Fundamentals of Nanotransistors

Unit 3: The Ballistic Nanotransistor

Lecture 3.3: More on Landauer

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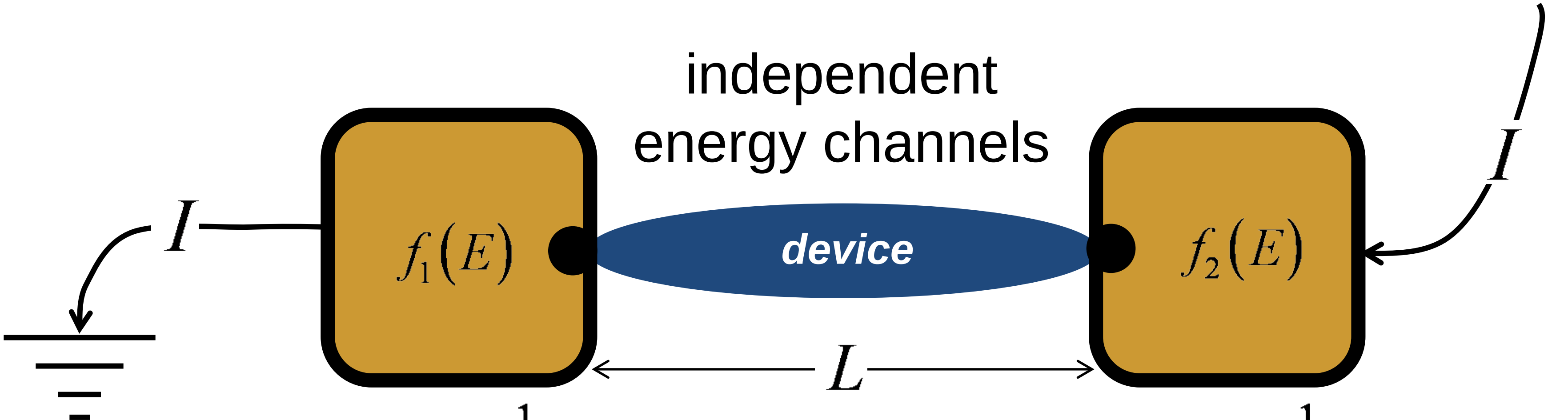
Nanodevice



$$f_1(E) = \frac{1}{1 + e^{(E - E_{F1})/k_B T}}$$

$$f_2(E) = \frac{1}{1 + e^{(E - E_{F2})/k_B T}}$$

$$E_{F2} = E_{F1} - qV$$

$$I = \frac{2q}{h}\int \mathcal{T}(E) M(E)\left(f_1(E) - f_2(E)\right) dE$$

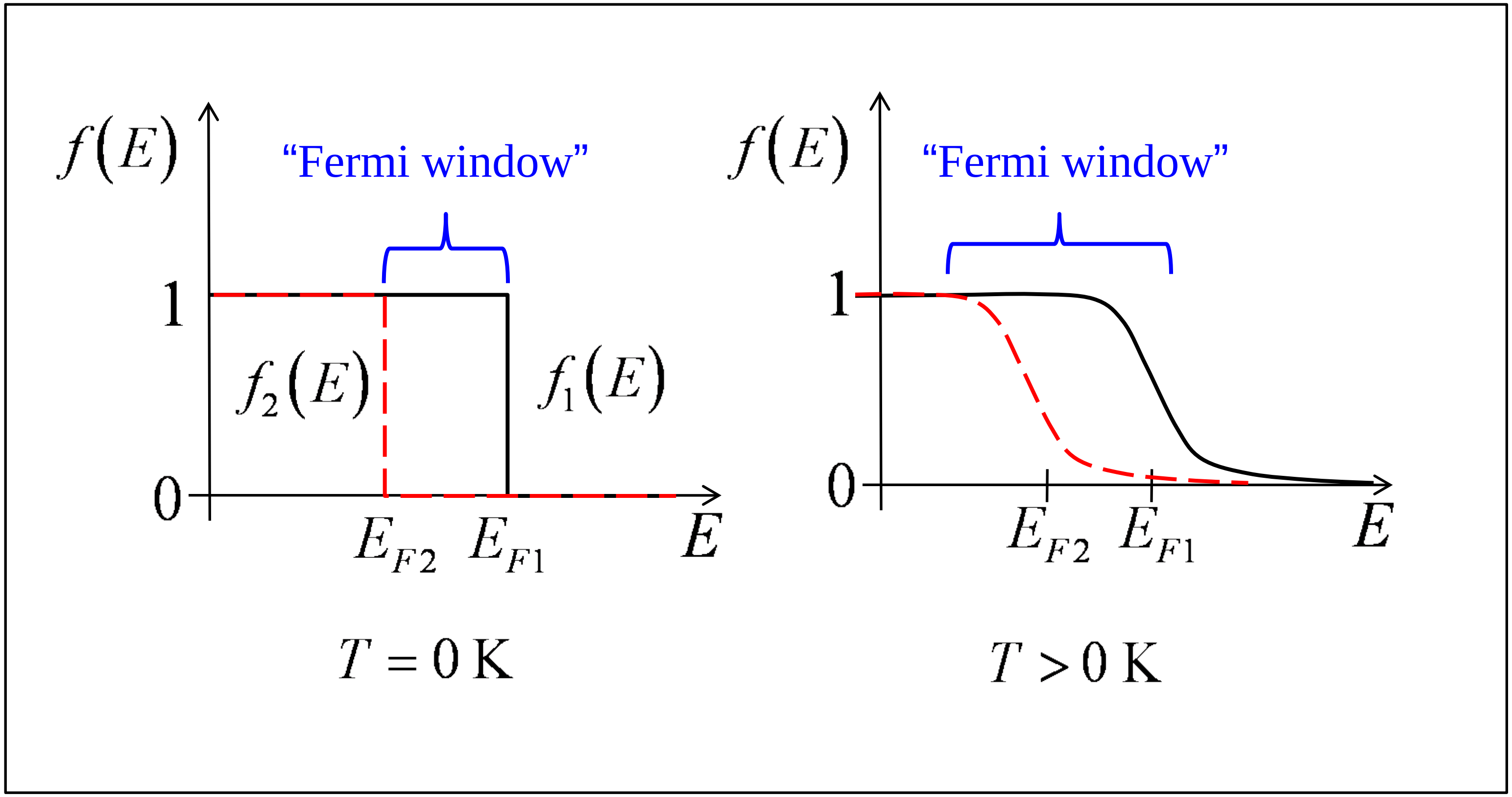
$$n_S = \int_{E_C}^{\infty}\left(\frac{D_{2D}(E)}{2} f_1(E) + \frac{D_{2D}(E)}{2} f_2(E)\right) dE \ \ \mathrm{m}^{-2}$$

Fermi window

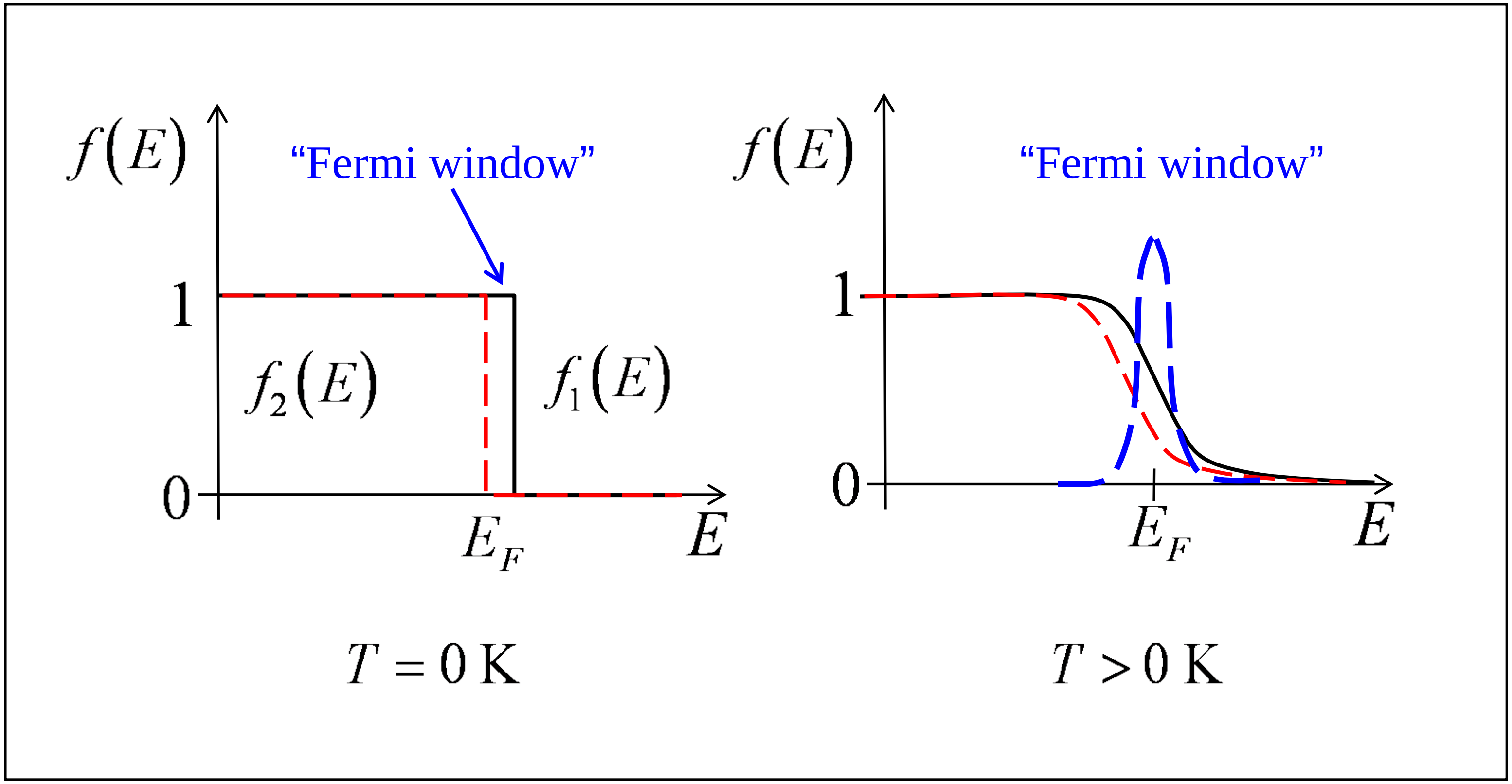
$$I = \frac{2q}{h} \int \mathcal{T}(E) M(E) \left(f_1 - f_2\right) dE$$

The range of energies over which $\left(f_1 - f_2\right) \neq 0$

Fermi window: Large bias



Fermi window: small bias



Small voltage (linear response)

$$I = \frac{2q}{h}\int \mathcal{T}(E)M(E)\left(\underline{f_1(E) - f_2(E)}\right)dE$$

$$f_1(E) = \frac{1}{1 + e^{(E - E_{F1})/k_B T}}$$

$$f_2(E) \approx f_1(E) + \frac{\partial f_1}{\partial E_F}\delta E_F$$

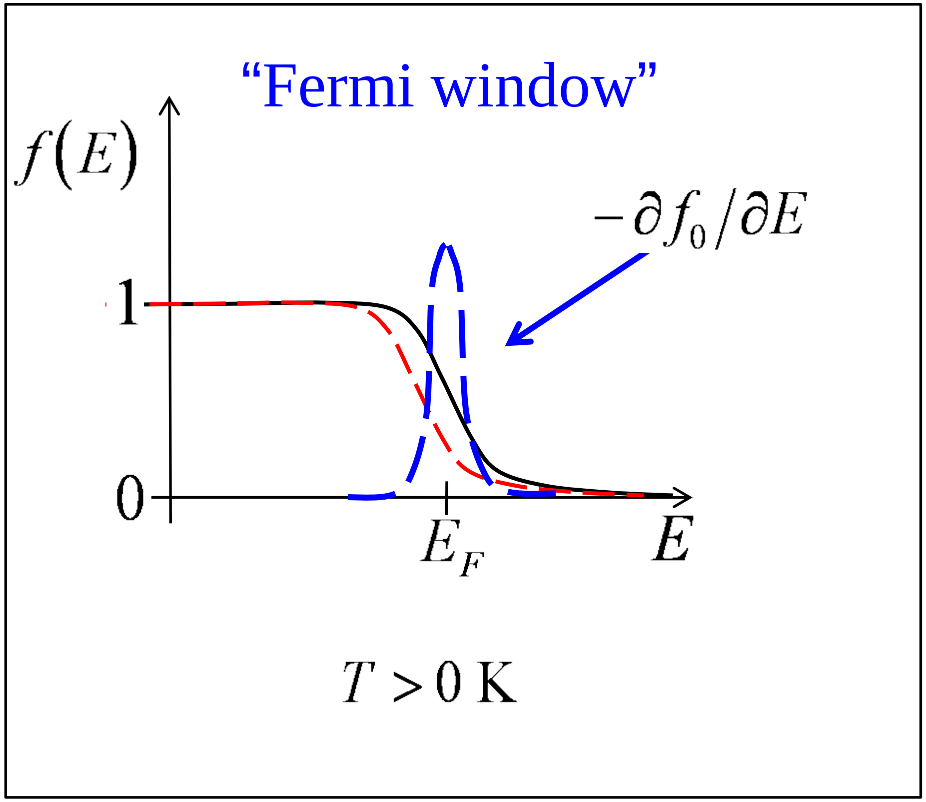
$$f_2(E) \approx f_1(E) - \left(-\frac{\partial f_1}{\partial E}\right)\delta E_F$$

$$f_2(E) \approx f_1(E) - \left(-\frac{\partial f_1}{\partial E}\right)(-qV)$$

$$f_1(E) - f_2(E) = \left(-\frac{\partial f_1}{\partial E}\right)(qV)$$

$$\boxed{\begin{gathered} I = GV \quad \mathrm{A} \\ G = \frac{2q^2}{h}\int \mathcal{T}(E)M(E)\left(-\frac{\partial f_0}{\partial E}\right)dE \quad \mathrm{S} \end{gathered}}$$

Fermi window: small bias



$$W_F(E) = \left(-\frac{\partial f_0}{\partial E}\right)$$

$$\int W_F(E)\,dE = 1$$

Quantum of conductance

$$G = \frac{2q^2}{h} \int \mathcal{T}(E) M(E) \left(-\frac{\partial f_0}{\partial E} \right) dE \;\; \mathrm{S}$$

$T = 0\,\mathrm{K}$:

$$\left(-\frac{\partial f_0}{\partial E} \right) = \delta(E_F)$$

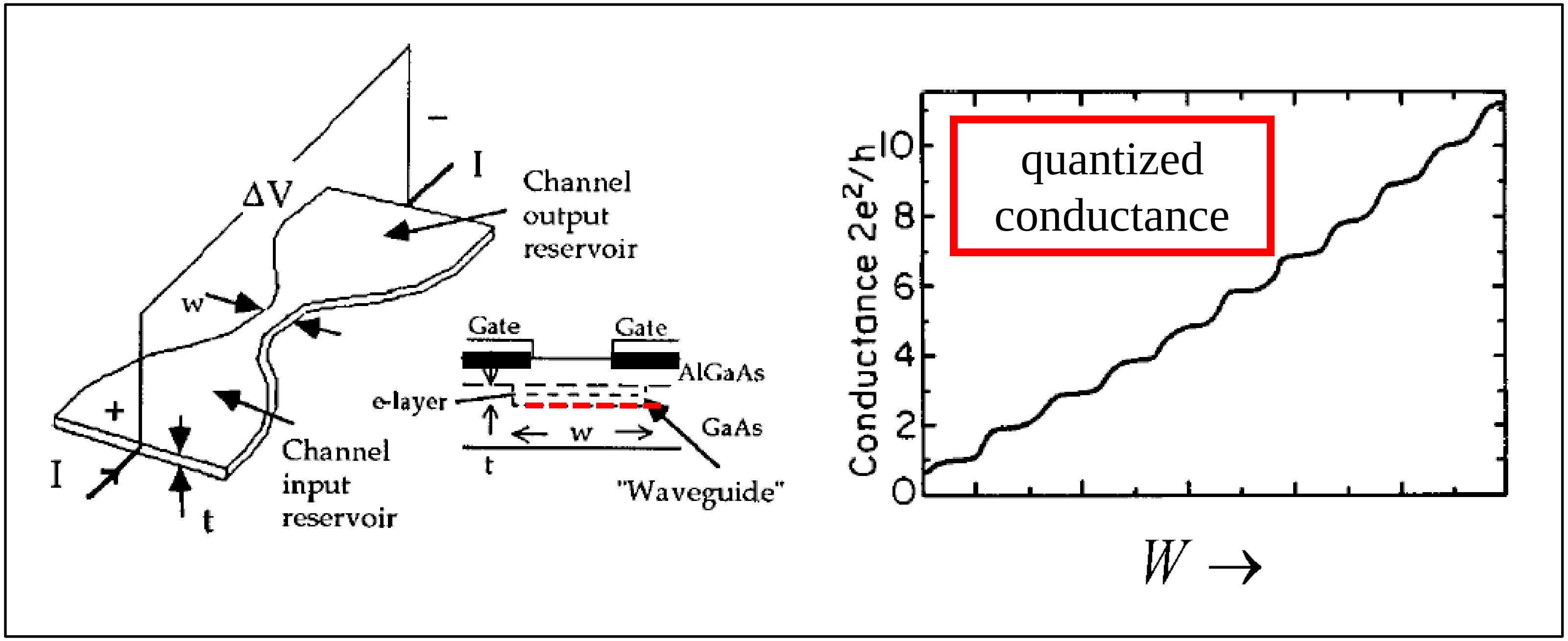
$$\boxed{G(T = 0\,\mathrm{K}) = \frac{2q^2}{h} \mathcal{T}(E_F) M(E_F)}$$

$$M(E) = W M_{2D}(E) = W g_v \frac{\sqrt{2m^* E}}{\pi \hbar}$$

For large *W*, *M* is ~ W

For small *W*, *M* comes in discrete units.

Quantized conductance



D. Holcomb, *American J. Physics*, **67**, pp. 278-297 1999.
Data from: B. J. van Wees, et al., *Phys. Rev. Lett*. **60**, 848851, 1988.

Conductance for T > 0 K

$$G = \frac{2q^2}{h} \int \mathcal{T}(E) M(E) \left(-\frac{\partial f_0}{\partial E} \right) dE \ \ \mathrm{S}$$

$$G = \frac{2q^2}{h} \frac{\int \mathcal{T}(E) M(E) \left(-\frac{\partial f_0}{\partial E} \right) dE}{\int M(E) \left(-\frac{\partial f_0}{\partial E} \right) dE} \int M(E) \left(-\frac{\partial f_0}{\partial E} \right) dE$$

$$\langle M \rangle \equiv \int M(E) \left(-\frac{\partial f_0}{\partial E} \right) dE$$ Number of channels in the Fermi window.

$$\langle\langle \mathcal{T} \rangle\rangle \equiv \frac{\int \mathcal{T}(E) M(E) \left(-\frac{\partial f_0}{\partial E} \right) dE}{\int M(E) \left(-\frac{\partial f_0}{\partial E} \right) dE}$$

Conductance

$$G = \frac{2q^2}{h} \int \mathcal{T}(E) M(E) \left(-\frac{\partial f_0}{\partial E} \right) dE \ \ \mathrm{S}$$

$$\boxed{\begin{aligned} G(T > 0\ \mathrm{K}) &= \frac{2q^2}{h} \langle\langle \mathcal{T} \rangle\rangle \langle M \rangle \\ G(T = 0\ \mathrm{K}) &= \frac{2q^2}{h} \mathcal{T}(E_F) M(E_F) \end{aligned}}$$

Ballistic conductance for $T > 0$ K

$$G = \frac{2q^2}{h} \int \mathcal{T}(E) M(E) \left(-\frac{\partial f_0}{\partial E} \right) dE \ \ \mathrm{S}$$

$$M(E) = W M_{2D}(E) = W g_v \frac{\sqrt{2m^* E}}{\pi \hbar} \qquad \mathcal{T}(E) = 1 \qquad f_0(E) = \frac{1}{1 + e^{(E - E_F)/k_B T}}$$

$$G_B = \frac{2q^2}{h} \int (1) \left(W g_v \frac{\sqrt{2m^* E}}{\pi \hbar} \right) \left(+ \frac{\partial}{\partial E_F} \right) \frac{1}{1 + e^{(E - E_F)/k_B T}} dE$$

Ballistic conductance for T > 0 K

$$G_B = \frac{2q^2}{h}\frac{\sqrt{\pi}}{2}\left(g_v\frac{\sqrt{2m^*k_BT}}{\pi\hbar}\right)\mathcal{F}_{-1/2}(\eta_F)W$$

simplify for non-degenerate statistics

1) $$G_B = \frac{2q^2}{h}\frac{\sqrt{\pi}}{2}\left(g_v\frac{\sqrt{2m^*k_BT}}{\pi\hbar}\right)e^{\eta_F}W$$

2) $$n_S = \frac{N_C^{2D}}{2}\left(e^{\eta_{F1}} + e^{\eta_{F2}}\right) = N_C^{2D}e^{\eta_F}$$

3) $$N_C^{2D} = g_v\frac{m^*k_BT}{\pi\hbar^2}$$

4) $$G_B = n_S q\left(\frac{\upsilon_T}{2k_BT/q}\right)W$$

5) $$\upsilon_T = \sqrt{\frac{2k_BT}{\pi m^*}}$$

6) $$G_B = n_S q\left(\frac{\upsilon_T L}{2k_BT/q}\right)\frac{W}{L}$$

Ballistic conductance for *T* > 0 K

$$G = \frac{2q^2}{h}\int \mathcal{T}(E)M(E)\left(-\frac{\partial f_0}{\partial E}\right)dE \ \ \mathrm{S}$$

$$\boxed{G_B = n_S q \mu_B \frac{W}{L} \qquad \mu_B \equiv \left(\frac{\upsilon_T L}{2 k_B T / q}\right)}$$

Large voltage

$$I = \frac{2q}{h}\int \mathcal{T}(E)M(E)\left(f_1(E) - f_2(E)\right)dE$$

$$f_1(E) = \frac{1}{1 + e^{(E - E_{F1})/k_B T}}$$

$$f_2(E) = \frac{1}{1 + e^{(E - E_{F2})/k_B T}}$$

$$E_{F2} = E_{F1} - qV$$

$$f_1(E) >> f_2(E)$$

$$I_{ON} = \frac{2q}{h}\int_{E_C}^{\infty} \mathcal{T}(E)M(E)f_1(E)dE$$

Ballistic on-current

$$I_{ON} = \frac{2q}{h}\int_{E_C}^{\infty} \mathcal{T}(E) M(E) f_1(E)\, dE$$

$$\mathcal{T}(E) = 1 \qquad M(E) = WM_{2D}(E) = Wg_v \frac{\sqrt{2m^*(E - E_C)}}{\pi\hbar} \qquad f_1(E) = \frac{1}{1 + e^{(E - E_{F1})/k_B T}}$$

$$I_{ON}^{ball} = \frac{2q}{h}\int_{E_C}^{\infty} (1) Wg_v \frac{\sqrt{2m^* E}}{\pi\hbar} \frac{1}{1 + e^{(E - E_{F1})/k_B T}}\, dE$$

Ballistic on-current

$$I_{ON}^{ball} = \frac{2q}{h}\int_{E_C}^{\infty}(1)Wg_v\frac{\sqrt{2m^*E}}{\pi\hbar}\frac{1}{1+e^{(E-E_{F1})/k_BT}}dE$$

$$I_{ON} = \frac{2q}{h}\sqrt{\frac{m^*}{2\pi\hbar^2}}\left(k_BT\right)^{3/2}\mathcal{F}_{1/2}\left(\eta_F\right)$$

$$\upsilon_T = \sqrt{\frac{2k_BT}{\pi m^*}}$$

$$I_{ON} = \upsilon_T\frac{N_C^{2D}}{2}\mathcal{F}_{1/2}\left(\eta_F\right)$$

$$N_{2D} = g_v\left(\frac{m^*k_BT}{\pi\hbar^2}\right)\ \mathrm{m}^{-2}$$

Ballistic on-current

$$I_{ON}^{ball} = \upsilon_T \frac{N_C^{2D}}{2} \mathcal{F}_{1/2}(\eta_F) \rightarrow \upsilon_T \frac{N_C^{2D}}{2} e^{\eta_F}$$

nondegenerate

$$n_S = \frac{N_{2D}}{2}\left(\mathcal{F}_0(\eta_{F1}) + \mathcal{F}_0(\eta_{F2})\right)\mathrm{m}^{-2} \rightarrow n_S = \frac{N_{2D}}{2}\left(e^{\eta_{F1}} + e^{\eta_{F2}}\right)$$

$$\eta_{F1} >> \eta_{F2}$$

$$\boxed{I_{ON}^{ball} = q n_S \upsilon_T}$$

$$n_S = \frac{N_{2D}}{2} e^{\eta_{F1}}$$

Ballistic summary

$$I = \frac{2q}{h}\int \mathcal{T}(E)M(E)\left(f_1(E) - f_2(E)\right)dE$$

$$\mathcal{T}(E) = 1$$

1) Low bias (linear transport): $f_1(E) \approx f_2(E)$

$$I = G_B V \qquad G_B = n_S q \mu_B \frac{W}{L} \qquad \mu_B \equiv \left(\frac{\upsilon_T L}{2k_B T/q}\right)$$

2) High bias: $f_1(E) >> f_2(E)$

$$I_{ON}^{ball} = q n_S \upsilon_T$$