Unit 3: The Ballistic Nanotransistor

Lecture 3.4: The Ballistic MOSFET

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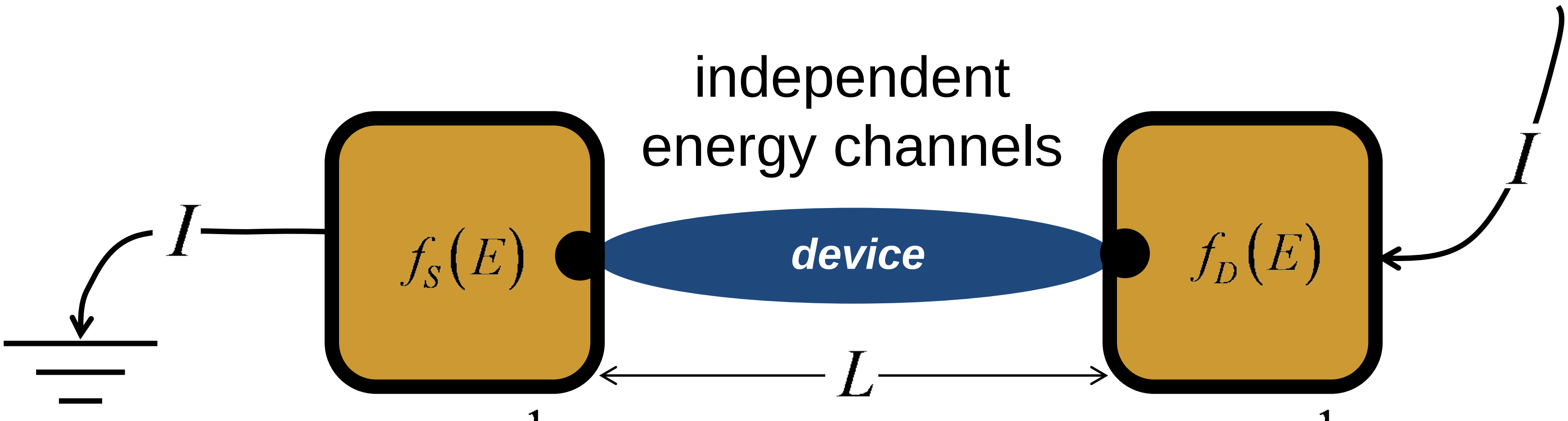
Nanodevice

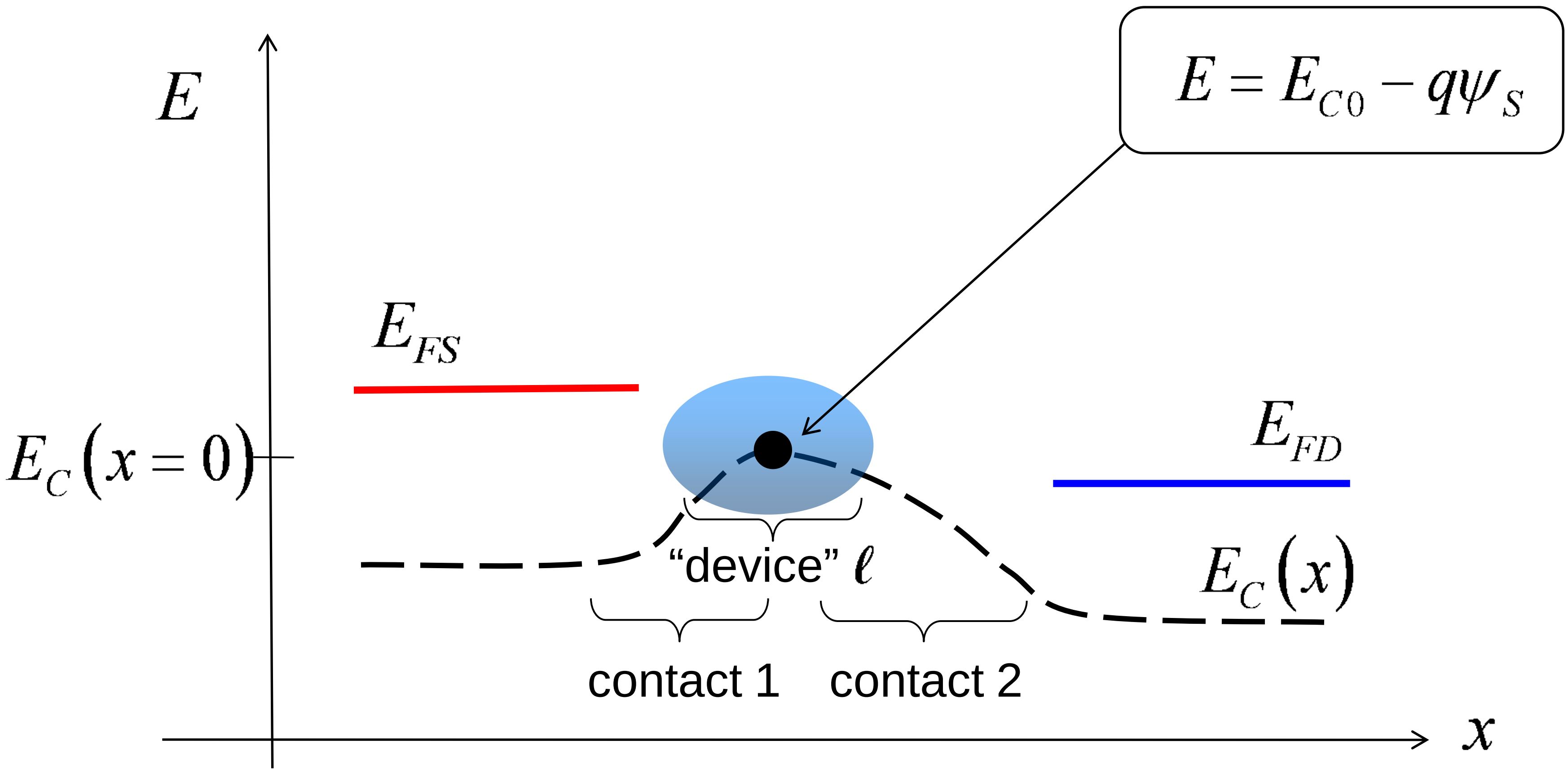


$$f_S(E) = \frac{1}{1 + e^{(E - E_{FS})/k_B T}}$$

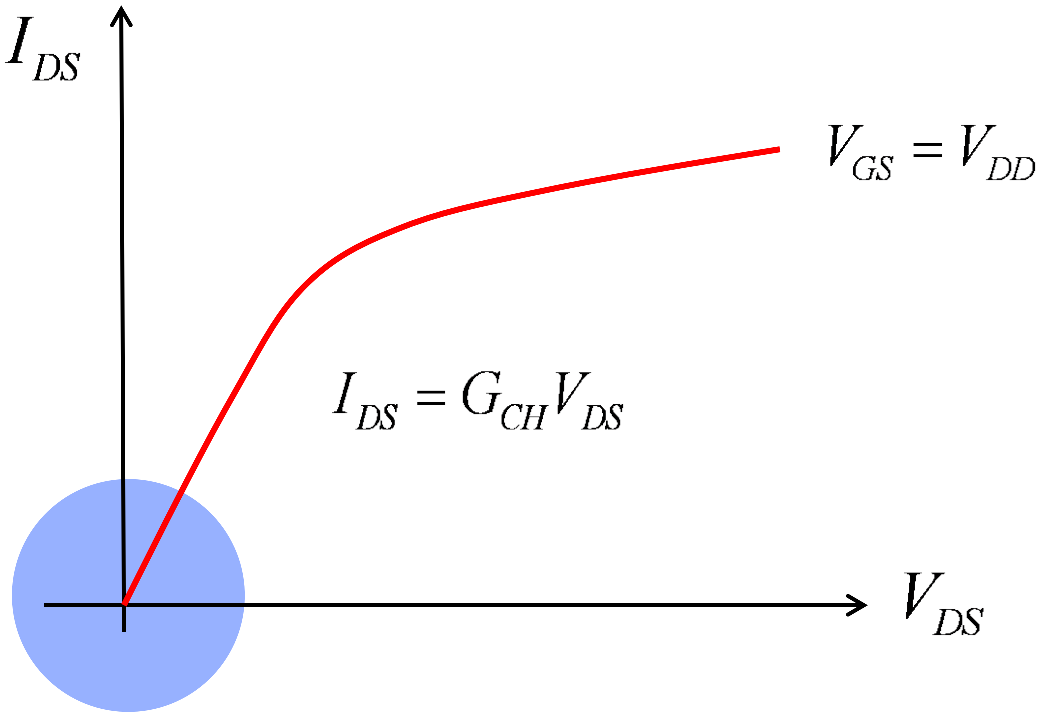
$$f_D(E) = \frac{1}{1 + e^{(E - E_{FD})/k_B T}}$$

$$E_{FD} = E_{FS} - qV_{DS}$$

Focus on the top of the barrier



Ballistic MOSFET: linear region



near-equilibrium $f_S \approx f_D$

Linear region with MB statistics (i)

$$G_{CH} = \frac{2q^2}{h}\int_{E_C}^{\infty} \mathcal{T}(E) M(E)\left(-\frac{\partial f_0}{\partial E}\right) dE$$

$$M(E) = W g_V \frac{\sqrt{2m^*\left(E - E_C\right)}}{\pi\hbar}$$

$$\mathcal{T}(E) = 1$$

$$f_0(E) = \frac{1}{1 + e^{(E - E_F)/k_B T}} = e^{(E_F - E)/k_B T}$$

$$n_S = N_{2D} e^{(E_F - E_C)/k_B T}$$

$$N_{2D} = \left(g_V \frac{m^*}{\pi\hbar^2} k_B T\right)$$

(See Sec. 12.7 – 12.9 of lecture notes for the complete derivation.)

Linear region with MB statistics (ii)

$$G_{CH} = \frac{2q^2}{h}\int_{E_C}^{\infty} \mathcal{T}(E)M(E)\left(-\frac{\partial f_0}{\partial E}\right)dE$$

$$-\partial f_0/\partial E = f_0/k_BT$$

$$G_{CH} = \frac{2q^2}{h}\int_{E_C}^{\infty} 1\left[g_V W\frac{\sqrt{2m^*(E-E_C)}}{\pi\hbar}\right]\left(\frac{f_0}{k_BT}\right)dE$$

$$G_{CH} = W(qn_S)\frac{\upsilon_T}{2(k_BT/q)} \qquad \upsilon_T = \sqrt{\frac{2k_BT}{\pi m^*}}$$

$$G_{CH} = -W\,Q_n(V_{GS},V_{DS})\frac{\upsilon_T}{2(k_BT/q)}$$ ✔

$$M(E) = g_V W\frac{\sqrt{2m^*(E-E_C)}}{\pi\hbar}$$

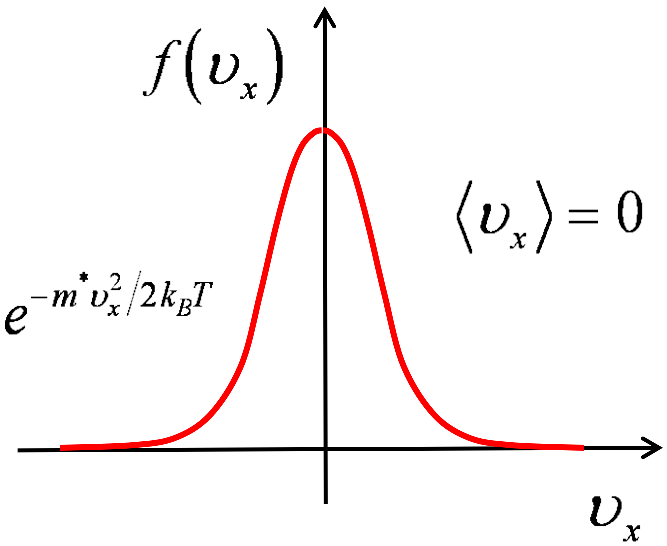
$$\mathcal{T}(E) = 1$$

$$f_0(E) = e^{(E_F-E)/k_BT}$$

$$n_S = N_{2D}e^{(E_F-E_C)/k_BT}$$

$$N_{2D} = \left(g_V\frac{m^*}{\pi\hbar^2}k_BT\right)$$

Equilibrium Maxwellian velocity distribution



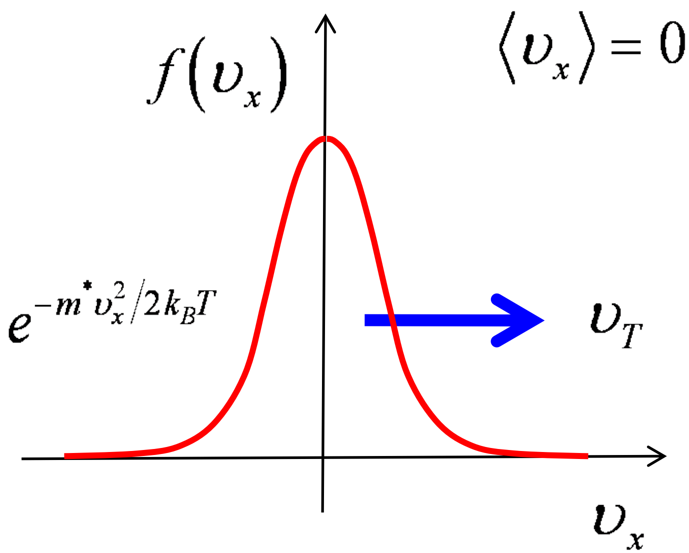
$$f_0(E) \propto e^{-m^*\left(\upsilon_x^2 + \upsilon_y^2\right)/2k_B T}$$

$$f_0(E) = \frac{1}{1 + e^{(E - E_F)/k_B T}} = e^{(E_F - E)/k_B T}$$

$$E = E_C + \frac{1}{2} m^* \upsilon^2$$

$$f_0(E) = e^{(E_F - E_C)/k_B T} \times e^{-m^* \upsilon^2 / 2k_B T}$$

Unidirectional thermal velocity



$$\upsilon_T = \sqrt{\frac{2k_B T}{\pi m^*}} = \left\langle \left\langle \upsilon_x^+ \right\rangle \right\rangle$$

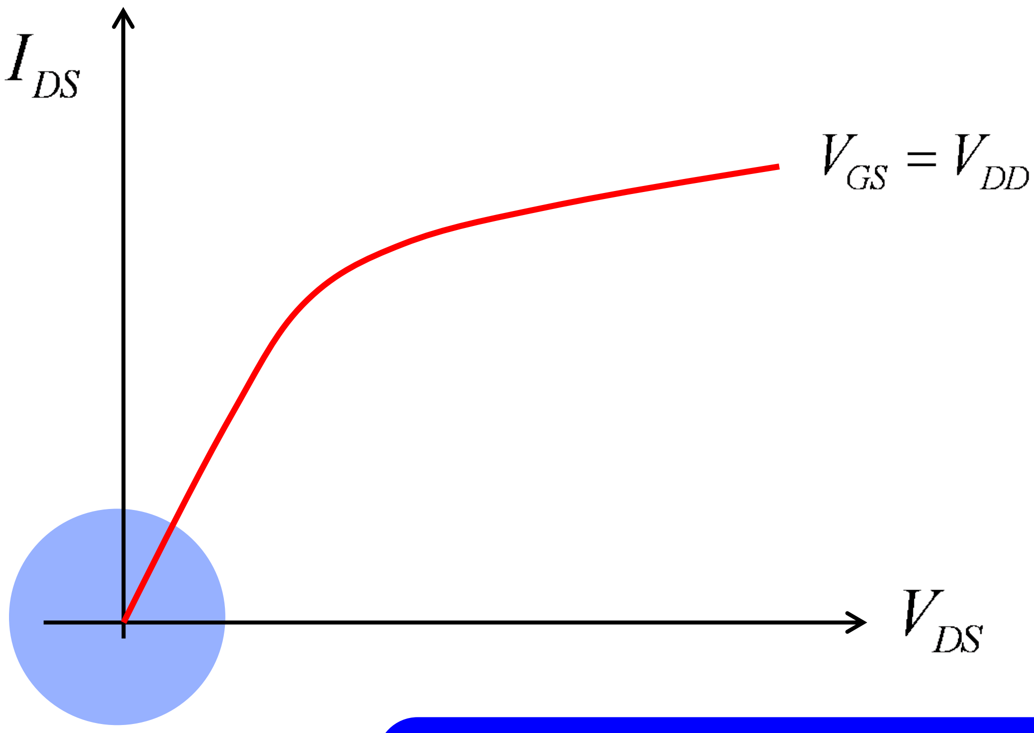
(nondegenerate)

$$\left\langle \upsilon_x^+(E) \right\rangle = \frac{2}{\pi} \upsilon(E)$$

(Lecture 3.2)

$$\left\langle \left\langle \upsilon_x^+(E) \right\rangle \right\rangle = \frac{\int_{E_C}^{\infty} \left\langle \upsilon_x^+(E) \right\rangle D_{2D}(E) f_0(E) dE}{\int_{E_C}^{\infty} D_{2D}(E) f_0(E) dE}$$

Ballistic MOSFET: linear region

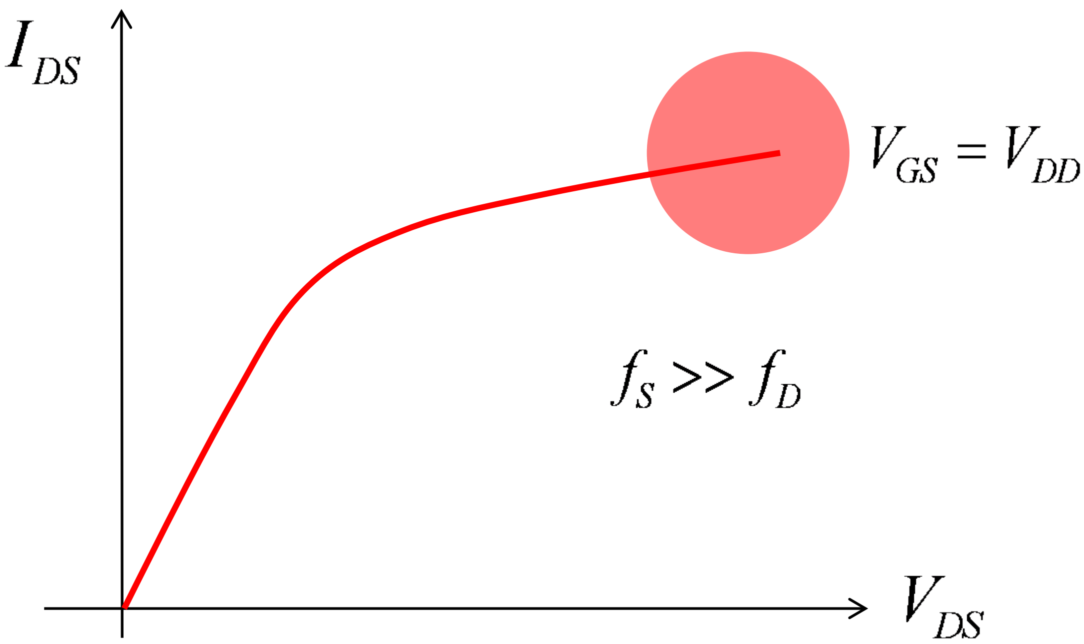


near-equilibrium: $f_S \approx f_D$

$$I_{DS} = G_{CH} V_{DS}$$

$$\boxed{I_{DS} = W \frac{\upsilon_T}{2k_B T / q} C_{inv} \left(V_{GS} - V_T\right) V_{DS}}$$

Ballistic MOSFET: saturation region



$$I_{DS} = \frac{2q}{h}\int \mathcal{T}(E)M(E)(f_S - f_D)dE \rightarrow I_{DS} = \frac{2q}{h}\int \mathcal{T}(E)M(E) f_S\, dE$$

Saturation region with MB statistics

$$I_{DS} = \frac{2q}{h}\int_{E_C}^{\infty} \mathcal{T}(E) M(E) f_S(E)\, dE$$

$$M(E) = g_V W \frac{\sqrt{2m^*(E - E_C)}}{\pi\hbar}$$

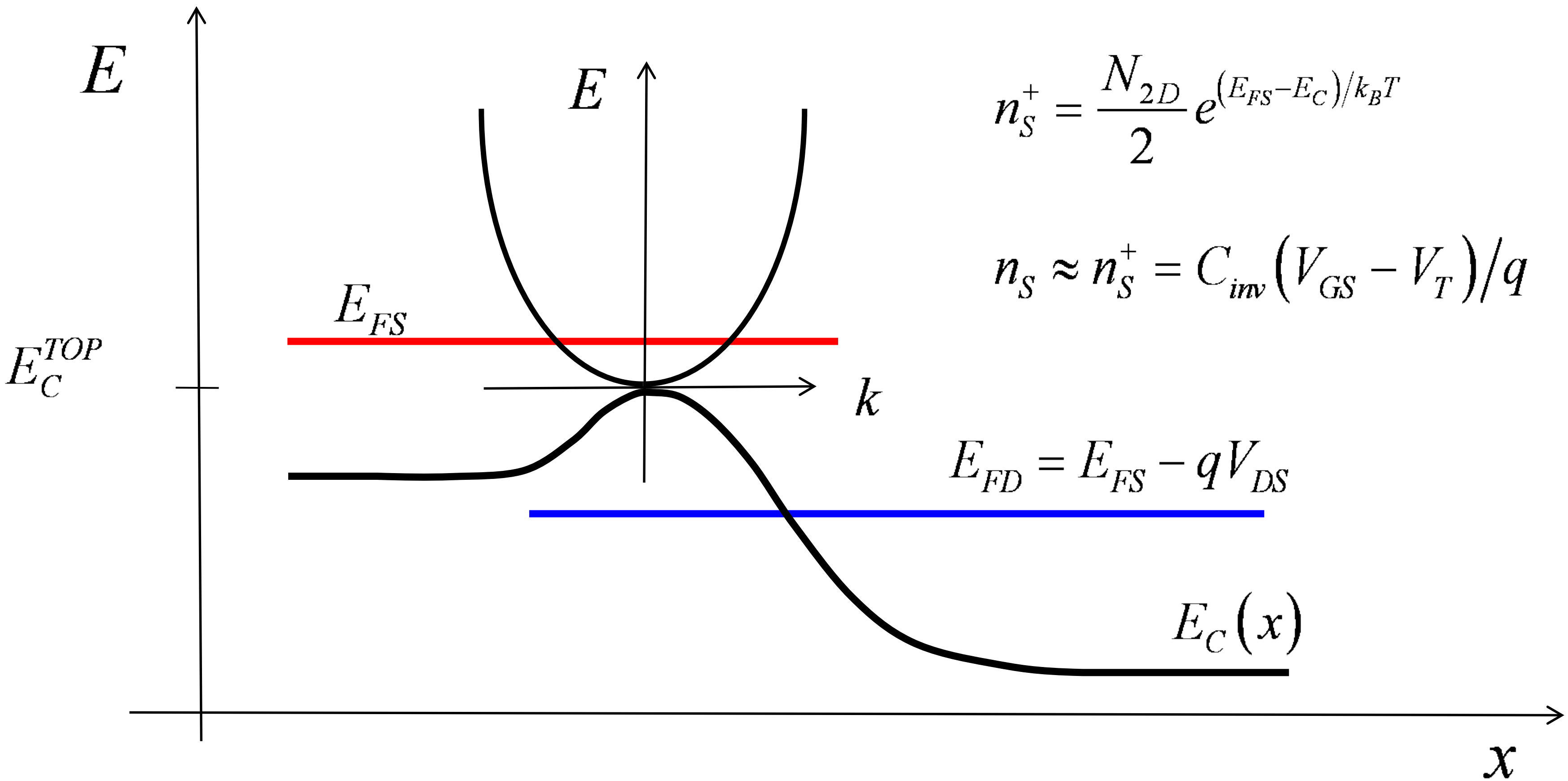
$$\mathcal{T}(E) = 1$$

$$f_0(E) = e^{(E_F - E)/k_B T}$$

$$\upsilon_T = \sqrt{2k_B T / \pi m^*}$$

$$n_S = \frac{N_{2D}}{2} e^{(E_F - E_C)/k_B T}$$

Carrier density at the top of the barrier



$$n_S^+ = \frac{N_{2D}}{2} e^{(E_{FS} - E_C)/k_B T}$$

$$n_S \approx n_S^+ = C_{inv}\left(V_{GS} - V_T\right)/q$$

Saturation region with MB statistics

$$I_{DS} = \frac{2q}{h} \int_{E_C}^{\infty} \mathcal{T}(E) M(E) f_S(E) dE$$

$$I_{DS} = \frac{2q^2}{h} \int_{E_C}^{\infty} 1 \left[g_V W \frac{\sqrt{2m^*(E - E_C)}}{\pi\hbar} \right] f_0(E) dE$$

$$I_{DS} = Wqn_S\upsilon_T$$

$$I_{DS} = -WQ_n(V_{GS}, V_{DS})\upsilon_T \quad ✔$$

$$M(E) = g_V W \frac{\sqrt{2m^*(E - E_C)}}{\pi\hbar}$$

$$\mathcal{T}(E) = 1$$

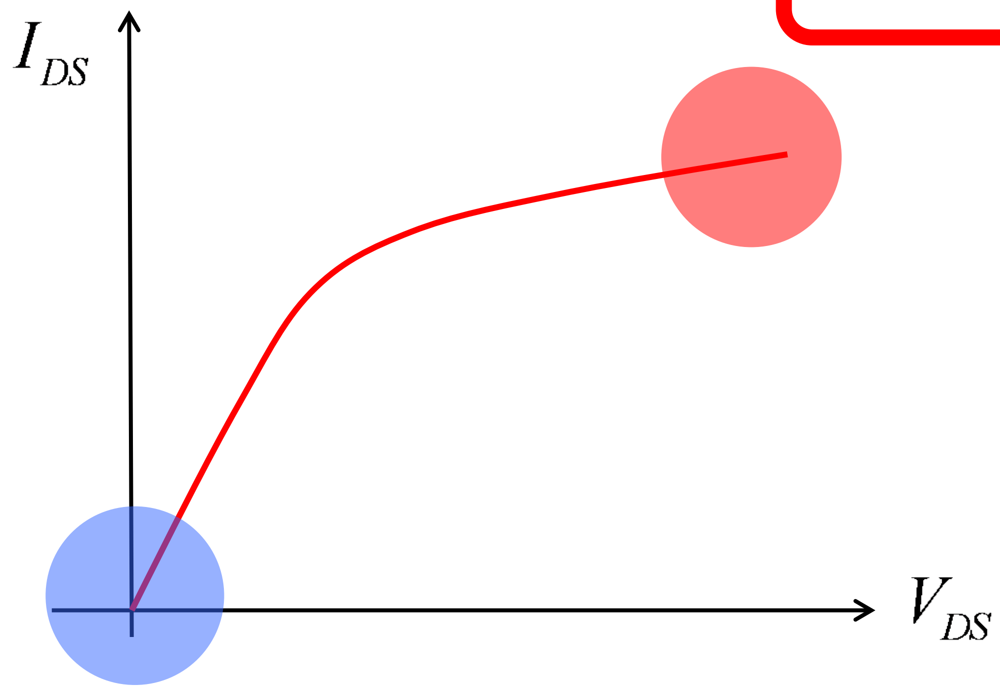
$$f_0(E) = e^{(E_F - E)/k_B T}$$

$$\upsilon_T = \sqrt{2k_B T / \pi m^*}$$

$$n_S = \frac{N_{2D}}{2} e^{(E_F - E_C)/k_B T}$$

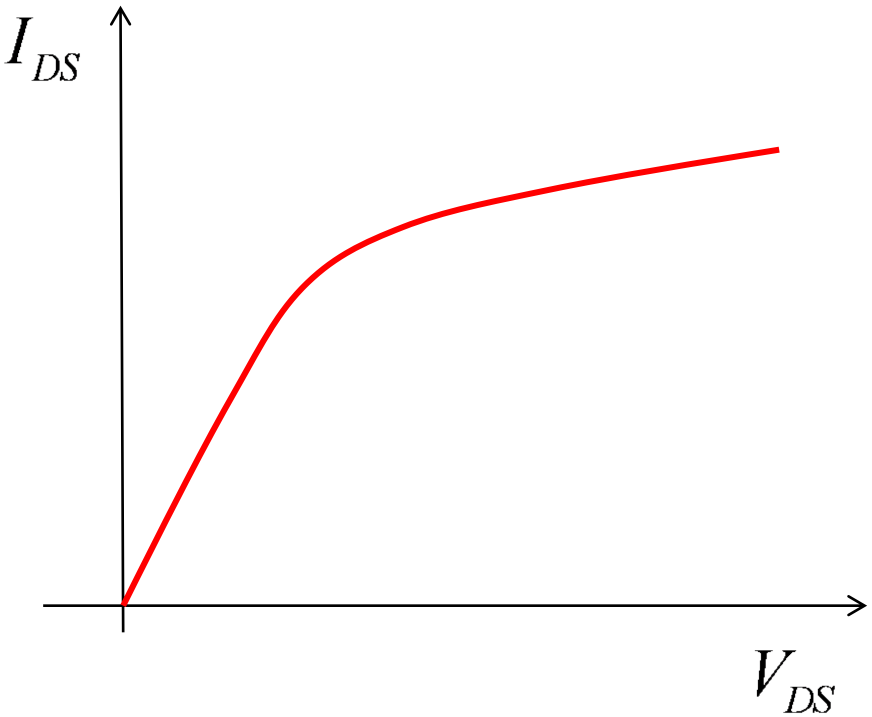
Ballistic MOSFET

$$I_{DS} = W\,C_{inv}\left(V_{GS} - V_T\right)\upsilon_T$$



$$I_{DS} = W\,\frac{\upsilon_T}{2k_BT/q}C_{inv}\left(V_{GS} - V_T\right)V_{DS}$$

Ballistic MOSFET: full V_{DS} range



$$I_{DS} = \frac{2q}{h}\int \mathcal{T}(E)M(E)\left(f_S - f_D\right)dE$$

$$I_{DS} = I_{D1} - I_{D2}$$

$$I_{D1} = -W\,Q_n^+\,\upsilon_T$$

$$I_{D2} = -W\,Q_n^-\,\upsilon_T$$

$$\begin{aligned} I_{DS} &= -W\,\upsilon_T\left(Q_n^+ - Q_n^-\right) \\ &= -W\,\upsilon_T Q_n^+\left(1 - Q_n^-/Q_n^+\right) \end{aligned}$$

Full V_{DS} expression

$$I_{DS} = -W\,\upsilon_T Q_n^+\left(1 - Q_n^- / Q_n^+\right)$$

$$Q_n = Q_n^+ + Q_n^- = Q_n^+\left(1 + Q_n^+ / Q_n^-\right)$$

$$Q_n^+ = \frac{Q_n}{\left(1 + Q_n^+ / Q_n^-\right)}$$

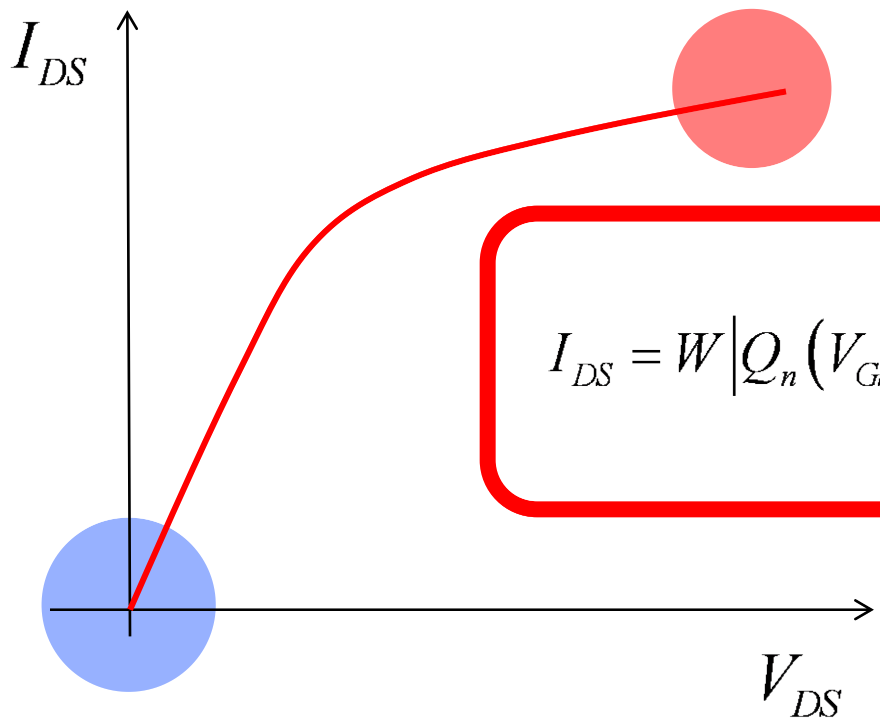
$$I_{DS} = -W\,\upsilon_T Q_n \frac{\left(1 - Q_n^- / Q_n^+\right)}{\left(1 + Q_n^- / Q_n^+\right)}$$

$$\frac{Q_n^-}{Q_n^+} = e^{-qV_{DS}/k_B T}$$

$$\boxed{I_{DS} = W\left|Q_n\left(V_{GS}, V_{DS}\right)\right|\upsilon_T\left(\frac{1 - e^{-qV_{DS}/k_B T}}{1 + e^{-qV_{DS}/k_B T}}\right)}$$

Full range ballistic model (nondegenerate)

$$I_{DS} = W\left|Q_n\left(V_{GS}, V_{DS}\right)\right|\upsilon_T$$



$$\boxed{I_{DS} = W\left|Q_n\left(V_{GS}, V_{DS}\right)\right|\upsilon_T\left(\frac{1-e^{-qV_{DS}/k_BT}}{1+e^{-qV_{DS}/k_BT}}\right)}$$

$$I_{DS} = W\frac{\upsilon_T}{2k_BT/q}\left|Q_n\left(V_{GS}, V_{DS}\right)\right|V_{DS}$$

Ballistic MOSFET with FD statistics

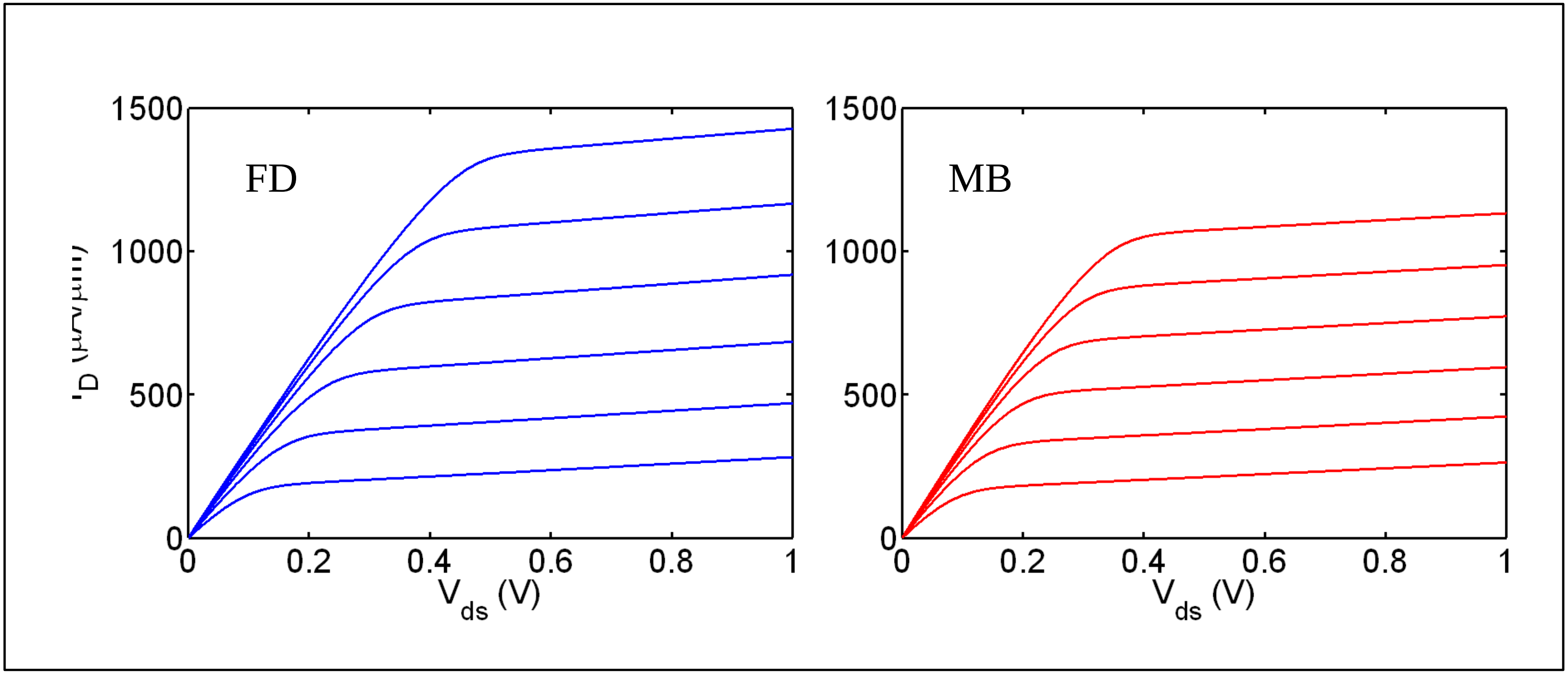
3) $$I_{DS} = W\left|Q_n\left(V_{GS},V_{DS}\right)\right|\upsilon_{inj}^{ball}\left[\frac{1-\mathcal{F}_{1/2}\left(\eta_{FD}\right)/\mathcal{F}_{1/2}\left(\eta_{FS}\right)}{1+\mathcal{F}_{0}\left(\eta_{FD}\right)/\mathcal{F}_{0}\left(\eta_{FS}\right)}\right]$$

1) $$Q_n\left(V_{GS},V_{DS}\right) = -q\frac{N_{2D}}{2}\left(\mathcal{F}_0\left(\eta_{FS}\right)+\mathcal{F}_0\left(\eta_{FD}\right)\right)\mathrm{m}^{-2}$$

2) $$\upsilon_{inj}^{ball} = \left\langle\left\langle \upsilon_x^+ \right\rangle\right\rangle = \upsilon_T\frac{\mathcal{F}_{1/2}\left(\eta_{FS}\right)}{\mathcal{F}_{0}\left(\eta_{FS}\right)} \qquad \upsilon_T = \sqrt{\frac{2k_BT}{\pi m^*}}$$

$$\eta_{FD} = \eta_{FS} - qV_{DS}/k_BT$$

FD vs. MB



Ballistic calculations using typical numbers for an ETSOI MOSFET. By Xingshu Sun, in *Fundamentals of Nanotransistors*, Lecture 13, World Scientific, 2015.

Next Lecture

The ballistic injection velocity

$$\upsilon_{inj}^{ball} = \langle\langle \upsilon_x^+ \rangle\rangle = \upsilon_T \frac{\mathcal{F}_{1/2}(\eta_{FS})}{\mathcal{F}_0(\eta_{FS})} \qquad \upsilon_T = \sqrt{\frac{2k_B T}{\pi m^*}}$$