Fundamentals of Nanotransistors

Unit 3: The Ballistic Nanotransistor

Lecture 3.2: Landauer Approach

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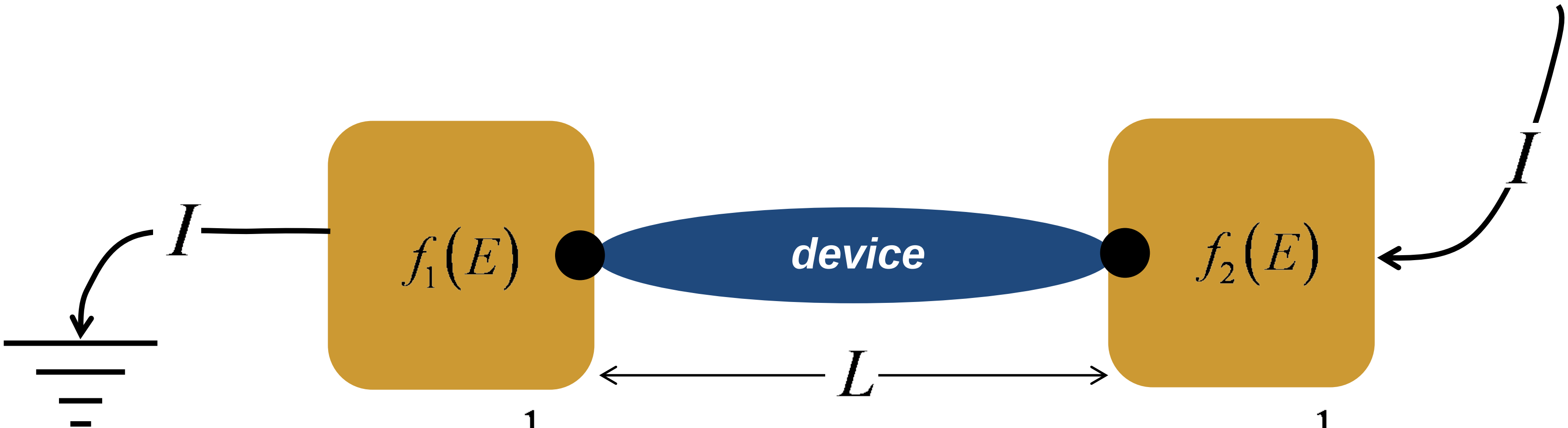
Nanodevice

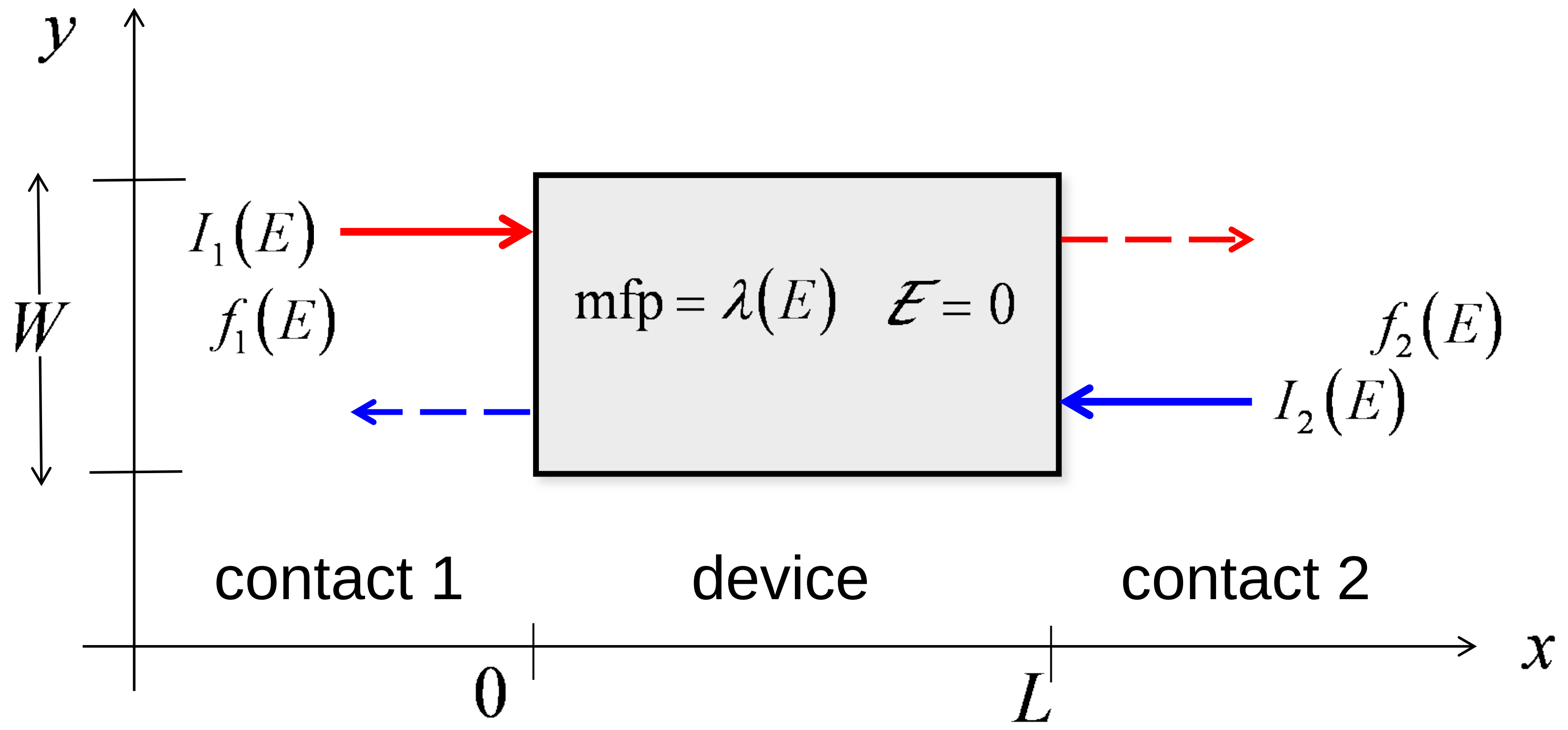


$$f_1(E) = \frac{1}{1 + e^{(E - E_{F1})/k_B T}} \qquad f_2(E) = \frac{1}{1 + e^{(E - E_{F2})/k_B T}}$$

$$I = \frac{2q}{h} \int \mathcal{T}(E) M(E) (f_1 - f_2) dE$$

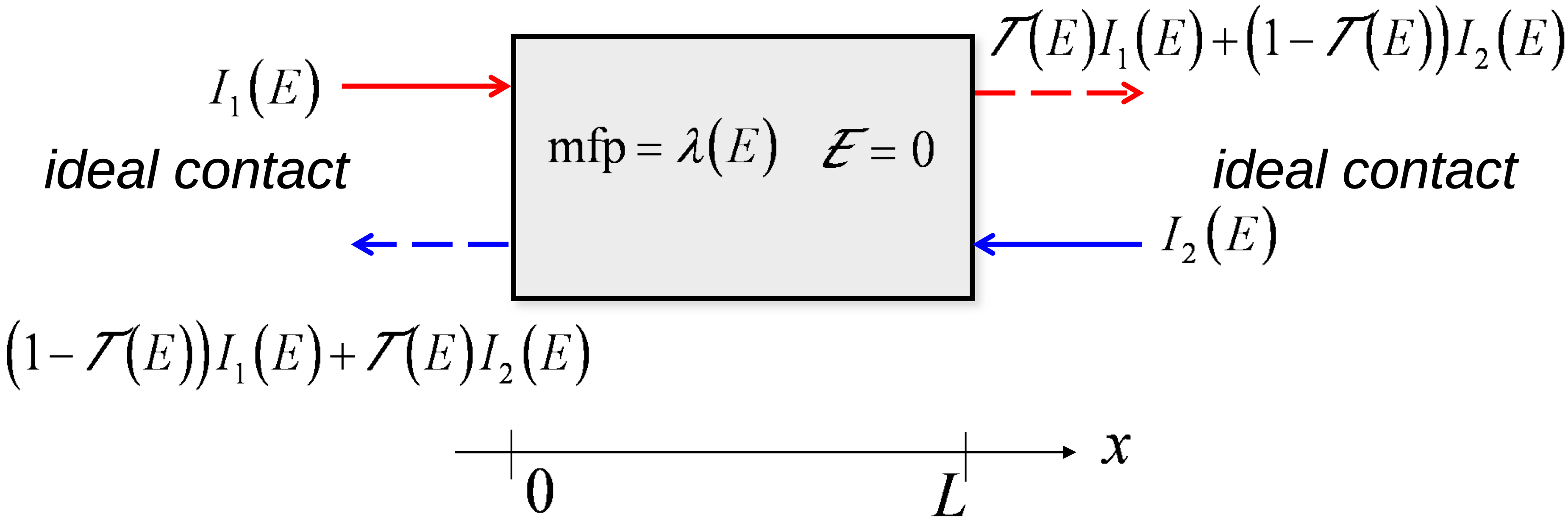
transmission, modes (channels), differences in Fermi functions

Derivation (2D)



1) Ideal ("Landauer") contacts
2) Independent energy channels

Derivation (i)



$$I(E) = \mathcal{T}(E)\left(I_1(E) - I_2(E)\right)$$

$$I = \int I(E)\,dE$$

$$\mathcal{T}_{12}(E) = \mathcal{T}_{21}(E)$$

Derivation (ii)

$$I(E) = \mathcal{T}(E)\big(I_1(E) - I_2(E)\big)$$

$$n^+(E)dE = \frac{D_{2D}(E)}{2} f_1(E)dE \text{ cm}^{-2}$$

$$I_1(E) = qW \langle \upsilon_x^+(E) \rangle n^+(E)dE$$

$$\boxed{\begin{aligned} I_1(E) &= qW \langle \upsilon_x^+(E) \rangle \frac{D_{2D}(E)}{2} f_1(E) \\ I_2(E) &= qW \langle \upsilon_x^+(E) \rangle \frac{D_{2D}(E)}{2} f_2(E) \end{aligned}}$$

Derivation (iii)

$$I(E) = \mathcal{T}(E)\left(I_1(E) - I_2(E)\right) \qquad I_{1,2}(E) = qW\left\langle \upsilon_x^+(E)\right\rangle \frac{D_{2D}(E)}{2} f_{1,2}(E)$$

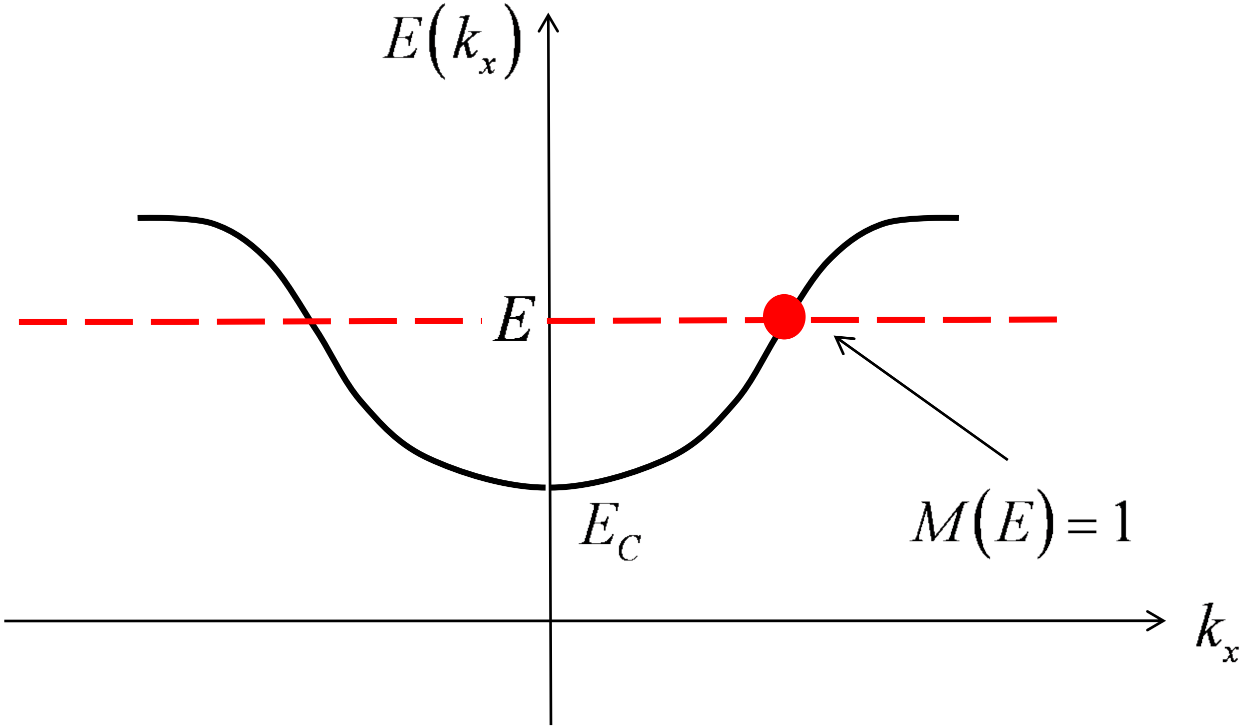
$$I(E) = \mathcal{T}(E)\left\{qW\left\langle \upsilon_x^+(E)\right\rangle \frac{D(E)}{2}\right\}\left(f_1(E) - f_2(E)\right)$$

Define: $$M(E) \equiv W\frac{h}{4}\left\langle \upsilon_x^+(E)\right\rangle D_{2D}(E)$$

Units: $$M(E) = (\mathrm{m})(\mathrm{J\text{-}s})\left(\frac{\mathrm{m}}{\mathrm{s}}\right)\left(\frac{\#}{\mathrm{J\text{-}m}^2}\right) = \#$$

Channels (modes) in 1D

$$M_{1D}(E) = \frac{M(E)}{W} \equiv \frac{h}{4}\left\langle \upsilon_x^+(E)\right\rangle D_{1D}(E)$$



(Easily generalized to arbitrary bandstructures in 1D, 2D, and 3D.)

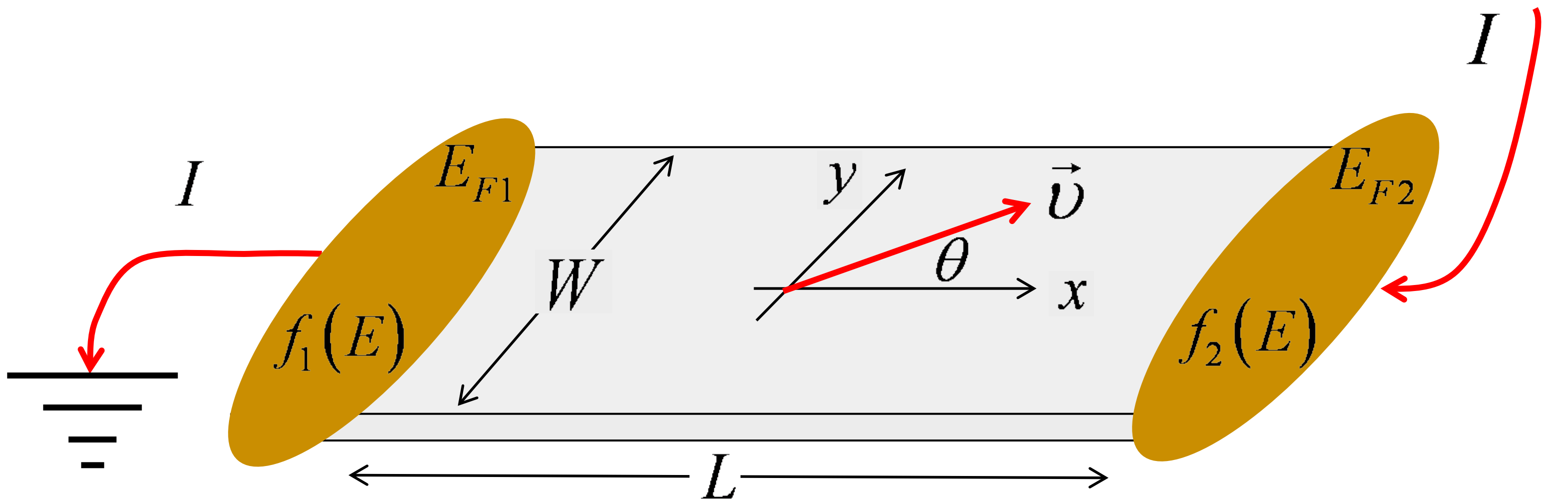
$M(E)$ in 2D

$$M(E) \equiv W\frac{h}{4}\left\langle \upsilon_x^+(E)\right\rangle D_{2D}(E) \qquad M_{2D}(E) \equiv M(E)/W = \frac{h}{4}\left\langle \upsilon_x^+(E)\right\rangle D_{2D}(E)$$

$$D_{2D}(E) = g_v\left(\frac{m^*}{\pi\hbar^2}\right)\ \mathrm{J^{-1}m^{-2}}$$

$$\left\langle \upsilon_x^+(E)\right\rangle = ?$$

$M(E)$ in 2D



$E = \frac{1}{2} m^* \upsilon^2$ (parabolic energy bands)

$$\upsilon(E) = \sqrt{\frac{2E}{m^*}}$$

$$\langle \upsilon_x(E) \rangle = \frac{\int_{-\pi/2}^{+\pi/2} \upsilon(E)\cos(\theta)\, d\theta}{\pi} = \frac{2}{\pi}\upsilon(E) \quad ✓$$

$$\upsilon_x(E) = \upsilon(E)\cos(\theta)$$

$M(E)$ in 2D

$$M(E) \equiv W\frac{h}{4}\left\langle \upsilon_x^+(E)\right\rangle D_{2D}(E) \qquad M_{2D}(E) \equiv M(E)/W = \frac{h}{4}\left\langle \upsilon_x^+(E)\right\rangle D_{2D}(E)$$

$$D_{2D}(E) = g_v\left(\frac{m^*}{\pi\hbar^2}\right) \text{ J}^{-1}\text{m}^{-2}$$

$$\left\langle \upsilon_x^+(E)\right\rangle = \frac{2}{\pi}\upsilon(E)$$

$$\upsilon(E) = \sqrt{\frac{2E}{m^*}}$$

$$\boxed{M_{2D}(E) = g_v\frac{\sqrt{2m^*E}}{\pi\hbar} \text{ m}^{-1}}$$

Derivation (iv)

$$I(E) = \mathcal{T}(E)\left(I_1(E) - I_2(E)\right)$$

$$I(E) = \mathcal{T}(E)\left\{qW\left\langle \upsilon_x^+(E)\right\rangle \frac{D(E)}{2}\right\}\left(f_1(E) - f_2(E)\right)$$

$$M(E) \equiv W\frac{h}{4}\left\langle \upsilon_x^+(E)\right\rangle D_{2D}(E)$$

$$I(E) = \frac{2q}{h}\mathcal{T}(E)M(E)\left(f_1(E) - f_2(E)\right)$$

$$\boxed{I = \int I(E)dE = \frac{2q}{h}\int \mathcal{T}(E)M(E)\left(f_1(E) - f_2(E)\right)dE}$$

Discussion

$$\boxed{I = \frac{2q}{h}\int \mathcal{T}(E)M(E)\left(f_1(E) - f_2(E)\right)dE}$$

Fundamental constants

Transmission:

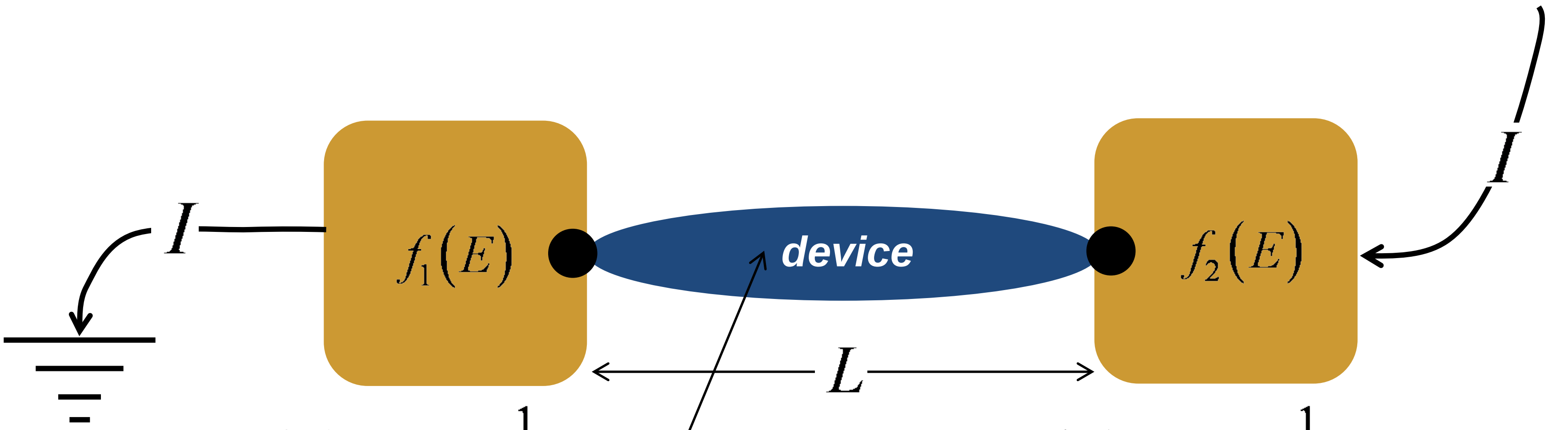
$\mathcal{T}(E) = 1$ ballistic

$\mathcal{T}(E) << 1$ diffusive

Channels
or
Modes

Ideal
“Landauer”
contacts

Carrier densities



$$f_1(E) = \frac{1}{1 + e^{(E - E_{F1})/k_B T}} \qquad f_2(E) = \frac{1}{1 + e^{(E - E_{F2})/k_B T}}$$

What is the carrier density?

Carrier density: Equilibrium

$$n_S = \int_{E_C}^{\infty} D_{2D}(E) f_0(E) dE \ \ \mathrm{m}^{-2}$$

$$f_0(E) = \frac{1}{1 + e^{(E-E_F)/k_B T}}$$

parabolic bands:

$$D_{2D}(E) = g_v \left(\frac{m^*}{\pi \hbar^2} \right) \ \mathrm{J}^{-1}\mathrm{m}^{-2}$$

$$n_S = g_v \left(\frac{m^*}{\pi \hbar^2} \right) \int_{E_C}^{\infty} \frac{dE}{1 + e^{(E-E_F)/k_B T}} \ \ \mathrm{m}^{-2}$$

$$n_S = N_{2D} \ln\left(1 + e^{\eta_F}\right) = N_{2D} \mathcal{F}_0(\eta_F) \mathrm{m}^{-2}$$

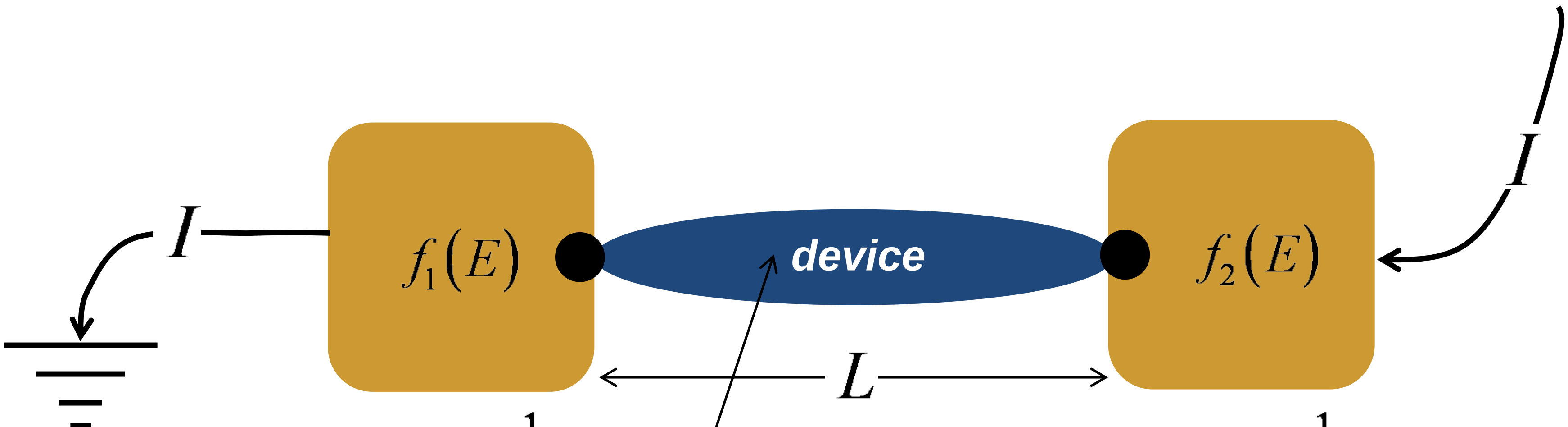
$$N_{2D} = g_v \left(\frac{m^* k_B T}{\pi \hbar^2} \right) \ \mathrm{m}^{-2}$$

$$\eta_F = \left(\frac{E_F - E_C}{k_B T} \right)$$

Non-degenerate conditions:

$$n_S = N_{2D} e^{\eta_F} \ \ \mathrm{m}^{-2}$$

Carrier densities



$$f_1(E) = \frac{1}{1 + e^{(E - E_{F1})/k_B T}}$$

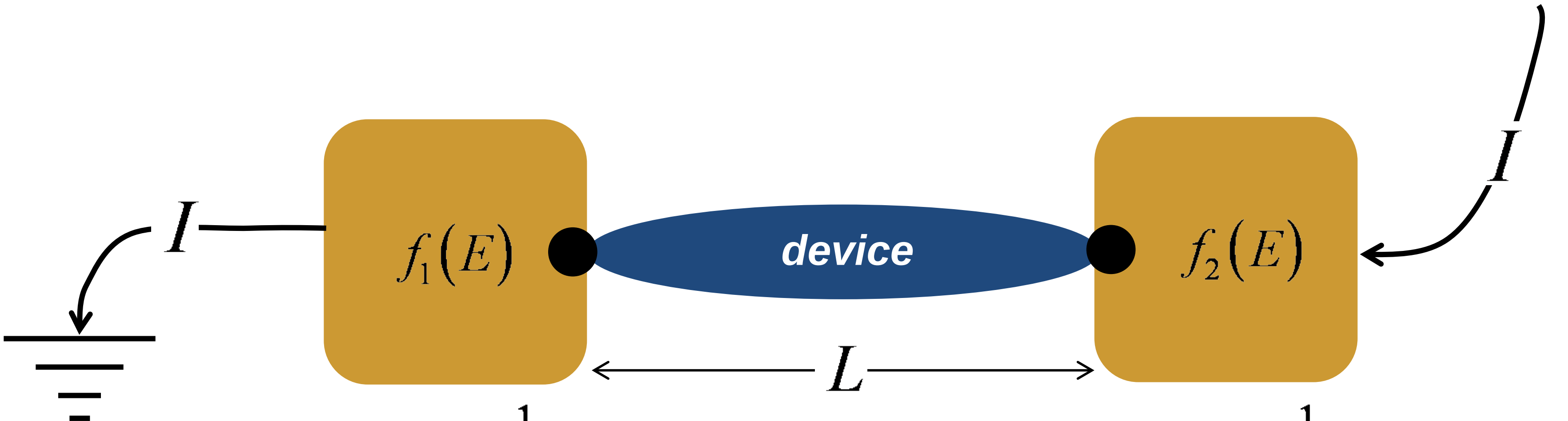
$$f_2(E) = \frac{1}{1 + e^{(E - E_{F2})/k_B T}}$$

$$E_{F2} = E_{F1} - qV$$

$$n_S = \int_{E_C}^{\infty} \left(\frac{D_{2D}(E)}{2} f_1(E) + \frac{D_{2D}(E)}{2} f_2(E) \right) dE + \mathrm{m}^{-2}$$

$$\boxed{n_S = \frac{N_{2D}}{2} \left(\mathcal{F}_0(\eta_{F1}) + \mathcal{F}_0(\eta_{F2}) \right) \mathrm{m}^{-2}}$$ (parabolic bands)

Final result



$$f_1(E) = \frac{1}{1 + e^{(E - E_{F1})/k_B T}}$$

$$f_2(E) = \frac{1}{1 + e^{(E - E_{F2})/k_B T}}$$

$$E_{F2} = E_{F1} - qV$$

$$I = \frac{2q}{h} \int \mathcal{T}(E) M(E) \left(f_1(E) - f_2(E) \right) dE$$

$$n_S = \int_{E_C}^{\infty} \left(\frac{D_{2D}(E)}{2} f_1(E) + \frac{D_{2D}(E)}{2} f_2(E) \right) dE + \mathrm{m}^{-2}$$

Next lecture

$$I = \frac{2q}{h}\int \mathcal{T}(E)M(E)\left(f_1(E) - f_2(E)\right)dE$$

$$n_S = \int_{E_C}^{\infty}\left(\frac{D_{2D}(E)}{2}f_1(E) + \frac{D_{2D}(E)}{2}f_2(E)\right)dE + \mathrm{m}^{-2}$$

Using these equations