Fundamentals of Nanotransistors

Unit 2: MOS Electrostatics

Lecture 2.8: The VS Model Revisited

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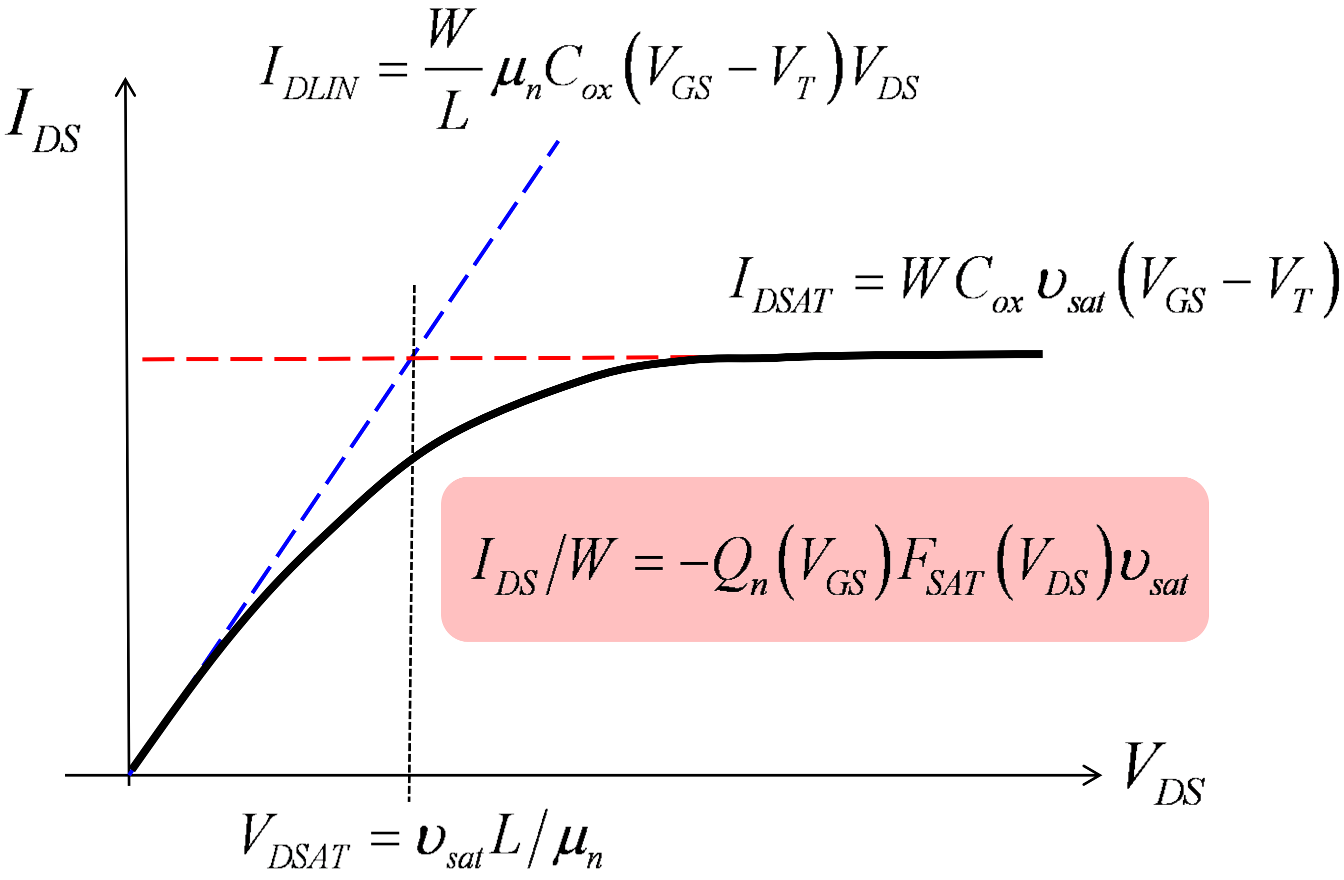
Recall: Lecture 1.7



"Level 0" VS model

1) $I_{DS}/W = -Q_n(V_{GS})\langle \upsilon(V_{DS})\rangle$

2) $Q_n(V_{GS}) = -C_{ox}(V_{GS} - V_T) \qquad (V_{GS} > V_T)$

$V_T = V_{T0} - \delta V_{DS}$

3) $\langle \upsilon(V_{DS})\rangle = F_{SAT}(V_{DS})\upsilon_{sat}$

4) $F_{SAT}(V_{DS}) = \dfrac{V_{DS}/V_{DSAT}}{\left[1+(V_{DS}/V_{DSAT})^{\beta}\right]^{1/\beta}}$

5) $V_{DSAT} = \dfrac{\upsilon_{sat} L}{\mu_n}$

There are only 8 device-specific parameters in this model:

$$C_{ox}, V_T, \delta, \upsilon_{sat}, \mu_n, L$$

$$R_{SD} = R_S + R_D, \beta$$

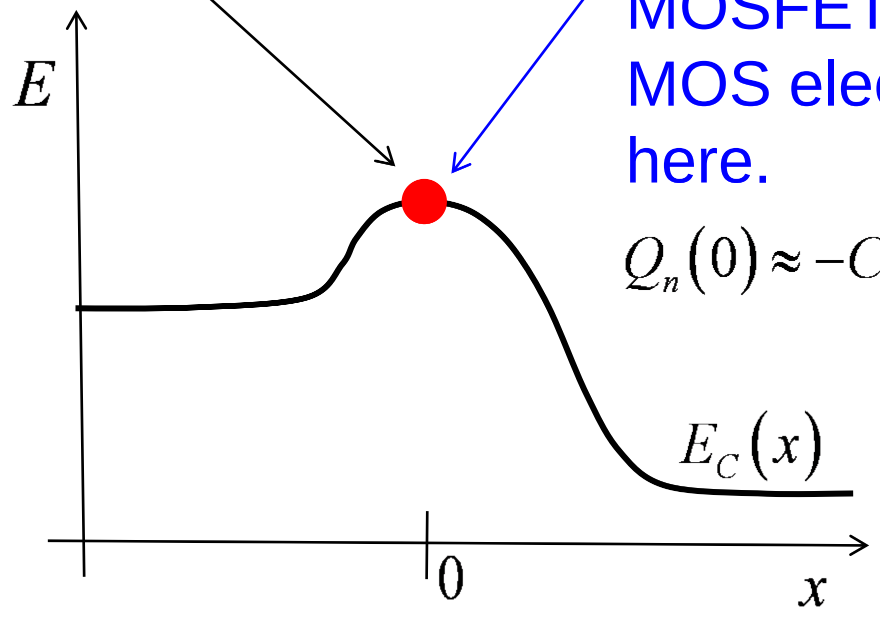
Why is it called the VS model?

$$I_{DS} = -W\,Q_n(0)\left\langle \upsilon_x(0)\right\rangle$$

"virtual source"

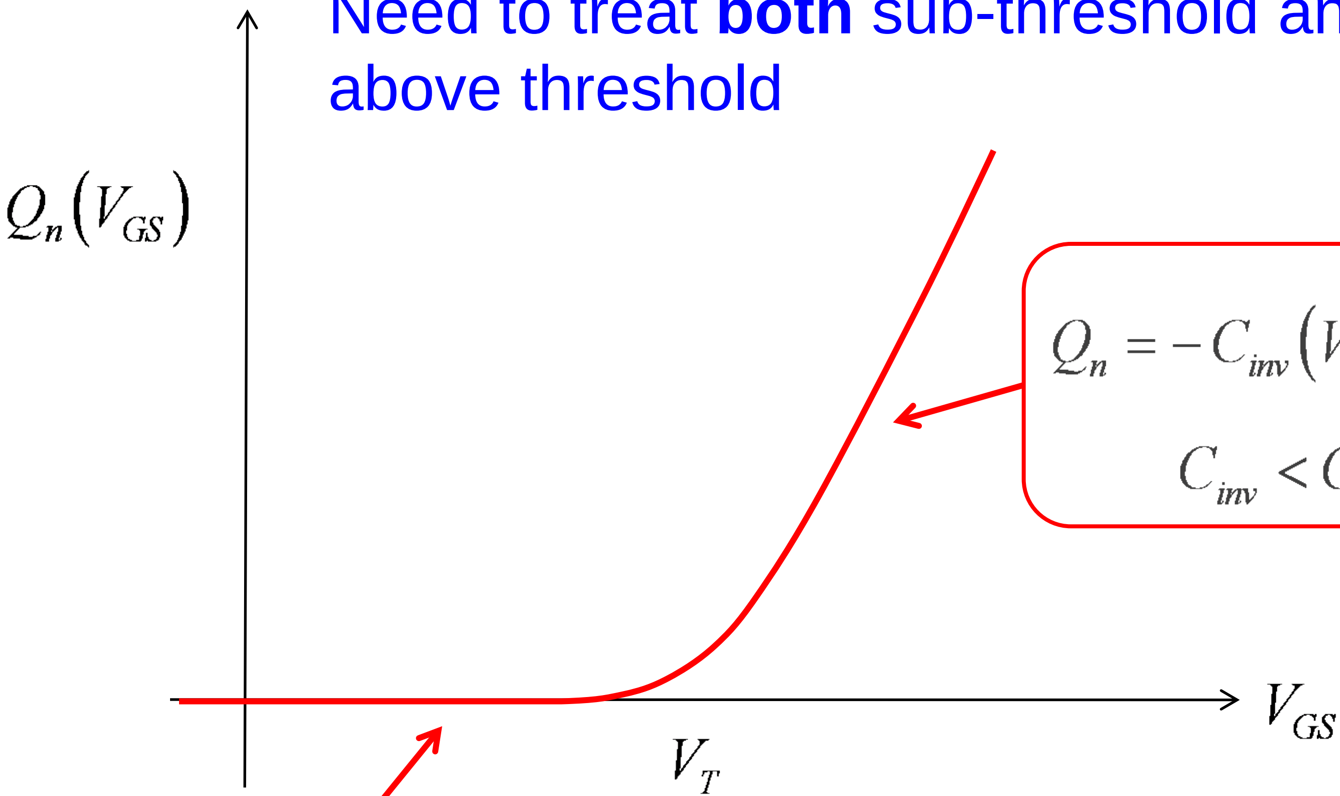
in a "well-tempered" MOSFET, near-ideal 1D MOS electrostatics apply here.

$$Q_n(0) \approx -C_{inv}\left(V_{GS} - V_T\left(V_{DS}\right)\right)$$



Improving the VS model: inversion charge

Need to treat **both** sub-threshold and above threshold

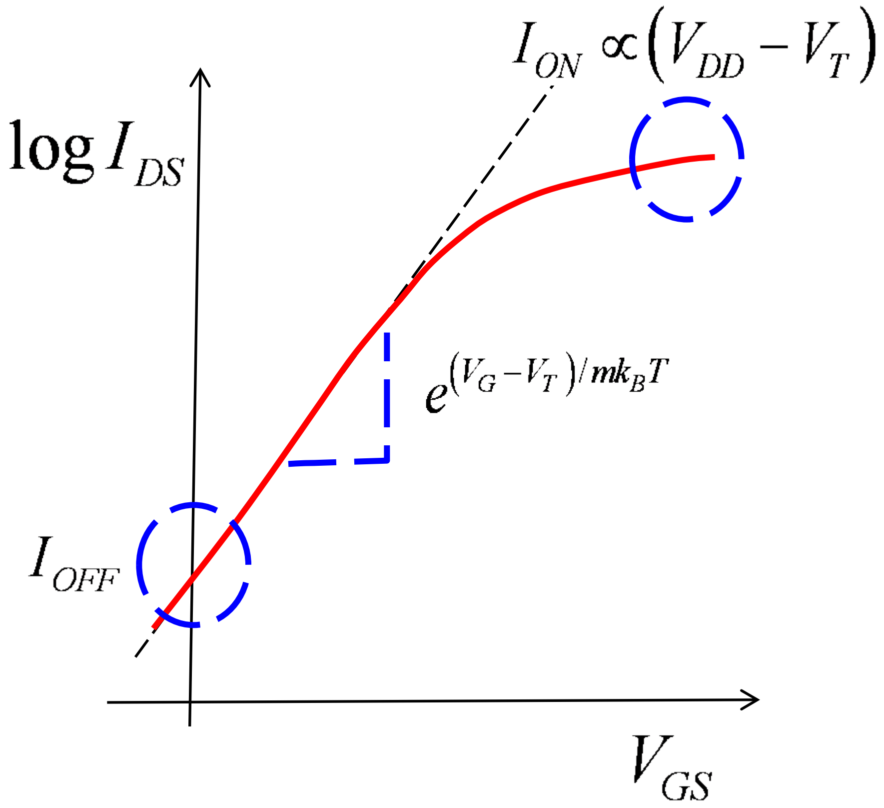


$$Q_n = -C_{inv}\left(V_{GS} - V_T\right)$$

$$C_{inv} < C_{ox}$$

$$Q_n(V_{GS}) = -(m-1)C_{ox}\left(\frac{k_B T}{q}\right)e^{q(V_{GS}-V_T)/mk_B T}$$

But first: the subthreshold current



1) subthreshold swing
2) off-current
3) on-current

$$Q_n(V_{GS}) \propto e^{q\psi_S/k_BT} \propto e^{q(V_{GS}-V_T)/mk_BT}$$

$$I_{DS} \propto Q_n(V_{GS})$$

$$I_{DS} \propto e^{q\psi_S/k_BT} \propto e^{q(V_{GS}-V_T)/mk_BT}$$

Subthreshold swing

$I_{DS} \sim e^{q\psi_S/k_BT}$ $\qquad$ $I_{DS} \sim e^{q(V_{GS}-V_T)/mk_BT}$ $\qquad$ $\psi_S = V_{GS}/m$ $\qquad$ $m \geq 1$

$$\ln I_{DS} = \frac{\psi_S}{(k_BT/q)} + c \qquad \log_{10} I_{DS} = \frac{\psi_S}{2.3(k_BT/q)} + \frac{c}{2.3}$$

$$\frac{\partial(\log_{10} I_{DS})}{\partial V_{GS}} = \frac{\partial(\log_{10} I_{DS})}{\partial \psi_S} \times \frac{\partial \psi_S}{\partial V_{GS}} = \frac{1}{2.3(k_BT/q)} \times \frac{1}{m} \qquad \frac{\text{Decades of } I_{DS}}{\text{Volts of } V_{GS}}$$

$$\boxed{SS = \left(\frac{\partial(\log_{10} I_D)}{V_{GS}}\right)^{-1} = 2.3m\left(k_BT/q\right)\ \frac{\text{mV}}{\text{dec}}}$$

Subthreshold swing

$$S = 2.3m\left(k_B T/q\right)\ \frac{\text{mV}}{\text{dec}} \qquad m \geq 1$$

$m \approx 1.1 - 1.4$ typically

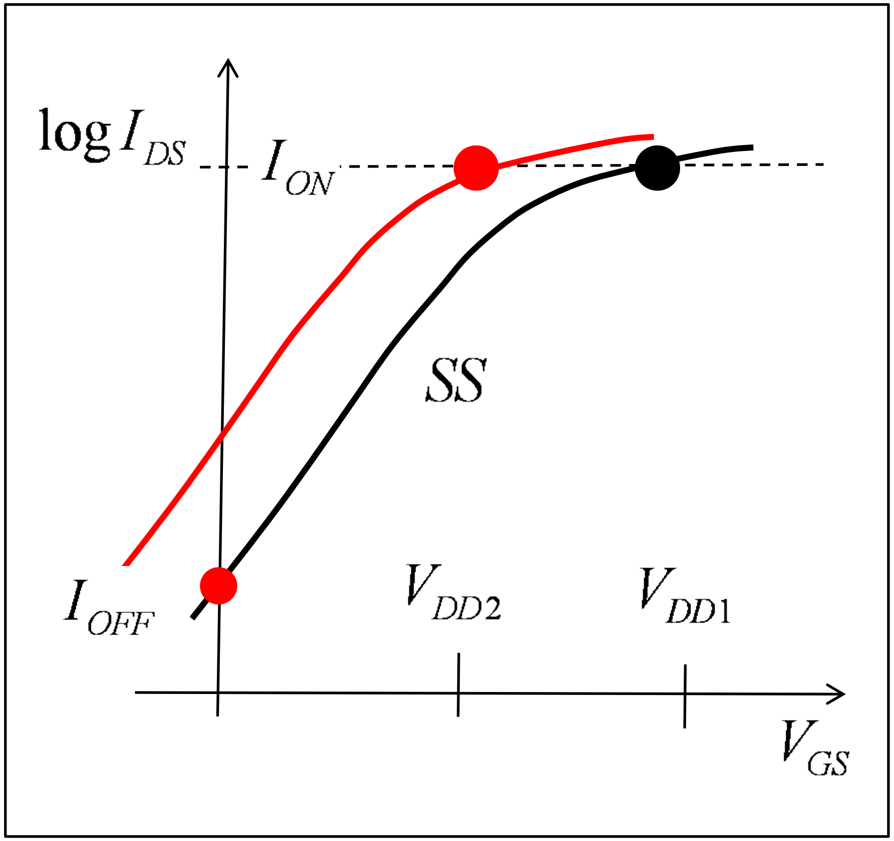
$$SS > 60\ \frac{\text{mV}}{\text{dec}}\ (T = 300K)$$

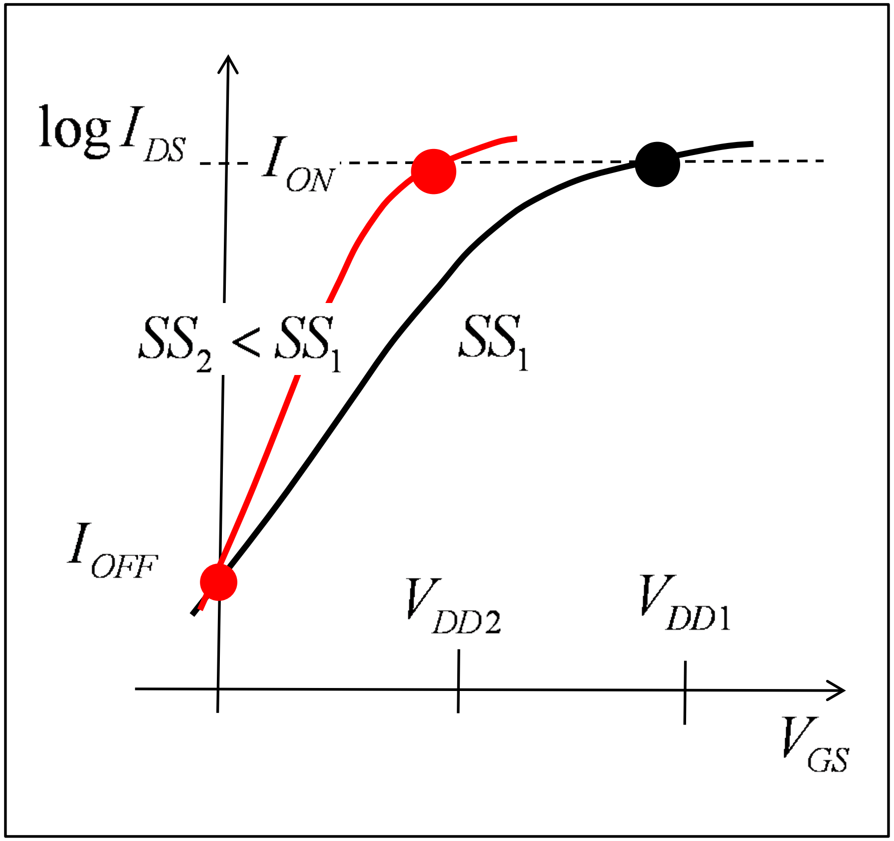
$$SS < 100\ \frac{\text{mV}}{\text{dec}}\ \text{(typically)}$$

Why is a small *SS* important?

$$P_D \propto V_{DD}^2$$

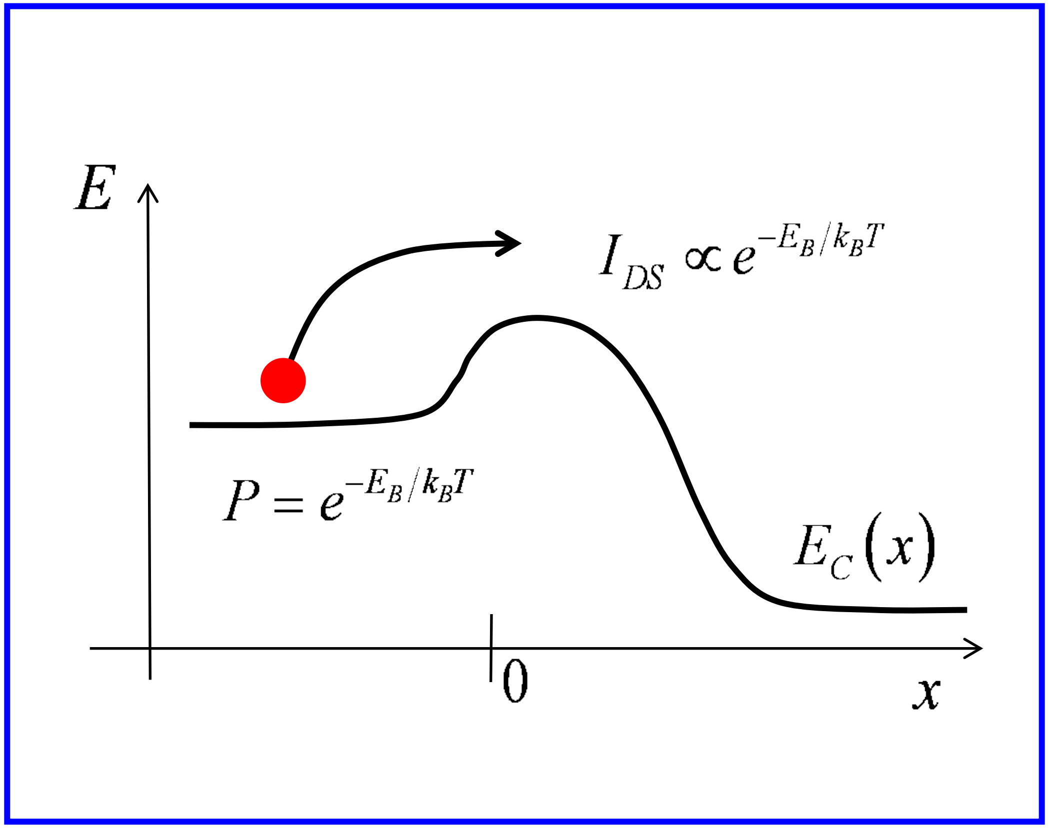
The need for a small SS





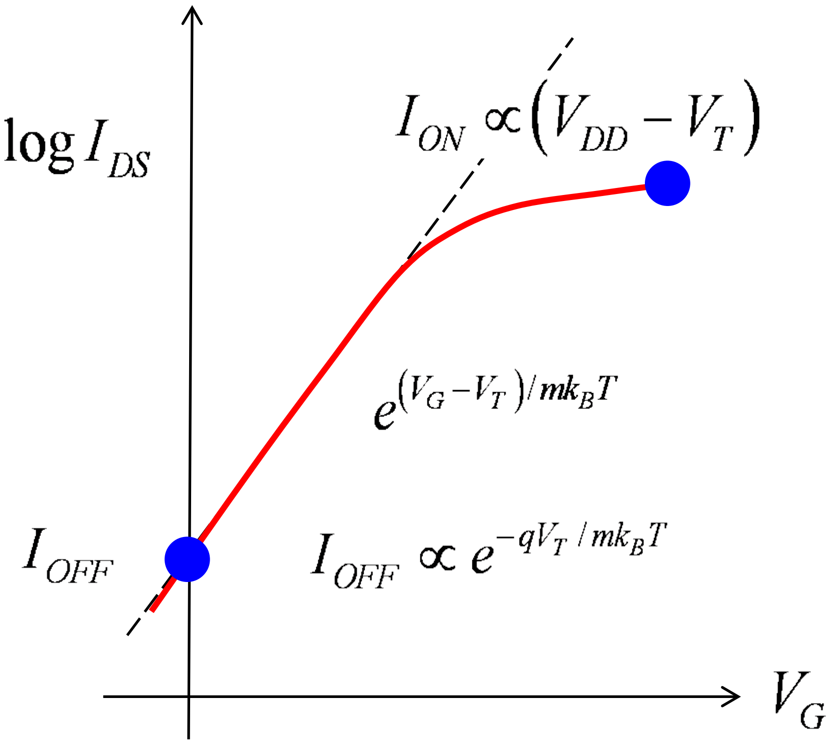
$$I_{ON} \propto \left(V_{DD} - V_T\right) \quad I_{OFF} \propto e^{-V_T/k_B T}$$

Why is S > 60 mV/decade?

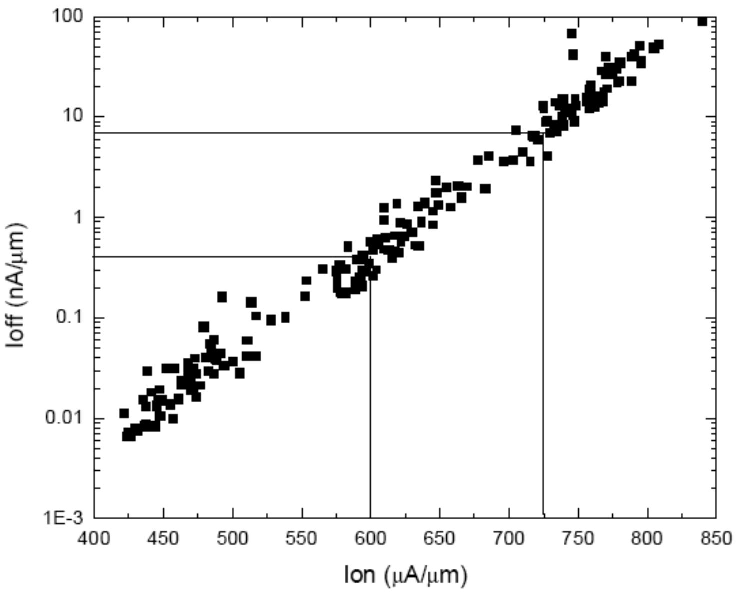


injection of thermal carriers over a barrier

Relation between I_{OFF} and I_{ON}



$$\ln I_{OFF} = c_1 + c_2 I_{ON}$$



A. Steegen, et al., "65nm CMOS Technology for low power applications," Intern. Electron Dev. Meeting, Dec. 2005.

Back to the VS model

Now let's return to the question at hand:

"How do we describe $Q_n(V_{GS})$ continuously below and above threshold?

Empirical treatment of inversion charge

$$Q_n(V_{GS}) = -C_{inv} m(k_B T/q) \ln\left(1 + e^{q(V_{GS}-V_T)/mk_BT}\right)$$

$V_{GS} << V_T$:

$$\ln(1+x) \approx x$$

$$Q_n(V_{GS}) \approx -C_{inv} m(k_B T/q) e^{q(V_{GS}-V_T)/mk_BT}$$

$$Q_n(V_{GS}) = -(m-1) C_{ox} (k_B T/q) e^{q(V_{GS}-V_T)/mk_BT}$$

correct result

G. T. Wright, "Threshold modelling of MOSFETs for CAD of CMOS VLSI," *Electron Lett.*, **21**, pp. 223–224, Mar. 1985.

Empirical treatment of inversion charge

$$Q_n\left(V_{GS}\right) = -C_{inv} m\left(k_B T/q\right)\ln\left(1 + e^{q\left(V_{GS}-V_T\right)/mk_B T}\right)$$

$V_{GS} > V_T$:

$$\ln\left(1+x\right) \approx \ln\left(x\right)$$

$$Q_n\left(V_{GS}\right) \approx -C_{inv}\left(V_{GS} - V_T\right)$$

$$Q_n\left(V_{GS}\right) = -C_{inv}\left(V_{GS} - V_T\right)$$

correct

G. T. Wright, "Threshold modelling of MOSFETs for CAD of CMOS VLSI," *Electron Lett.*, **21**, pp. 223–224, Mar. 1985.

Empirical treatment of inversion charge

expression used in the MIT VS Model

$$Q_n(V_{GS}) = -C_{inv} m (k_B T/q) \ln\left(1 + e^{q\left(V_{GS} - V_T + \alpha(k_B T_L/q) F_f\right)/m k_B T}\right)$$

$$V_T = V_{T0} - \delta V_{DS}$$

DIBL

different V_T's in strong and weak inversion

Ali Khakifirooz, Osama M. Nayfeh, and Dimitri Antoniadis, "A Simple Semi-empirical Short-Channel MOSFET Current–Voltage Model Continuous Across All Regions of Operation and Employing Only Physical Parameters," *IEEE Trans. Electron Devices*, **56**, pp. 1674-1680, 2009.

Level 1 VS model

1) $I_{DS}/W = -Q_n(V_{GS})\langle \upsilon(V_{DS})\rangle$

2) $Q_n(V_{GS}) = -C_{inv} m(k_B T/q)\ln\left(1 + e^{q\left(V_{GS} - V_T + \alpha(k_B T_L/q)F_f\right)/mk_B T}\right)$

$V_T = V_{T0} - \delta V_{DS}$

3) $\langle \upsilon(V_{DS})\rangle = F_{SAT}(V_{DS})\upsilon_{sat}$

4) $F_{SAT}(V_{DS}) = \dfrac{V_{DS}/V_{DSAT}}{\left[1 + (V_{DS}/V_{DSAT})^{\beta}\right]^{1/\beta}}$

5) $V_{DSAT} = \dfrac{\upsilon_{sat} L}{\mu_n}$

Only 10 device-specific parameters in this model:

$C_{inv}, V_T, \delta, m, \upsilon_{sat}, \mu_n,$
$L, R_{SD} = R_S + R_D,$
α, β

Discussion

With this extension (subthreshold to above threshold conduction), the VS model accurately describes modern transistors providing:

1) The high-field saturation velocity is viewed as an empirical, fitting parameter.

2) The mobility of carriers in the inversion is viewed as an empirical, fitting parameter.

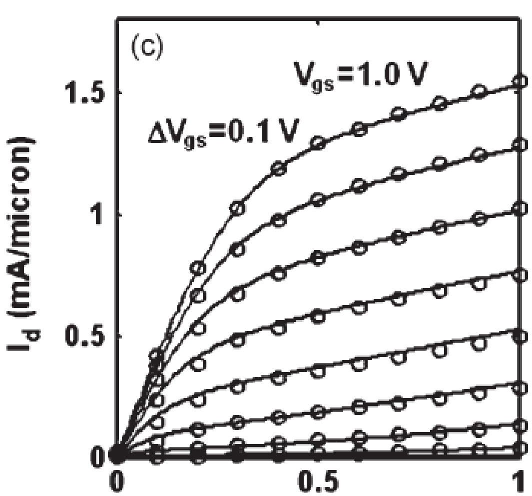
3) **But** we will see later, that these empirical parameters can be given a clear, physical interpretation**.**

MIT VS Model

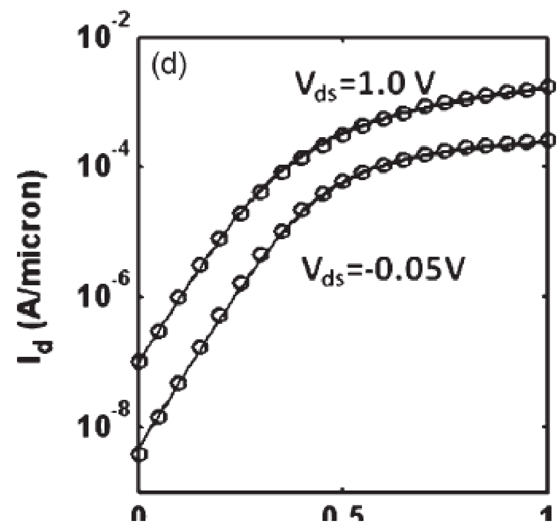
1674 IEEE TRANSACTIONS ON ELECTRON DEVICES, VOL. 56, NO. 8, AUGUST 2009

A Simple Semiempirical Short-Channel MOSFET Current–Voltage Model Continuous Across All Regions of Operation and Employing Only Physical Parameters

Ali Khakifirooz, *Member, IEEE*, Osama M. Nayfeh, *Member, IEEE*, and Dimitri Antoniadis, *Fellow, IEEE*



← 32 nm technology →



Download the MVS model at: https://nanohub.org/publications/15

Unit 2 → Unit 3

transport

$$I_{DS}/W = -Q_n\left(V_{GS}, V_{DS}\right)\left\langle \upsilon\left(V_{DS}\right)\right\rangle$$

electrostatics

In **Unit 2**, we have discussed MOS electrostatics. In **Unit 3**, we will begin our discussion of transport in nanotransistors.