Fundamentals of Nanotransistors

Unit 2: MOS Electrostatics

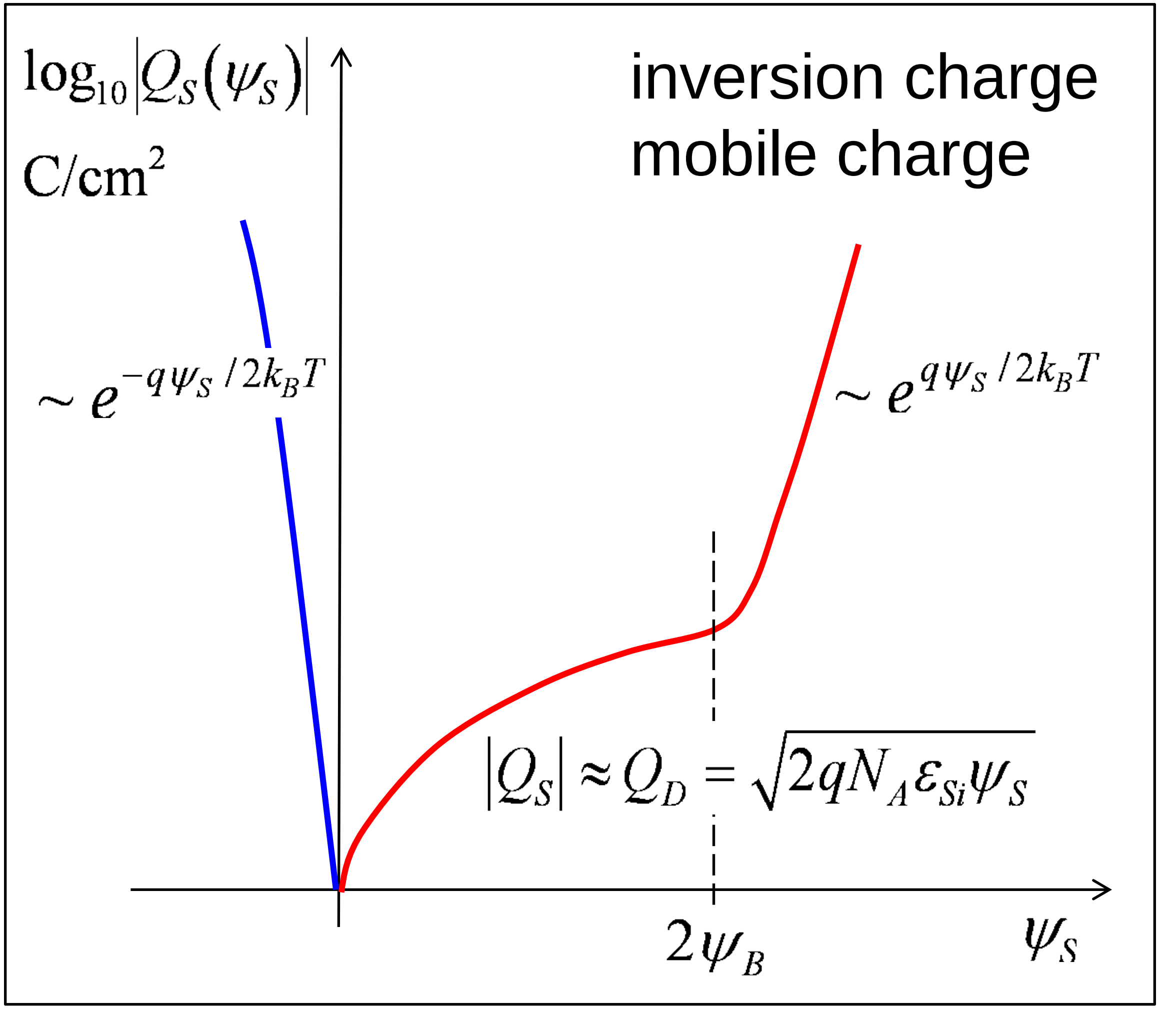
Lecture 2.5: Mobile Charge: Bulk MOS

Mark Lundstrom

lundstro@purdue.edu
Electrical and Computer Engineering
Birck Nanotechnology Center
Purdue University, West Lafayette, Indiana USA

PURDUE UNIVERSITY

Where we are



$$\psi_S = 2\psi_B$$

$$\psi_B = \frac{k_BT}{q}\ln\left(\frac{N_A}{n_i}\right)$$

$$V_G = V_{FB} - \frac{Q_S(\psi_S)}{C_{ox}} + \psi_S$$

MOSFET drain current

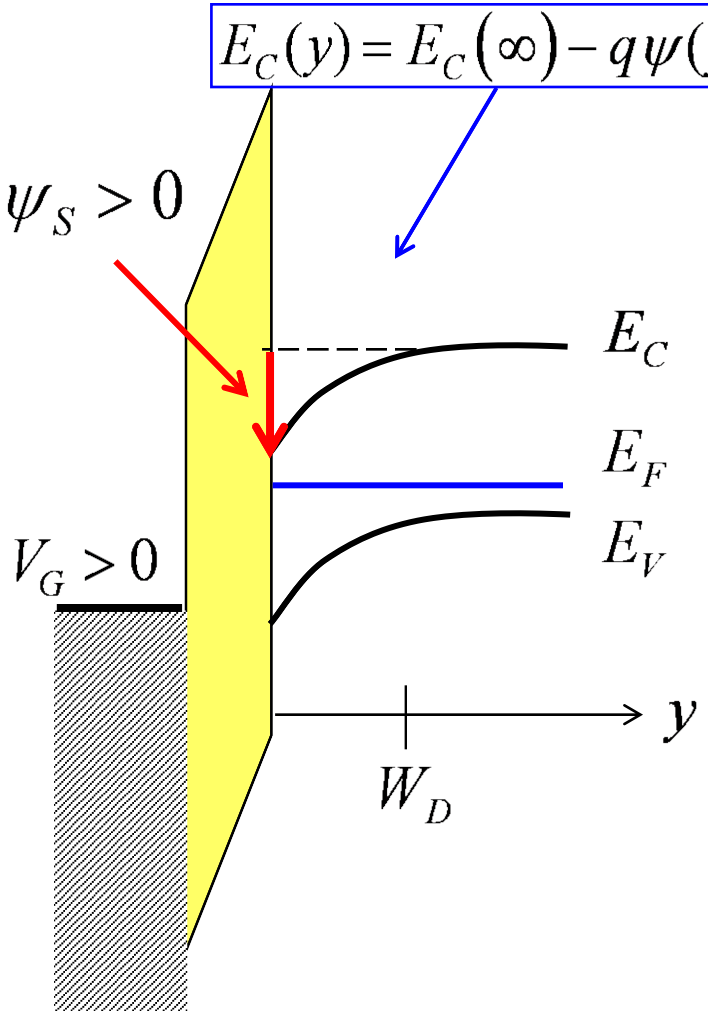
$$I_{DS}/W = -Q_n\left(V_{GS}\right)\left\langle \upsilon\left(V_{DS}\right)\right\rangle$$

We have been discussing Q_S and Q_D, but we need $\boldsymbol{Q_n}$ as a function of surface potential and gate voltage.

$$Q_S\left(\psi_S\right) = Q_D\left(\psi_S\right) + Q_n\left(\psi_S\right)$$

$$Q_S\left(V_G\right) = Q_D\left(V_G\right) + Q_n\left(V_G\right)$$

Mobile charge (per cm^3)



$$Q_n = -q\int_0^{\infty} n(y)\,dy = -qn_S \quad \text{C/cm}^2$$

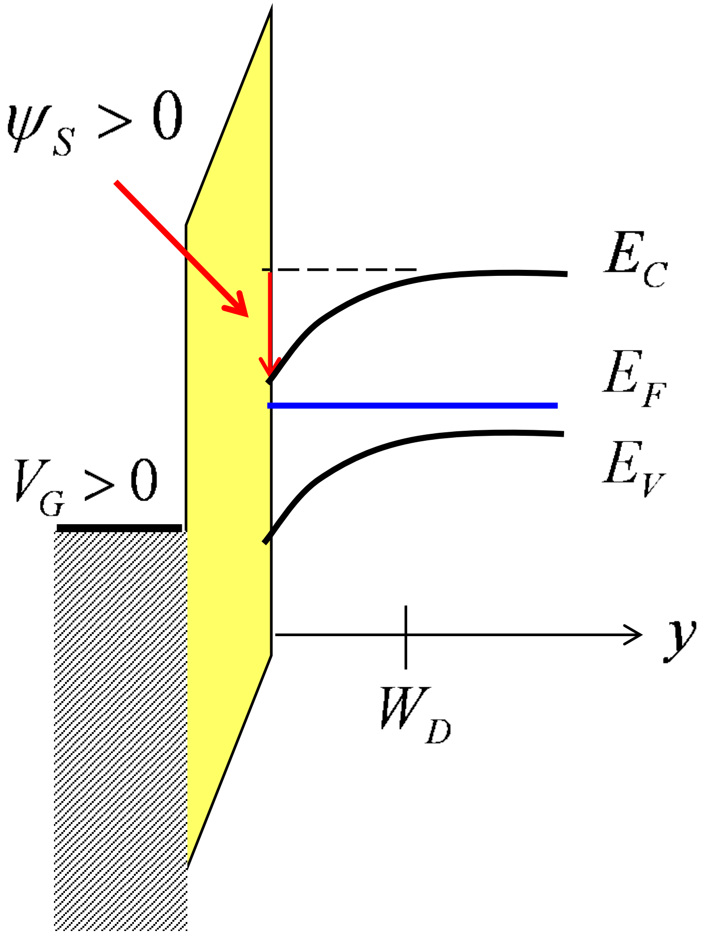
$$n_0(y) = N_C e^{(E_F - E_C(y))/k_BT}$$

$$n_0(y) = N_C\, e^{(E_F - E_C(\infty))/k_BT} \times e^{q\psi(y)/k_BT}$$

$$n_0(y) = n_B \times e^{q\psi(y)/k_BT}$$

$$n_B = \frac{n_i^2}{N_A}$$

Mobile sheet charge (per cm²)



$$Q_n = -q\int_0^\infty n(y)\,dy$$

$$= -q\int_0^\infty n_B\, e^{q\psi(y)/k_BT}\, dy$$

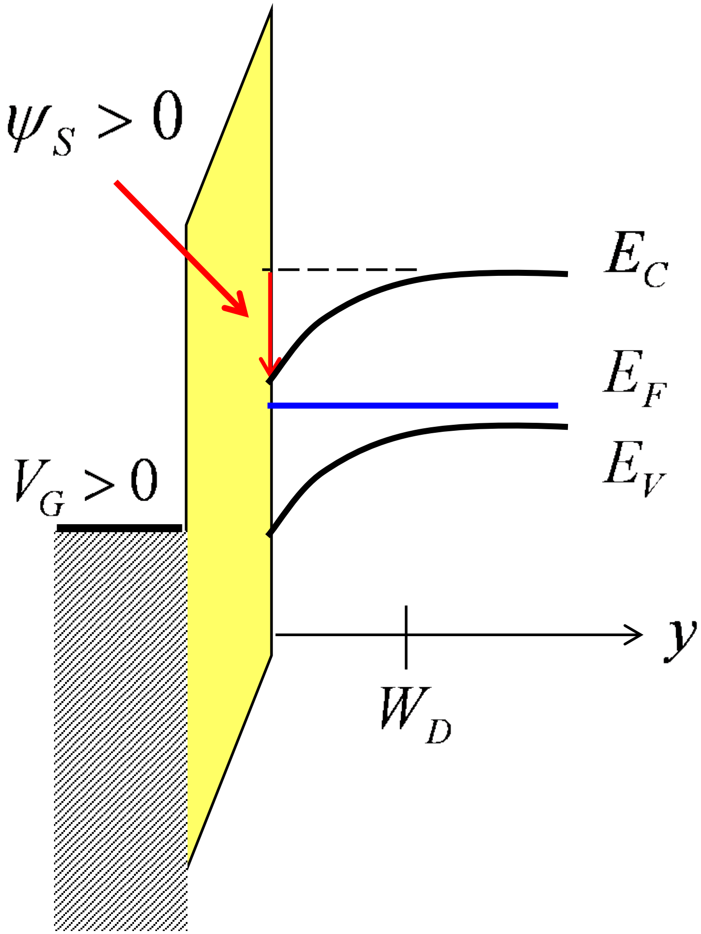
$$= -qn_B \int_0^{\psi_S} e^{q\psi(y)/k_BT}\, \frac{dy}{d\psi}\, d\psi$$

$$\approx \frac{qn_B}{\mathcal{E}_S}\int_0^{\psi_S} e^{q\psi(y)/k_BT}\, d\psi$$

$$Q_n \approx -qn_B e^{q\psi_S/k_BT}\left(\frac{k_BT/q}{\mathcal{E}_S}\right)$$

$$Q_n \approx -qn(0)\times t_{inv}$$

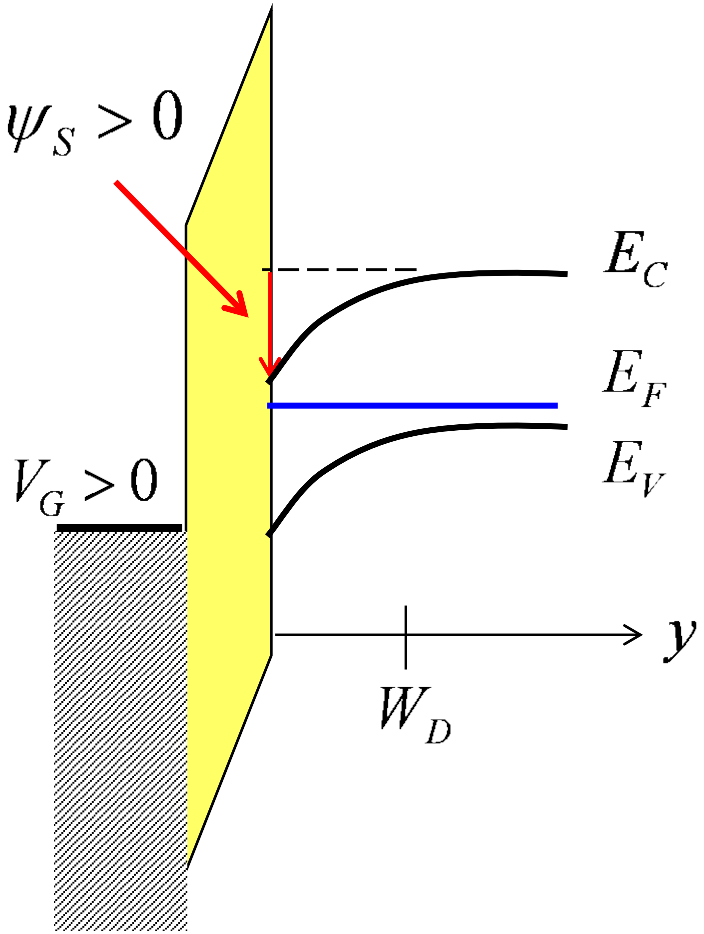
Mobile sheet charge (per cm^2)



$$Q_n(\psi_S) \approx -q n_B e^{q\psi_S/k_BT}\left(\frac{k_BT/q}{\mathcal{E}_S}\right)$$

valid above ***or*** below threshold

Mobile sheet charge: below threshold



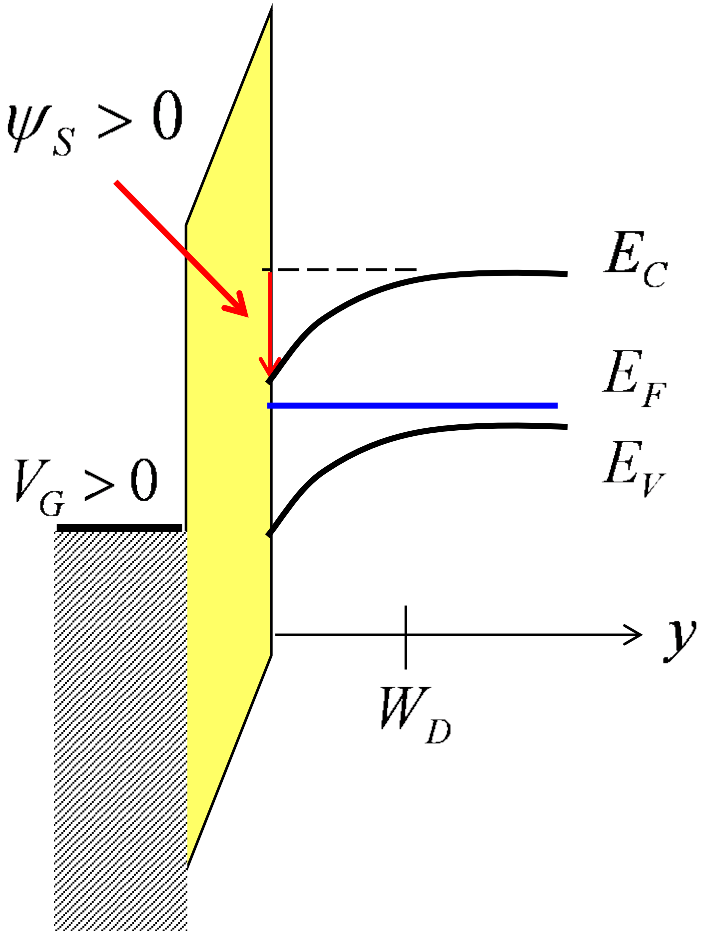
$$Q_n(\psi_S) \approx -qn_B e^{q\psi_S/k_BT}\left(\frac{k_BT/q}{\mathcal{E}_S}\right)$$

$$\mathcal{E}_S = \left(2qN_A\psi_S/\varepsilon_S\right)^{1/2}$$

$$Q_n(\psi_S) \approx -\left(\frac{n_i^2 k_BT/N_A}{\sqrt{\left(2qN_A\psi_S/\varepsilon_S\right)^{1/2}}}\right)e^{q\psi_S/k_BT}$$

$$Q_n(\psi_S) \propto e^{q\psi_S/k_BT}$$

Mobile sheet charge: above threshold



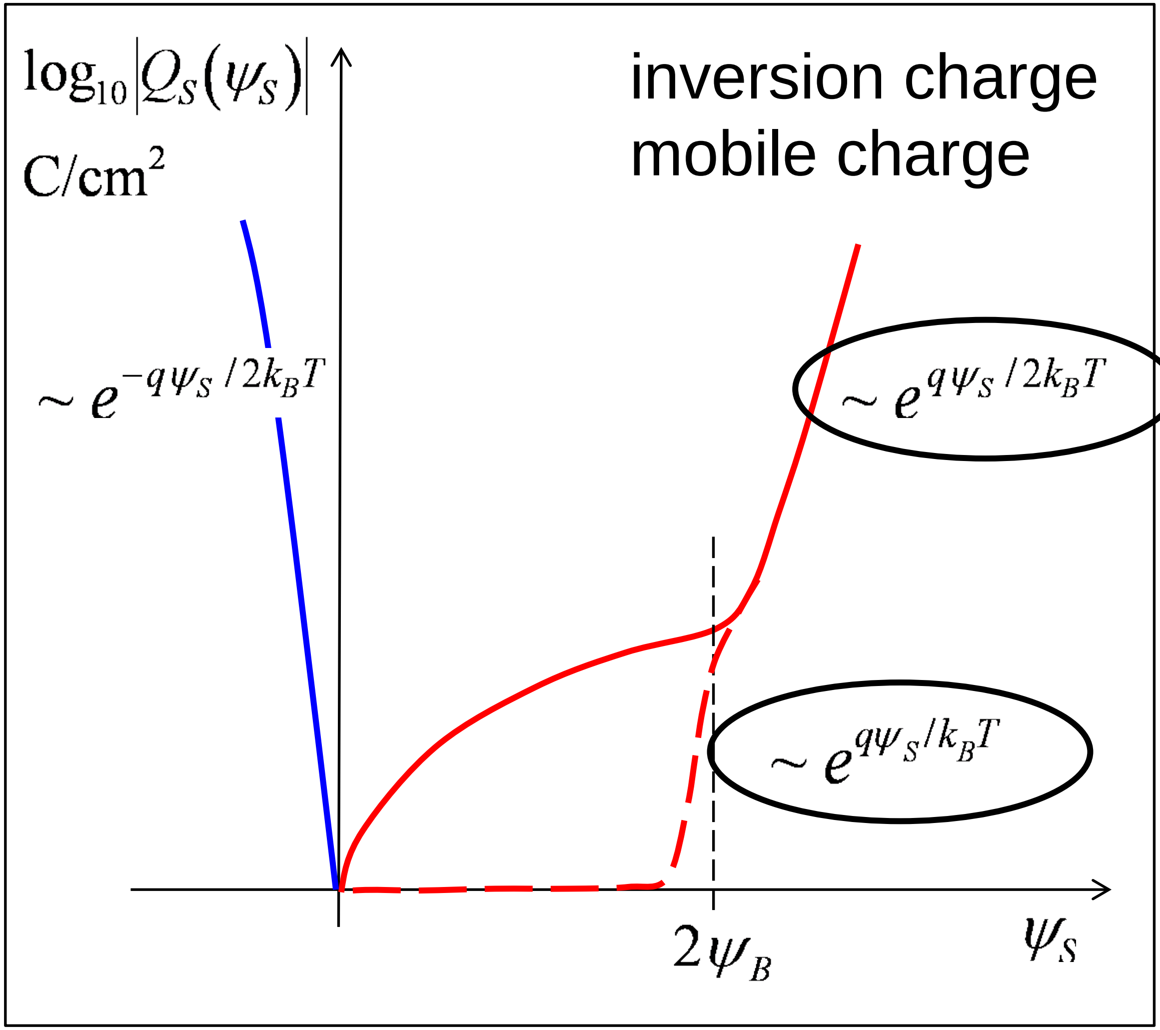
$$Q_n(\psi_S) \approx -q n_B e^{q\psi_S/k_BT}\left(\frac{k_BT/q}{\bar{\mathcal{E}}_S}\right)$$

$$\bar{\mathcal{E}}_S = \frac{Q_n(\psi_S)}{2\varepsilon_S}$$

$$Q_n(\psi_S) = -\sqrt{2\varepsilon_S k_B T\left(n_i^2/N_A\right)} \times e^{q\psi_S/2k_BT}$$

$$Q_n(\psi_S) \propto e^{q\psi_S/2k_BT}$$

Mobile charge vs. surface potential



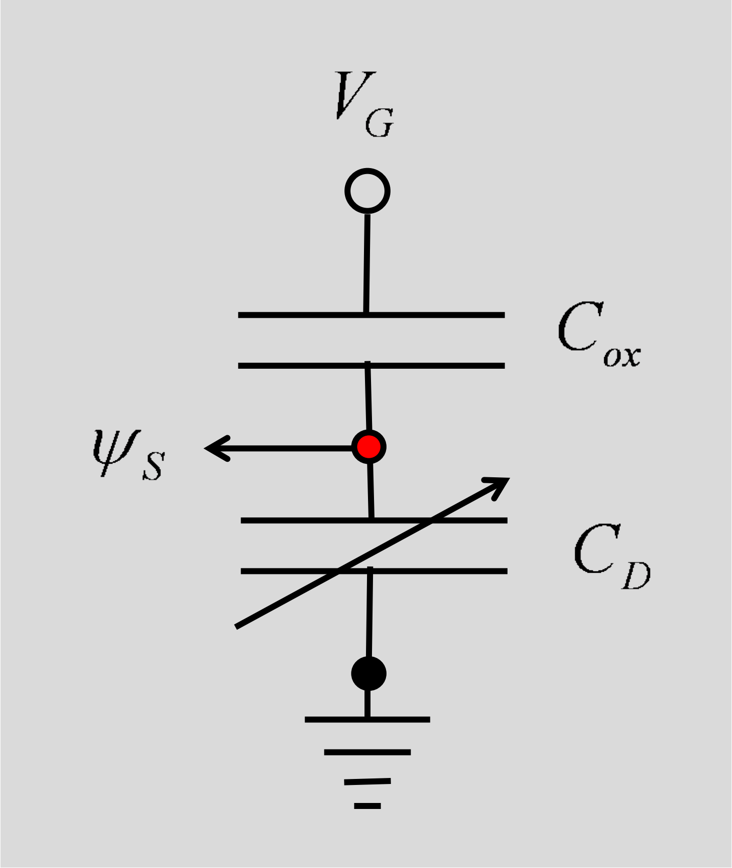
Subthreshold charge vs. **gate voltage**

$$\psi_S = \frac{V_G}{m}$$

$$m = 1 + C_D / C_{ox}$$

(See Lecture 2.3 slide 11)

$$Q_n(\psi_S) = -qn_B e^{q\psi_S/k_BT}\left(\frac{k_BT/q}{\mathcal{E}_S}\right)$$

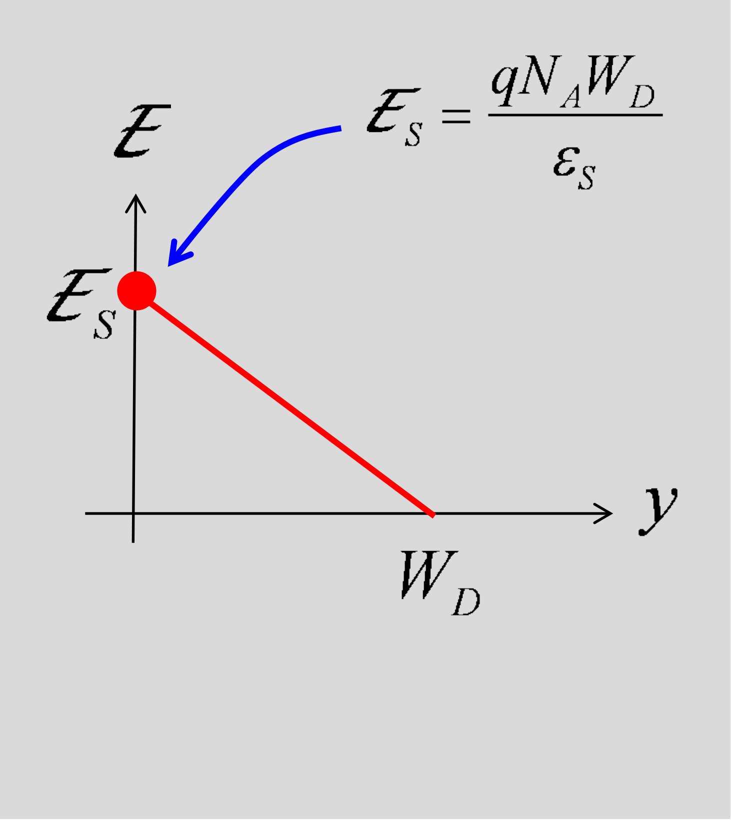


Subthreshold charge vs. gate voltage

$$\mathcal{E}_S = \frac{qN_A W_D}{\varepsilon_S} = \frac{qN_A}{C_D} \qquad C_D = \frac{\varepsilon_S}{W_D}$$

$$m = 1 + C_D / C_{ox} \qquad (m-1)C_{ox} = C_D$$

$$\mathcal{E}_S = \frac{qN_A}{(m-1)C_{ox}}$$



Subthreshold charge vs. gate voltage

$$Q_n(\psi_S) = -q n_B e^{q\psi_S/k_BT}\left(\frac{k_BT/q}{\mathcal{E}_S}\right)$$

$$\psi_S = \frac{V_G}{m} \qquad m = 1 + C_D/C_{ox}$$

$$\mathcal{E}_S = \frac{qN_A}{(m-1)C_{ox}} \qquad n_B = n_i^2/N_A$$

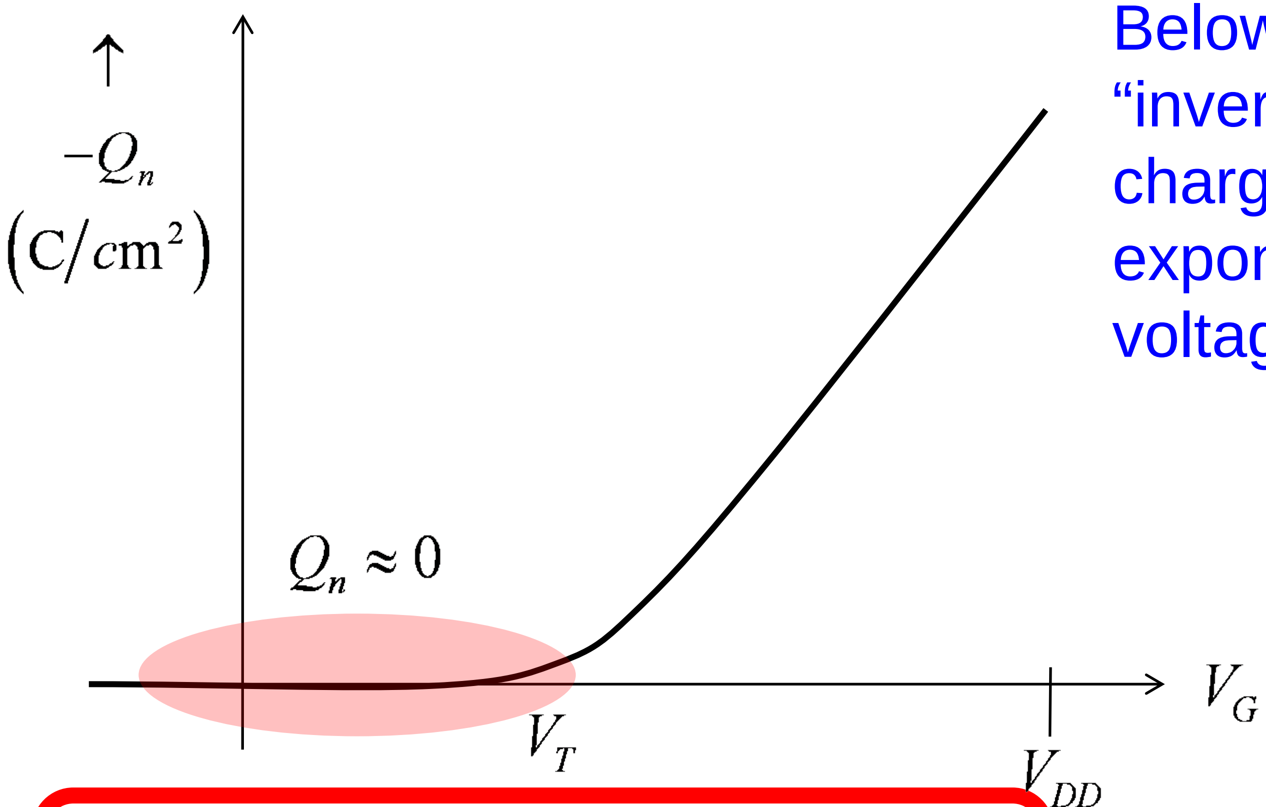
$$\psi_B = \frac{k_BT}{q}\ln\left(\frac{N_A}{n_i}\right)$$

$$2\psi_B = \frac{V_T}{m}$$

$$\left(\frac{n_i}{N_A}\right)^2 = e^{-qV_T/mk_BT}$$

$$Q_n(\psi_S) = -(m-1)C_{ox}\left(\frac{n_i}{N_A}\right)^2 e^{qV_{GS}/mk_BT}\left(\frac{k_BT}{q}\right)$$

Subthreshold charge vs. gate voltage



Below threshold, the "inversion" layer charge increases exponentially with gate voltage.

$$Q_n(V_G) = -(m-1)C_{ox}\left(\frac{k_B T}{q}\right)e^{q(V_G - V_T)/mk_B T}$$

Above threshold charge vs. gate voltage

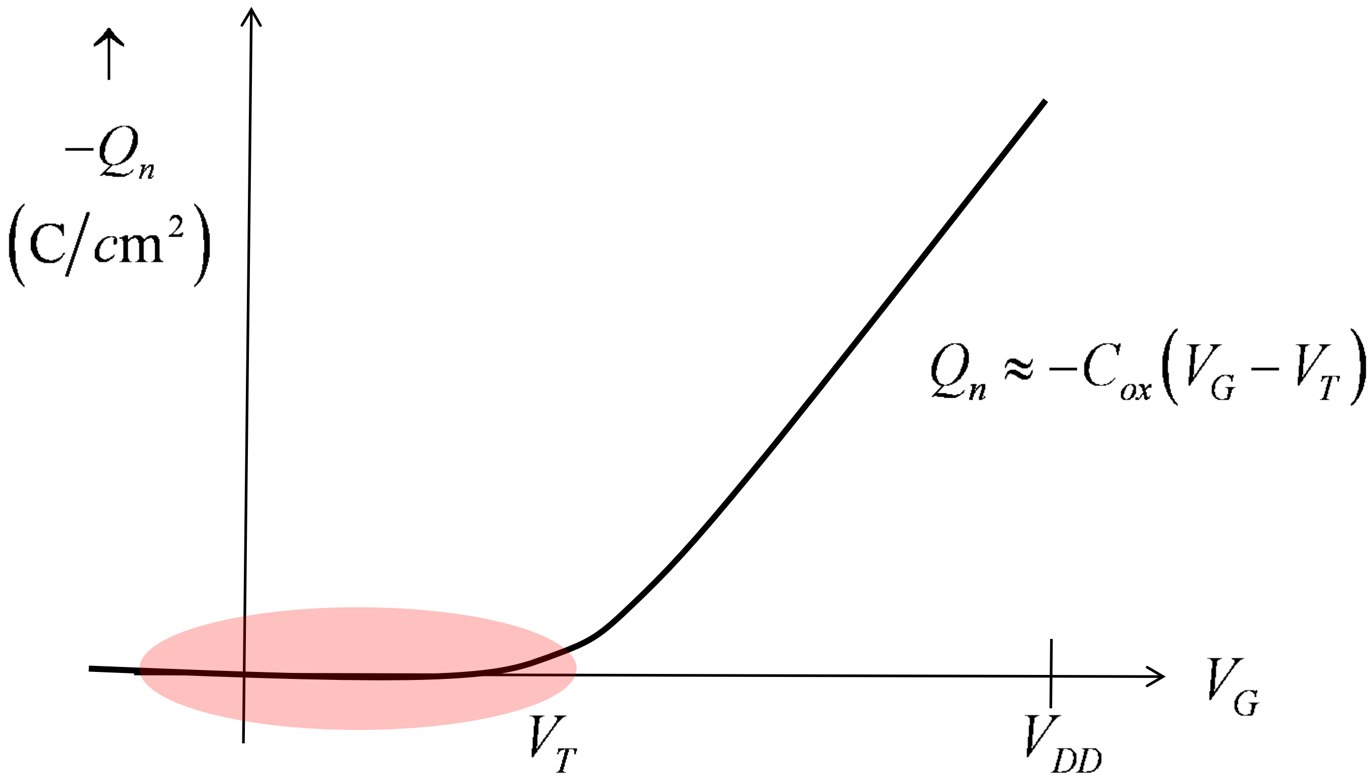
$$V_G = V_{FB} - \frac{Q_S(\psi_S)}{C_{ox}} + \psi_S \qquad V_G = V_{FB} - \frac{Q_D(\psi_S)}{C_{ox}} - \frac{Q_n(\psi_S)}{C_{ox}} + \psi_S$$

$$V_T = V_{FB} - \frac{Q_D(2\psi_B)}{C_{ox}} + 2\psi_B$$

$$V_G = V_{FB} - \frac{Q_D(2\psi_B + \delta\psi_S)}{C_{ox}} - \frac{Q_n(2\psi_B + \delta\psi_S)}{C_{ox}} + 2\psi_B + \delta\psi_S$$

$$V_G - V_T \approx -\frac{Q_n}{C_{ox}} \qquad Q_n \approx -C_{ox}(V_G - V_T)$$

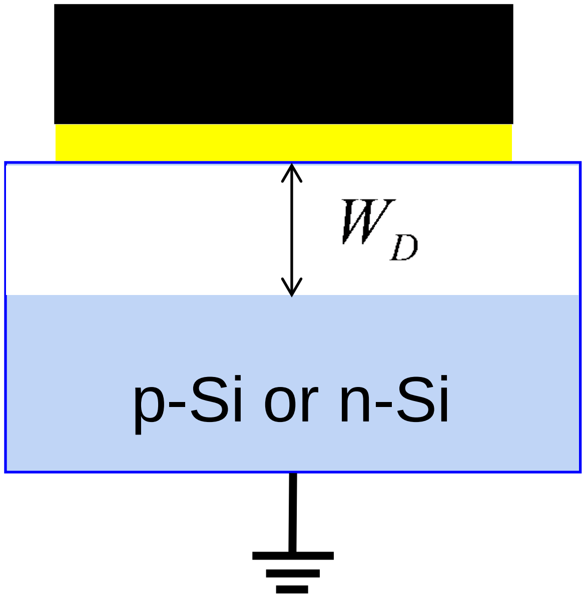
Mobile charge vs. gate voltage



$$Q_n\left(V_G\right) = -\left(m-1\right)C_{ox}\left(\frac{k_B T}{q}\right)e^{q\left(V_G - V_T\right)/mk_B T}$$

Gate capacitance

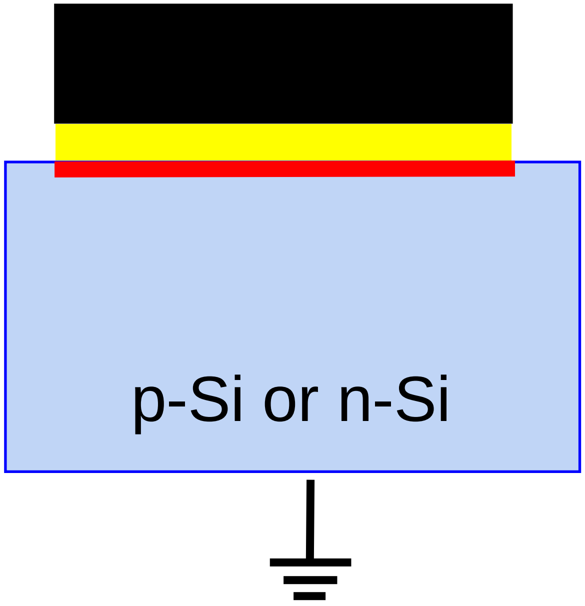
$$V_G < V_T$$



$$\frac{1}{C_G} = \frac{1}{C_{ox}} + \frac{1}{C_S} \qquad C_S = C_D = \frac{\varepsilon_S}{W_D}$$

$$C_S \equiv \frac{d(-Q_S)}{d\psi_S} = \frac{\varepsilon_S}{W_D}$$

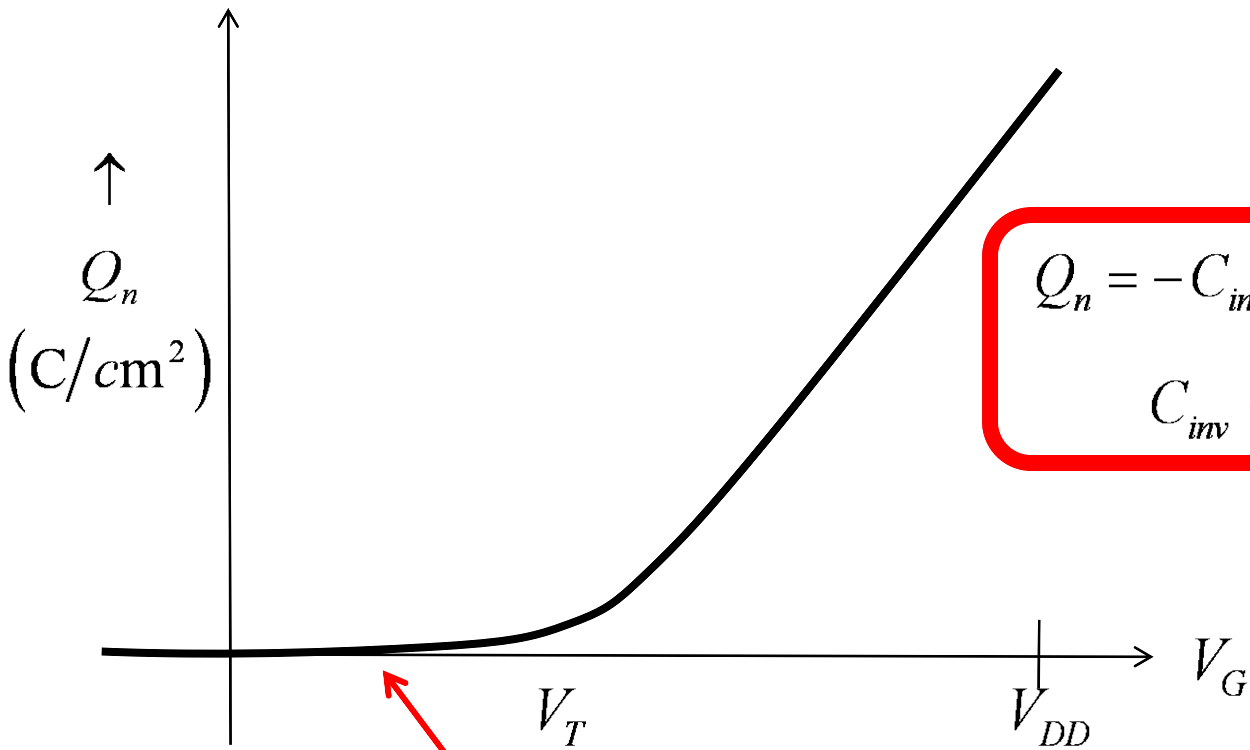
$$V_G > V_T$$



$$\frac{1}{C_G} = \frac{1}{C_{ox}} + \frac{1}{C_S} \qquad C_S = C_S(\text{inv}) = \frac{\varepsilon_S}{t_{inv}}$$

$$C_S \equiv \frac{d(-Q_S)}{d\psi_S}$$

Mobile charge vs. gate voltage



$$Q_n = -C_{inv}\left(V_G - V_T\right)$$

$$C_{inv} < C_{ox}$$

$$Q_n\left(V_G\right) = -\left(m-1\right)C_{ox}\left(\frac{k_B T}{q}\right)e^{q\left(V_G - V_T\right)/mk_B T}$$

Example

On-current conditions: $Q_n = -q \times 10^{13}\ \mathrm{C/cm^2}$

Oxide thickness: $t_{ox} = 1\ \mathrm{nm} = 10^{-7}\ \mathrm{cm}$ $\kappa_{ox} = 3.9$

$$C_{ox} = \frac{\varepsilon_{ox}}{t_{ox}} = 3.45 \times 10^{-6}\ \mathrm{F/cm^2}$$

$$C_S(\mathrm{inv}) = \frac{-Q_n}{2(k_B T/q)} = 30.8 \times 10^{-6}\ \mathrm{F/cm^2}$$

$$\frac{1}{C_G(\mathrm{inv})} = \frac{1}{C_{ox}} + \frac{1}{C_S(\mathrm{inv})} = \frac{1}{3.45 \times 10^{-6}} + \frac{1}{30.8 \times 10^{-6}}$$

$$C_G(\mathrm{inv}) = 3.10 \times 10^{-6}\ \mathrm{F/cm^2} = 0.90 C_{ox}$$

CET

$$\frac{1}{C_G} = \frac{1}{C_S(\text{inv})} + \frac{1}{C_{ox}}$$

$$C_G = \frac{C_{ox} C_S(\text{inv})}{C_{ox} + C_S(\text{inv})} < C_{ox}$$

$$C_G \equiv \frac{\kappa_{ox} \varepsilon_0}{CET}$$

Why is it hard to bend the bands more than $2\psi_B$?

below threshold:

$$C_S = \frac{d(-Q_D)}{d\psi_S} = C_D = \frac{\varepsilon_S}{W_D} \approx C_{ox}$$

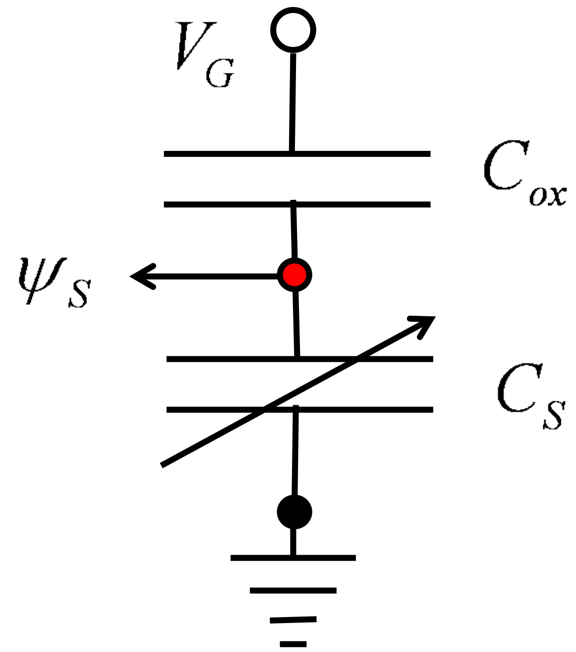
$$m \approx 1$$

above threshold:

$$C_S = \frac{d(-Q_n)}{d\psi_S} = C_S(\text{inv}) \equiv \frac{\varepsilon_S}{t_{inv}} >> C_{ox}$$

$$m >> 1$$

$$\Delta\psi_S = \Delta V_G \frac{C_{ox}}{C_{ox} + C_S} = \frac{\Delta V_G}{m}$$



Mobile charge vs. gate voltage

$V_G << V_T$:

$$Q_n(V_G) = -(m-1)C_{ox}\left(\frac{k_B T}{q}\right)e^{q(V_G - V_T)/mk_B T}$$

$V_G >> V_T$:

$$Q_n = -C_{inv}(V_G - V_T) \qquad C_{inv} < C_{ox}$$

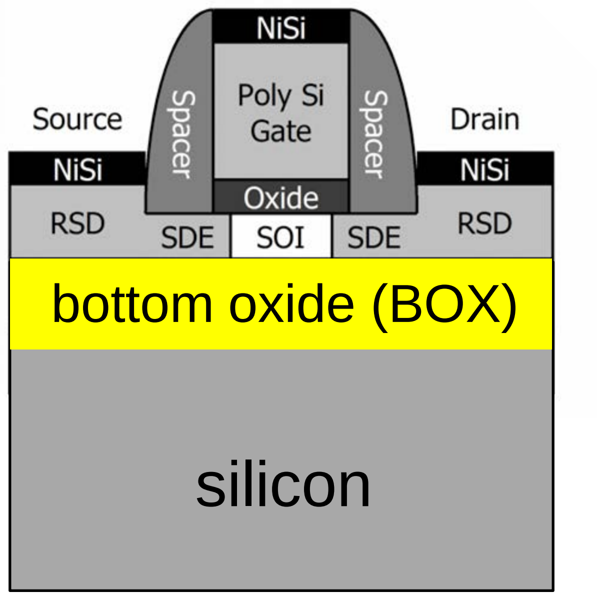
Is there a single expression that works both below and above threshold?

Yes – a numerical one – the "surface potential model"
Yes – an empirical one – with some fitting parameters

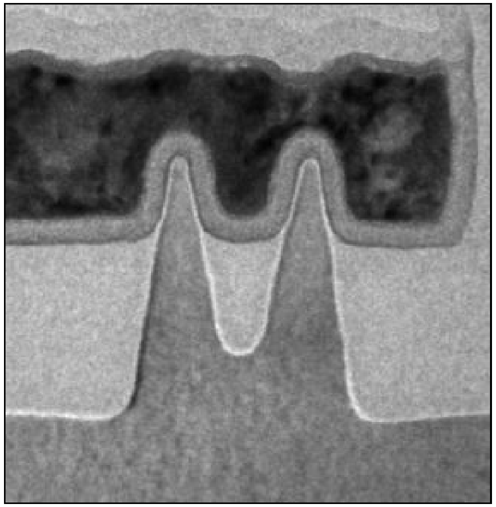
Next lecture

For bulk MOSFETs, we now understand how the mobile charge varies with surface potential and with gate voltage – both above threshold and below.

What happens for modern (fully depleted) structures?



(ETSOI: Source: IBM, 2009)



(FinFET: Source: Intel, 2015)