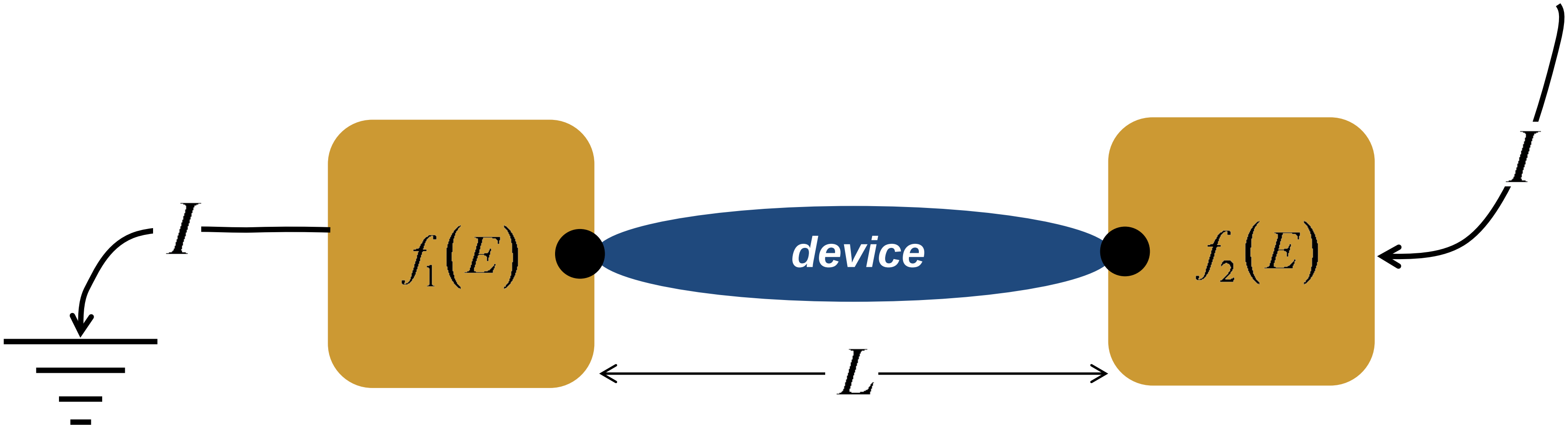
$$\mu_B \equiv \frac{\upsilon_T L}{2(k_B T/q)} \quad \mathrm{cm^2/V\text{-}s}$$

"ballistic mobility"

$$\boxed{\langle \upsilon(x=0) \rangle = \mu_B \mathcal{E}_x}$$



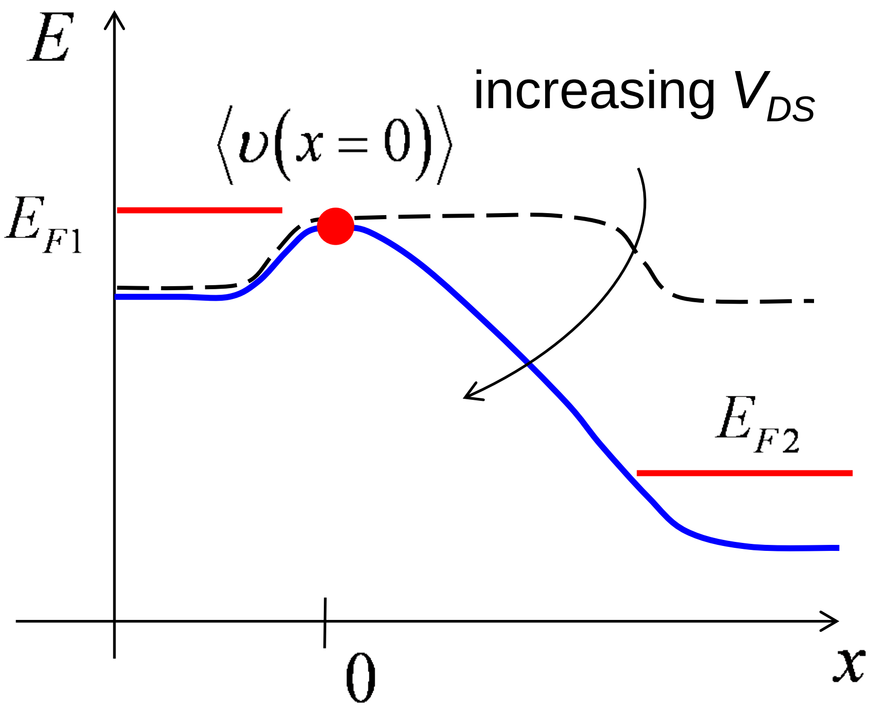
The meaning of "ballistic mobility"

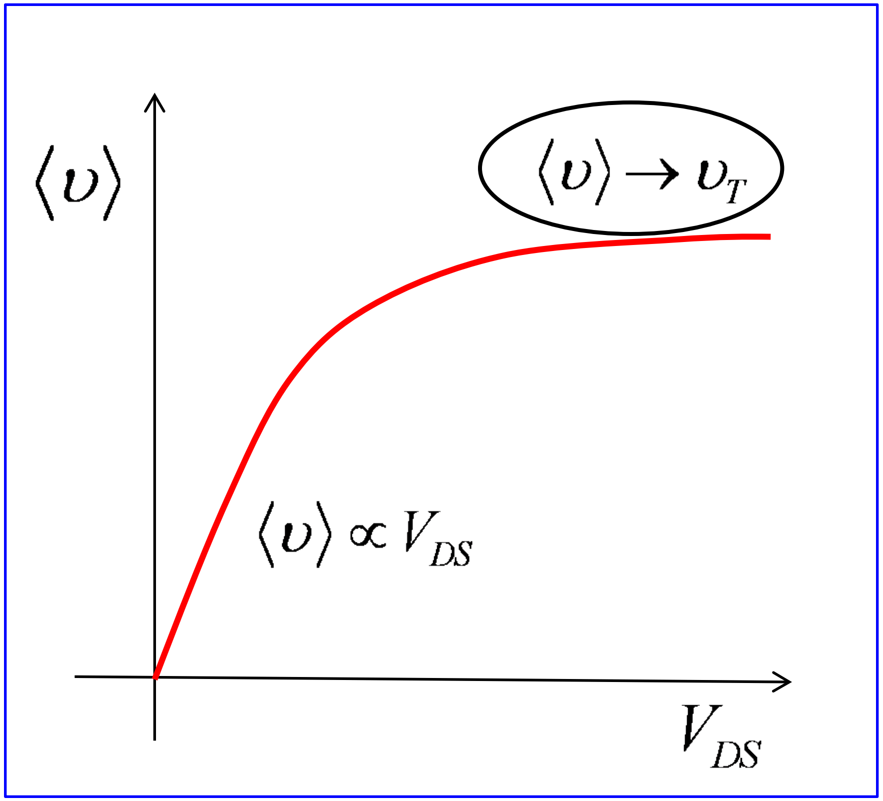


$$\mu_n = \frac{\upsilon_T \lambda_0}{2 k_B T / q} \ \ \mathrm{cm^2/V\text{-}s} \qquad\qquad \mu_B = \frac{\upsilon_T L}{2 k_B T / q} \ \ \mathrm{cm^2/V\text{-}s}$$

λ_0: average mean-free-path for backscattering $\qquad \lambda_0 \rightarrow L$

Velocity vs. V_{DS}

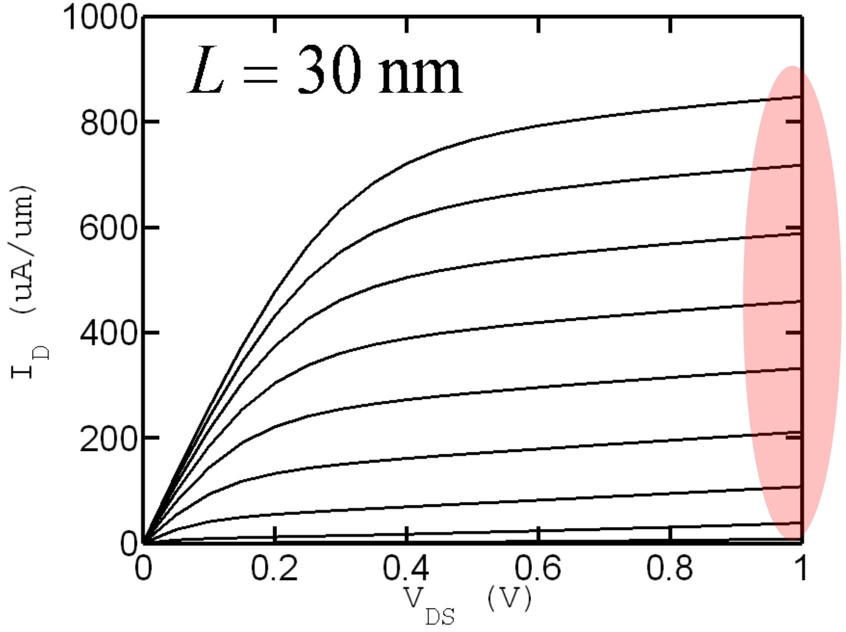




$$\langle \upsilon_x(x=0) \rangle = \upsilon_T \frac{\left(1 - e^{-qV_{DS}/k_BT}\right)}{\left(1 + e^{-qV_{DS}/k_BT}\right)}$$

The velocity ***saturates*** in a ballistic MOSFET but **at the top of the barrier**, where *E*-field = 0.

"Signature" of velocity saturation



Traditional (diffusive) model:

$$I_{DSAT} = W\left|Q_n\left(V_{GS}, V_{DS}\right)\right|\upsilon_{sat}$$

Ballistic model:

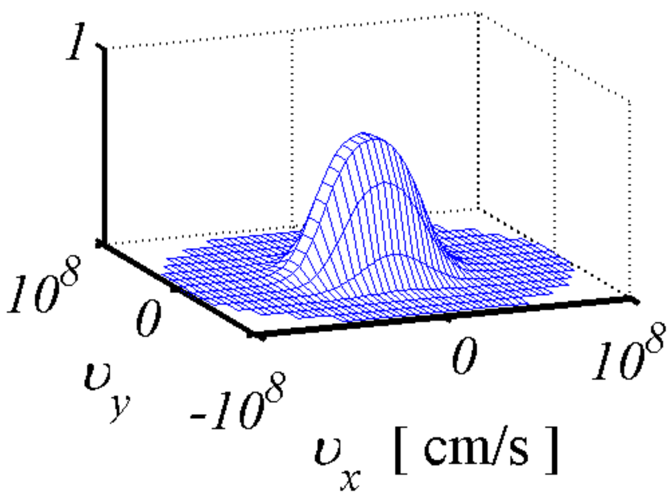
$$I_{DSAT} = W\left|Q_n\left(V_{GS}, V_{DS}\right)\right|\upsilon_T$$

ETSOI MOSFET data provided by
A. Majumdar, IBM Research, 2015.

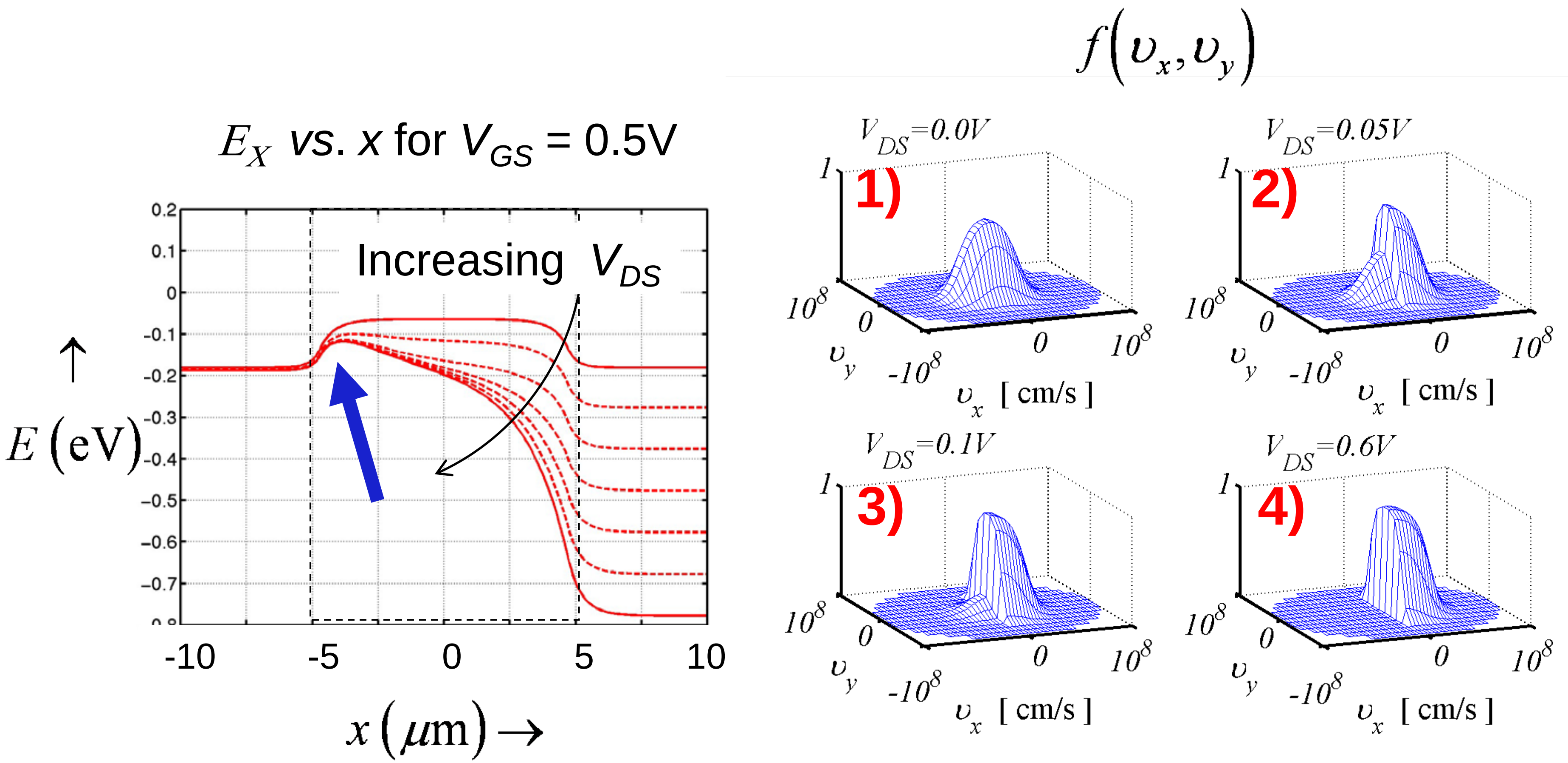
Maxwellian velocity distribution

$$f_0(E) = \frac{1}{1+e^{(E-E_F)/k_BT}} \qquad f_0(E) \approx e^{-(E-E_F)/k_BT} \qquad E = E_C + \frac{m^*\upsilon^2}{2} \qquad \upsilon^2 = \upsilon_x^2 + \upsilon_y^2$$

$$f_0(\upsilon_x, \upsilon_y) \propto e^{-m^*(\upsilon_x^2+\upsilon_y^2)/2k_BT}$$



Filling states at the top of the barrier



(Numerical simulations of an L = 10 nm double gate Si MOSFET from J.-H. Rhew and M.S. Lundstrom, *Solid-State Electron.*, **46**, 1899, 2002)

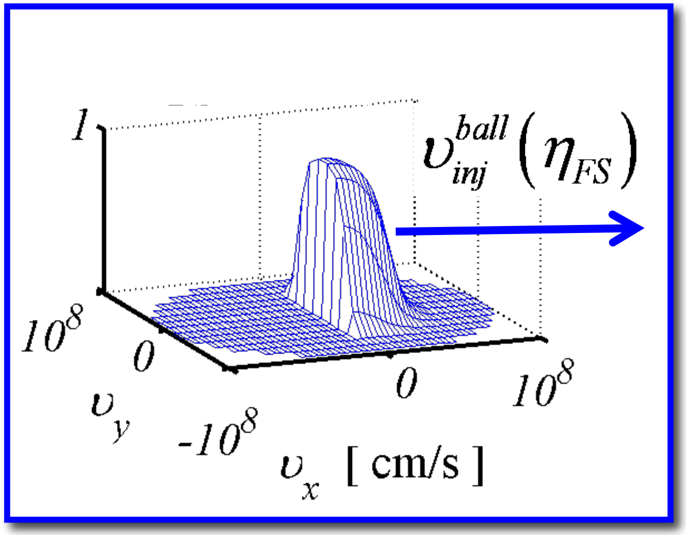
Ballistic injection velocity

$$I_{DS} = W\left|Q_n(x=0)\right| \upsilon_{inj}^{ball}(\eta_{FS}) \left[\frac{1 - \mathcal{F}_{1/2}(\eta_{F2}) / \mathcal{F}_{1/2}(\eta_{F1})}{1 + \mathcal{F}_0(\eta_{F2}) / \mathcal{F}_0(\eta_{F1})} \right]$$

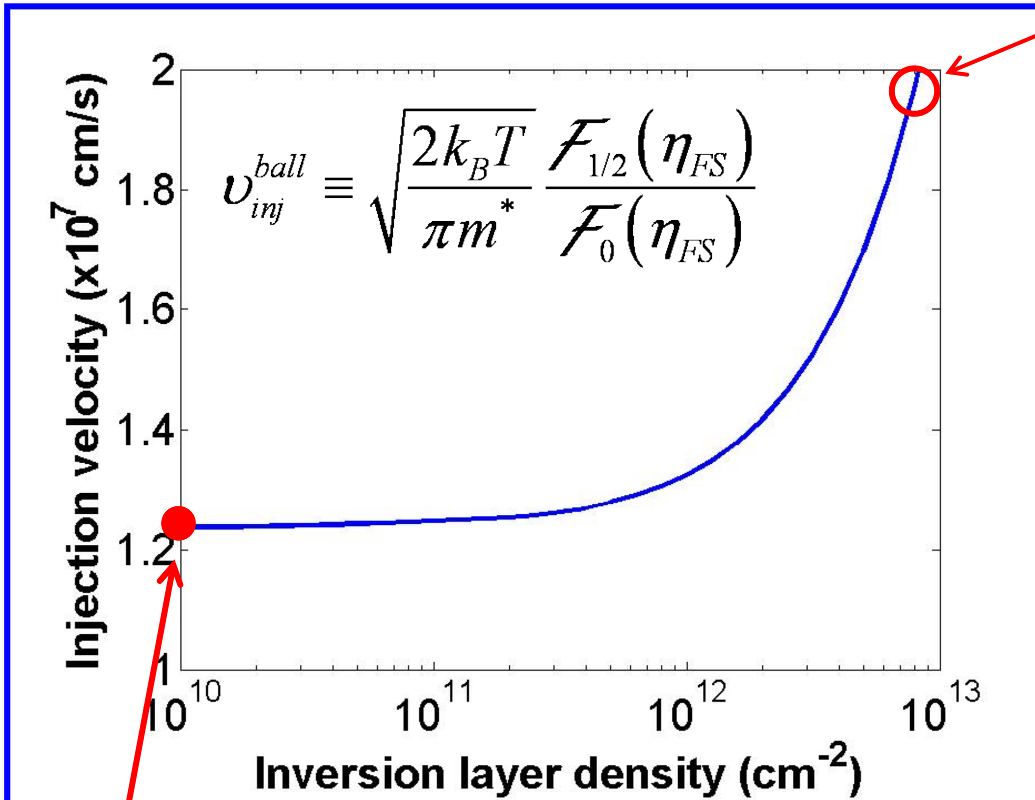
$$I_{ON} = W\left|Q_n(0)\right| \upsilon_{inj}^{ball}(\eta_{FS})$$ High drain bias

$$\langle\langle \upsilon_x(0) \rangle\rangle = \upsilon_{inj}^{ball}(\eta_{FS}) = \upsilon_T \frac{\mathcal{F}_{1/2}(\eta_{FS})}{\mathcal{F}_0(\eta_{FS})}$$

ballistic injection velocity
(2D carriers)



Ballistic injection velocity



$$\upsilon_{inj}^{ball} \rightarrow \frac{4\upsilon_F}{3\pi}$$

$$\upsilon_F = \sqrt{\frac{2(E_F - E_C)}{m^*}}$$

fully degenerate

$$\boxed{\upsilon_T = \sqrt{2k_BT/\pi m^*}}$$

non-degenerate

Maxwell-Boltzmann vs. Fermi-Dirac statistics

Maxwell-Boltzmann statistics

$$\upsilon_{inj}^{ball} = \upsilon_T = \sqrt{\frac{2k_B T}{\pi m^*}}$$

Fermi-Dirac statistics

$$\upsilon_{inj}^{ball} = \left\langle\left\langle \upsilon_x^+ \right\rangle\right\rangle = \upsilon_T \frac{\mathcal{F}_{1/2}\left(\eta_{FS}\right)}{\mathcal{F}_0\left(\eta_{FS}\right)}$$

$$\eta_{FS} = \frac{E_{FS} - E_C}{k_B T}$$

$$Q_n\left(V_{GS}, V_{DS}\right) = -q \frac{N_{2D}}{2} \mathcal{F}_0\left(\eta_{FS}\right) \mathrm{m}^{-2}$$

(2D carriers and parabolic energy bands)

Key points

- $I_{DSAT} \sim (V_{GS} - V_T)^1$ is the "signature" of velocity saturation.
- The velocity saturates in a ballistic MOSFET, but at the top of the barrier where the electric field is zero – not near the drain where the electric field is large.
- The ballistic injection velocity increases with increasing electron density (increasing Fermi level).

Next lecture

Time to revisit the VS model.