Fundamentals of Nanotransistors

Unit 2: MOS Electrostatics

Lecture 2.3: Gate Voltage and Surface Potential

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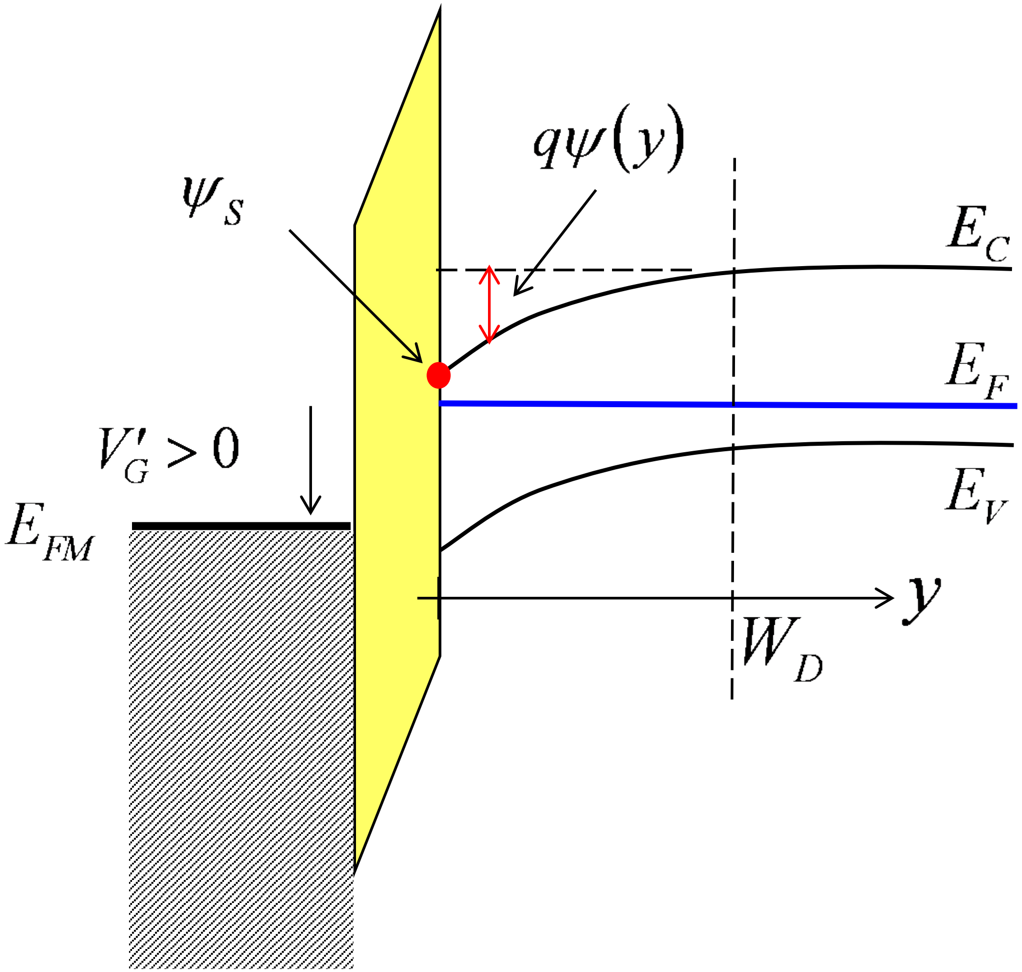
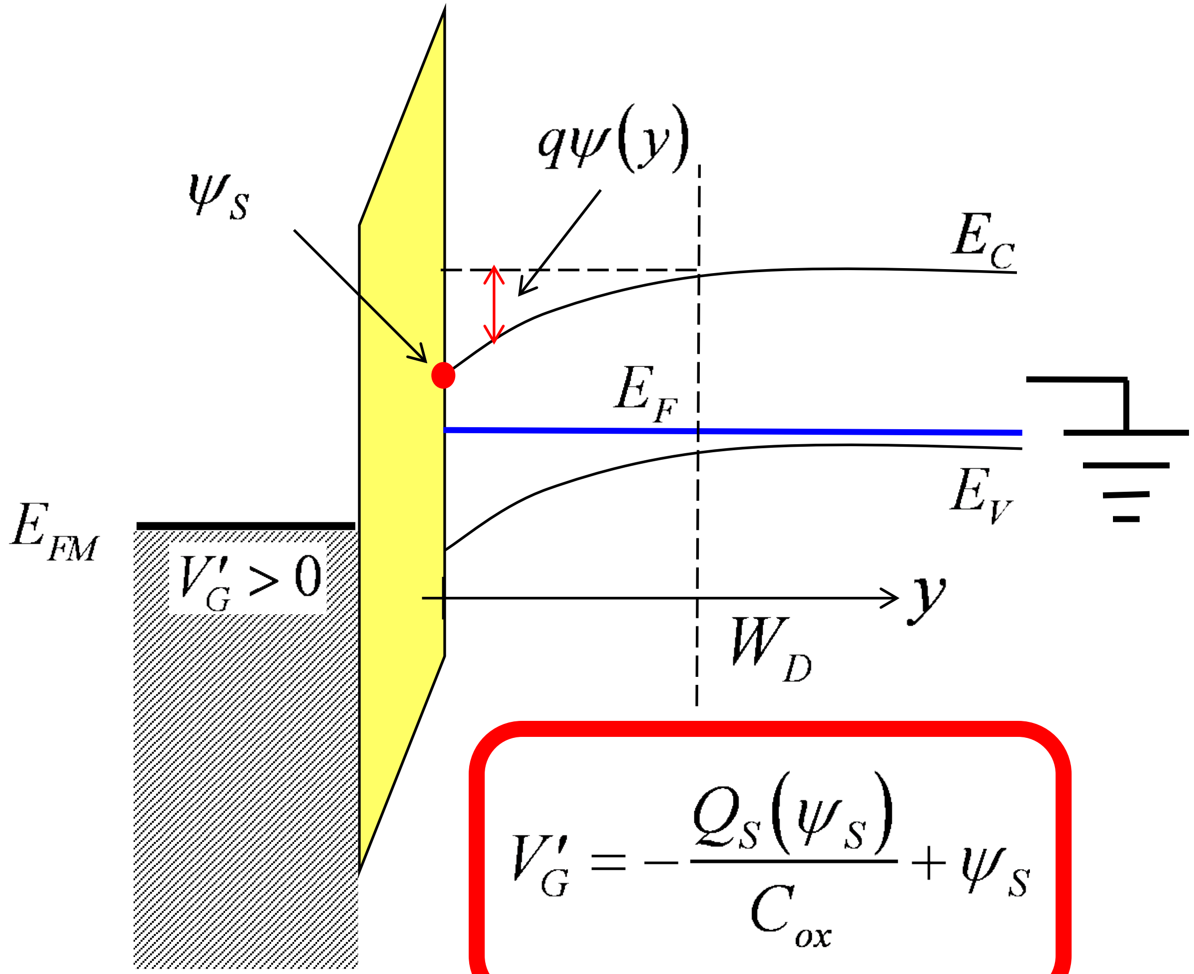
Band bending depends on surface potential



$$\psi(y \rightarrow \infty) = 0$$

What gate voltage produced this surface potential?

Gate voltage and surface potential



$$V_G' = -\frac{Q_S(\psi_S)}{C_{ox}} + \psi_S$$

$$V_G = \Delta V_{ox} + \Delta V_{Si}$$

$$\Delta V_{Si} = \psi_S$$

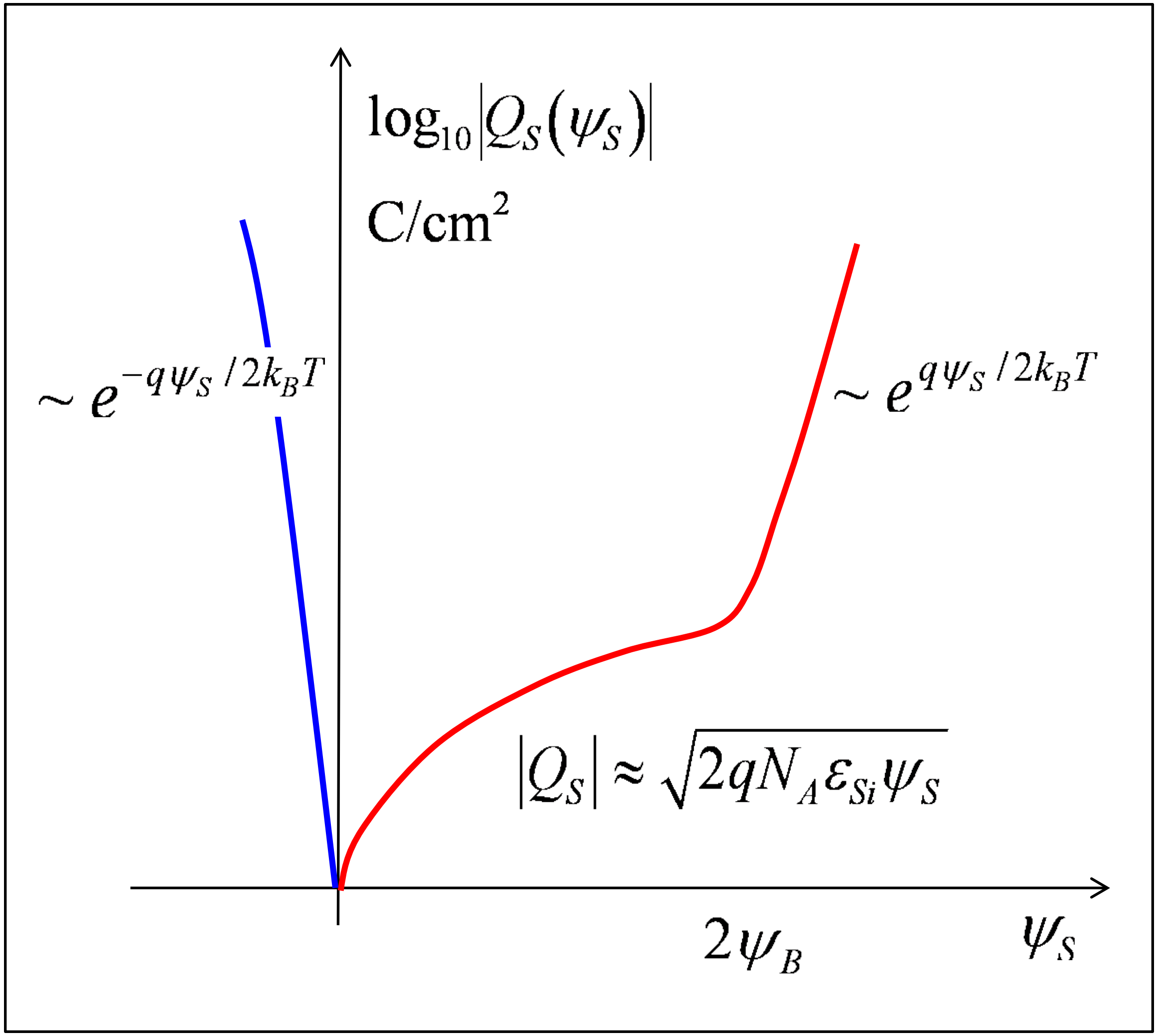
$$\Delta V_{ox} = \mathcal{E}_{ox} t_{ox}$$

$$\varepsilon_{ox}\mathcal{E}_{ox} = -Q_S(\psi_S)$$

$$\Delta V_{ox} = -Q_S(\psi_S)\frac{t_{ox}}{\varepsilon_{ox}}$$

$$C_{ox} = \frac{\varepsilon_{ox}}{t_{ox}} \quad \text{F/cm}^2$$

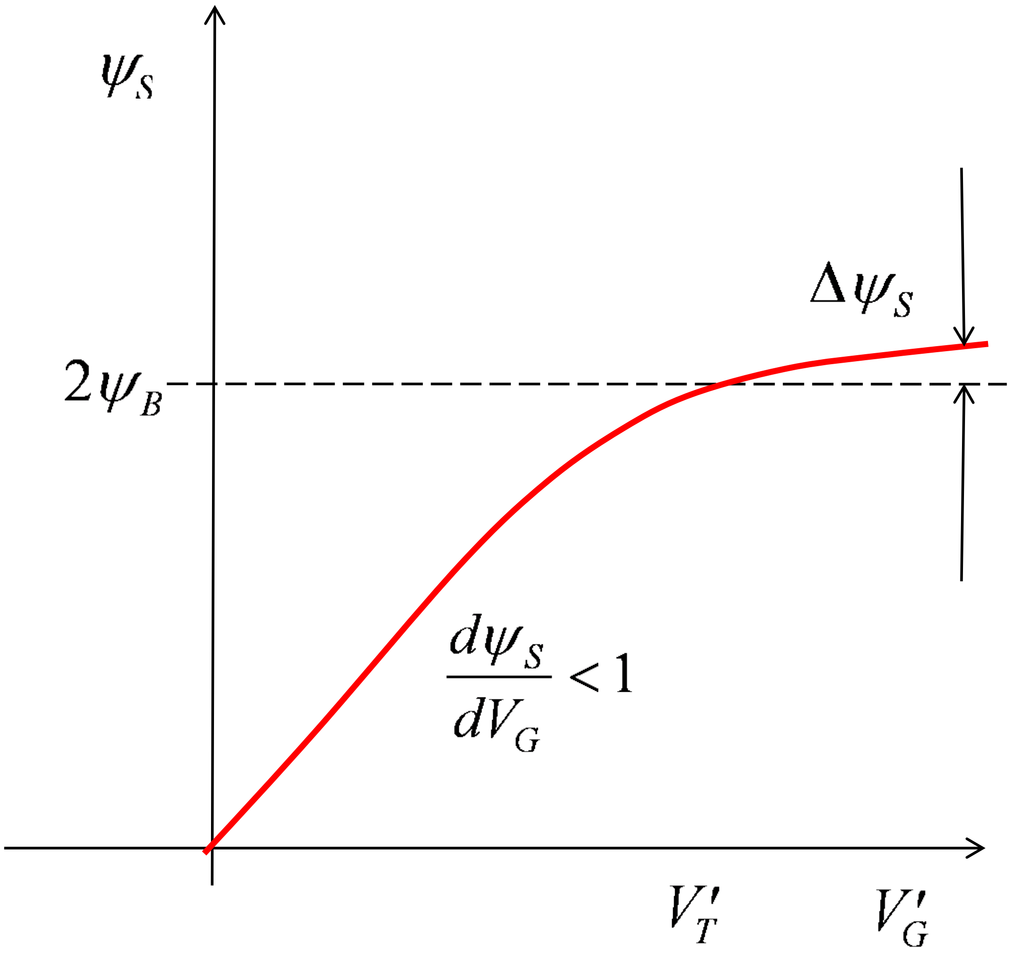
MOS electrostatics



$$V_G' = -\frac{Q_S(\psi_S)}{C_{ox}} + \psi_S$$

- given ψ_S
- determine Q_S
- find V_G

Surface potential vs. **gate voltage**



$$V_G' = -\frac{Q_S(\psi_S)}{C_{ox}} + \psi_S$$

Threshold voltage

The gate voltage needed to make: $\psi_S = 2\psi_B$

$$V_G' = -\frac{Q_S(\psi_S)}{C_{ox}} + \psi_S$$

$$V_G' = V_T' = -\frac{Q_S(2\psi_B)}{C_{ox}} + 2\psi_B$$

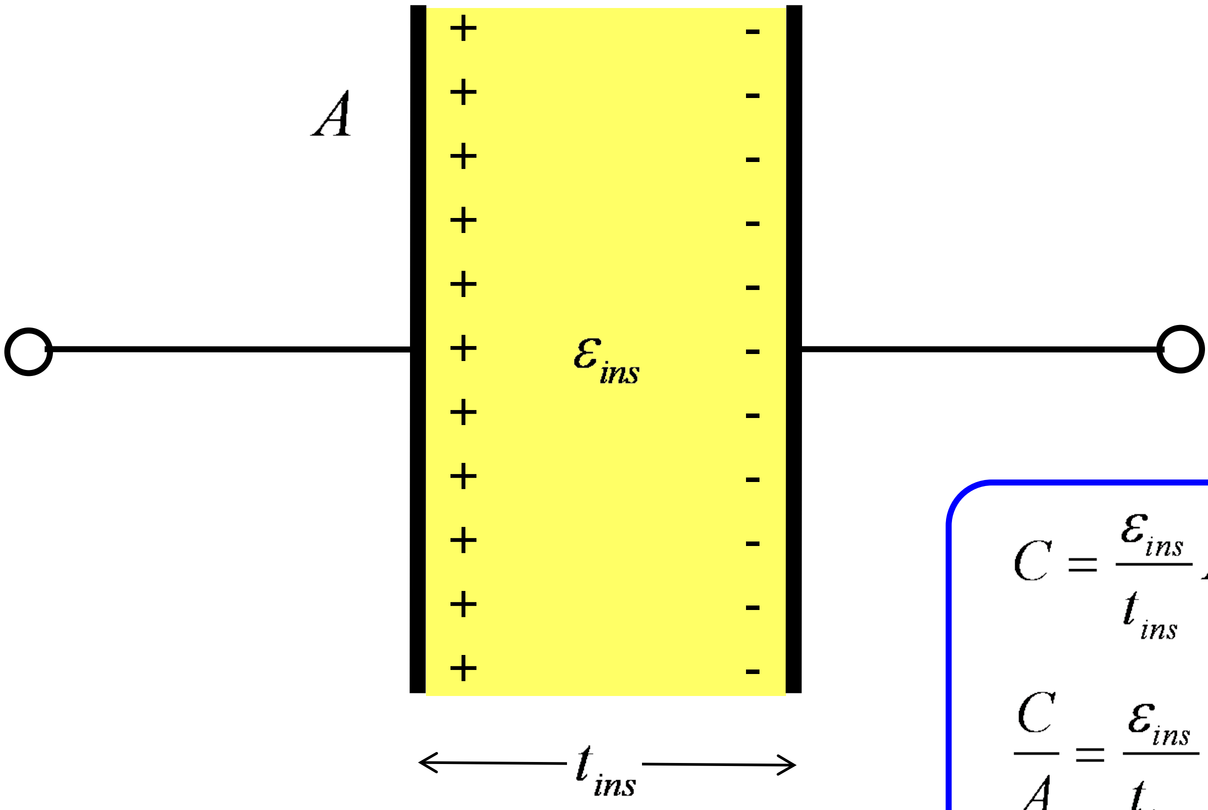
$$\begin{aligned} Q_S(2\psi_B) &= Q_D(2\psi_B) + Q_n(2\psi_B) \\ &\approx Q_D(2\psi_B) \\ &= -qN_A W_D(2\psi_B) \\ &= -qN_A W_T \end{aligned}$$

$$V_T' = \frac{qN_A W_T}{C_{ox}} + 2\psi_B$$

$$V_T' = \frac{\sqrt{2qN_A\varepsilon_S(2\psi_B)}}{C_{ox}} + 2\psi_B$$

$$\psi_B = \frac{k_B T_L}{q}\ln\left(\frac{N_A}{n_i}\right)$$

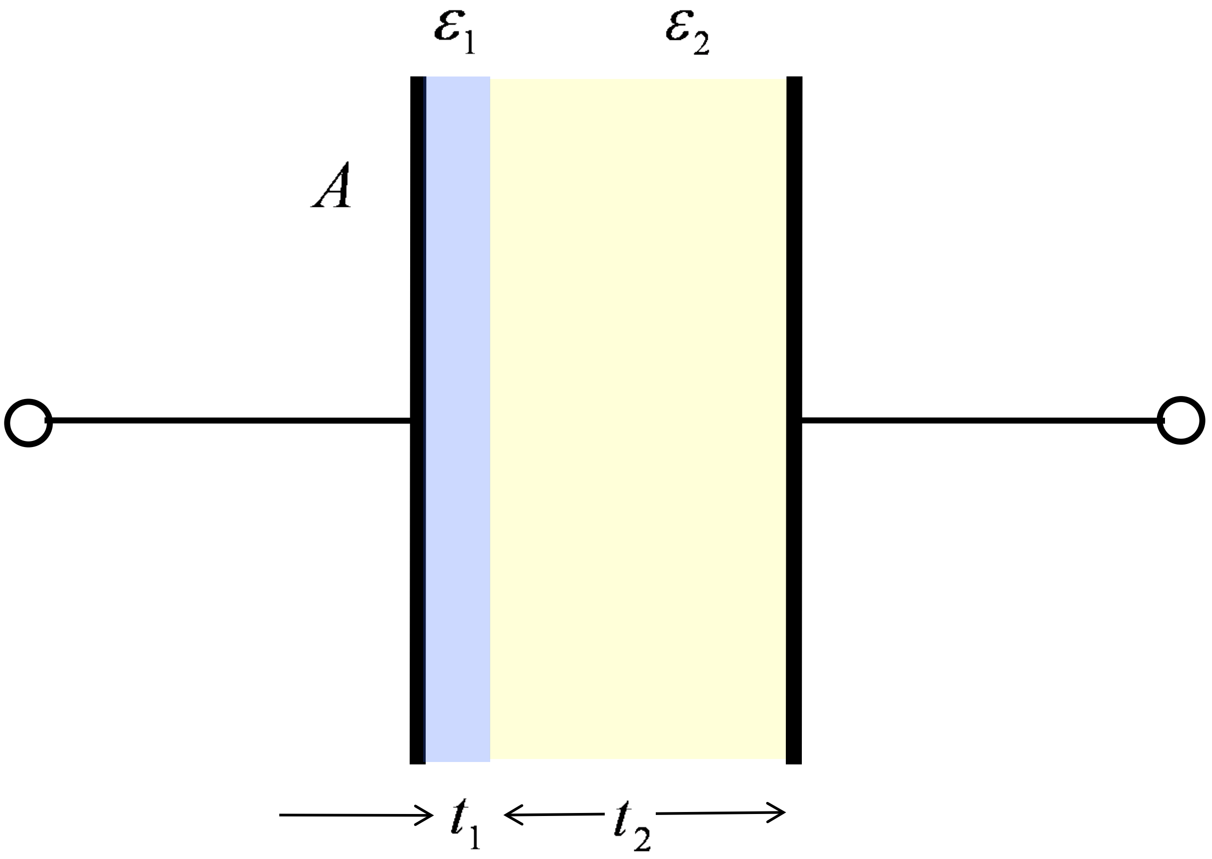
A word about capacitance



$$C = \frac{\varepsilon_{ins}}{t_{ins}} A \quad \text{F}$$

$$\frac{C}{A} = \frac{\varepsilon_{ins}}{t_{ins}} \quad \text{F/cm}^2$$

Capacitor with two dielectrics

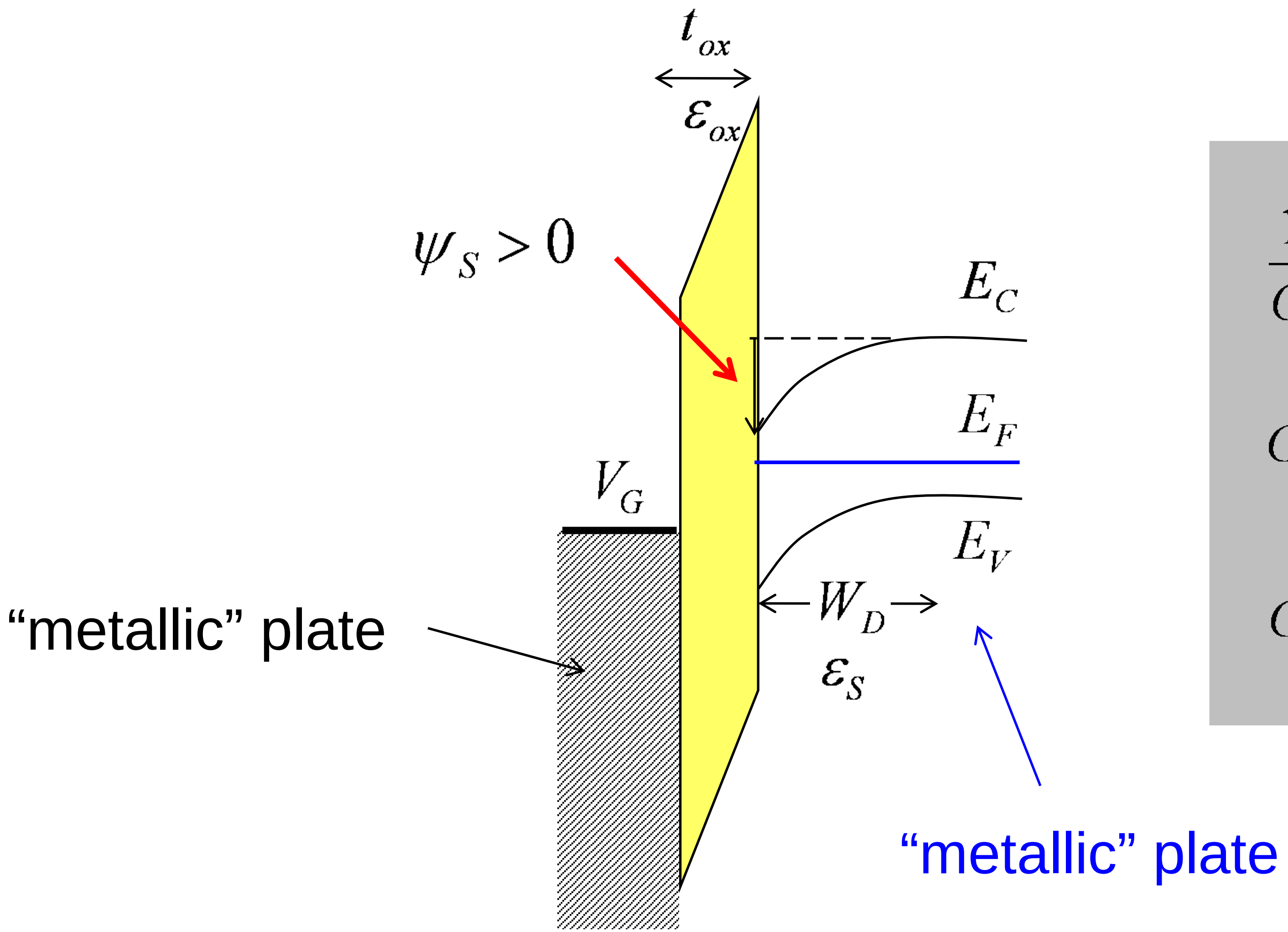


$$\frac{1}{C} = \frac{1}{C_1} + \frac{1}{C_2}$$

$$C_1 = \frac{\varepsilon_1}{t_1} A$$

$$C_2 = \frac{\varepsilon_2}{t_2} A$$

Depletion capacitance

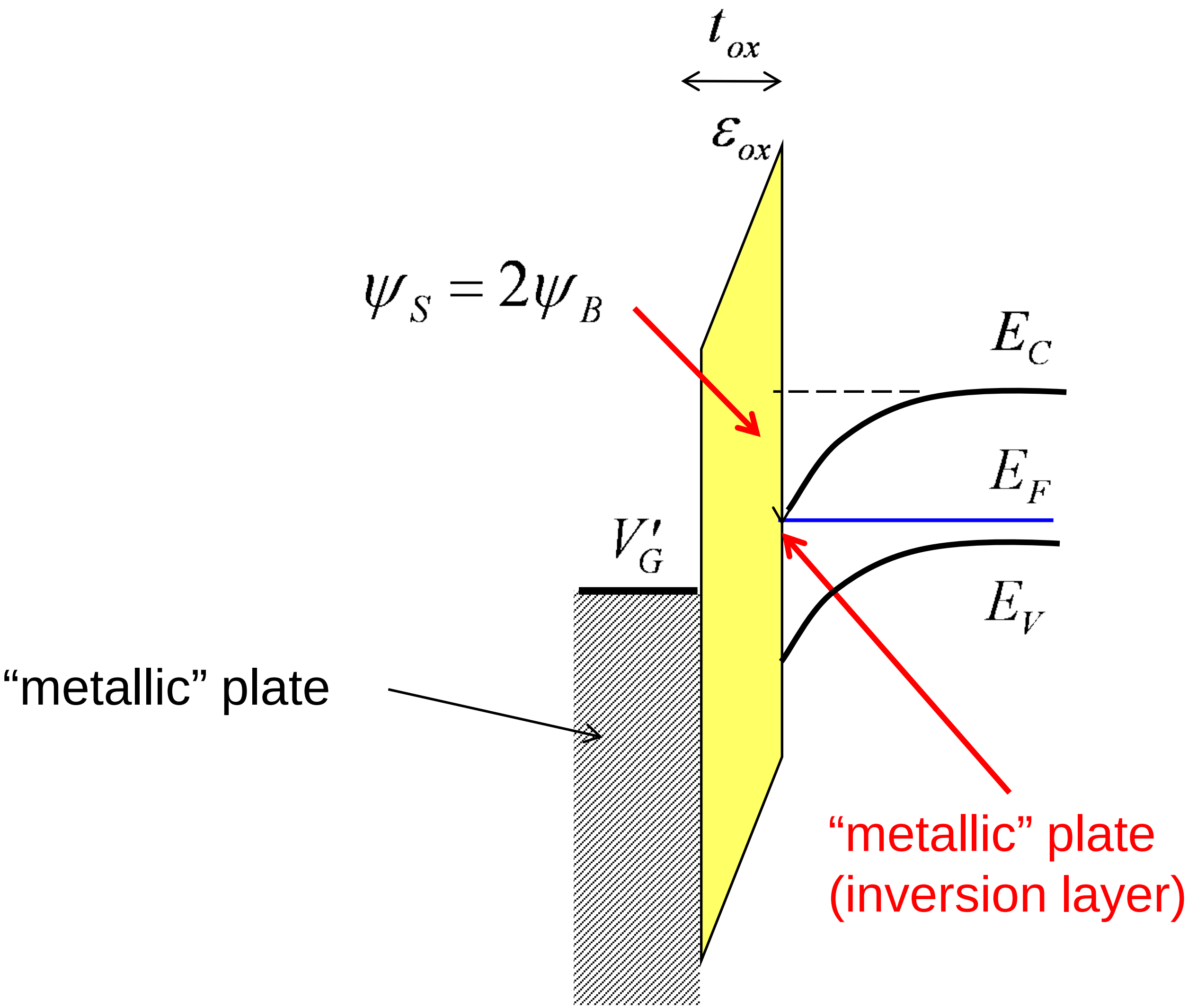


$$\frac{1}{C} = \frac{1}{C_{ox}} + \frac{1}{C_D}$$

$$C_{ox} = \frac{\varepsilon_{ox}}{t_{ox}}$$

$$C_D = \frac{\varepsilon_S}{W_D}$$

Inversion capacitance



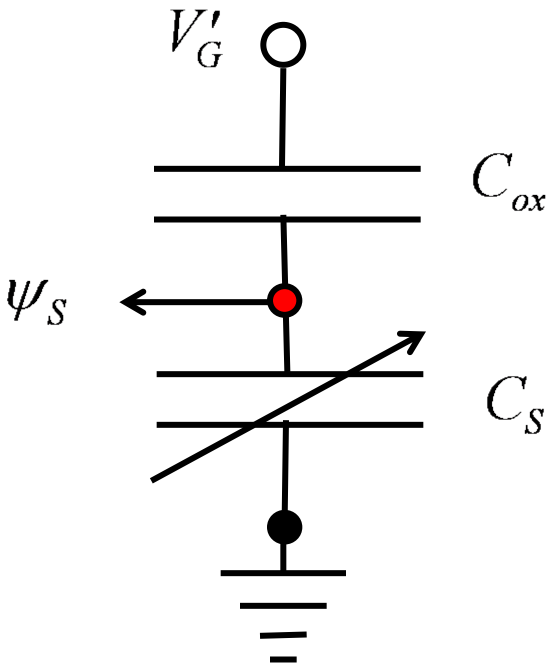
$$C \approx C_{ox}$$

(Assuming that the inversion layer can respond to the small signal a.c. voltage.)

Approximate ψ_S vs. V_G relation

$$V_G' = -\frac{Q_S(\psi_S)}{C_{ox}} + \psi_S$$

approximate solution



Below threshold:

$$C_S \approx C_D = \frac{\varepsilon_S}{W_D}$$

$$\psi_S \approx V_G \frac{C_{ox}}{C_{ox} + C_D} = \frac{V_G'}{1 + C_D / C_{ox}}$$

$$\psi_S \approx \frac{V_G'}{m}$$

$$m = 1 + C_D / C_{ox}$$

(depletion)

Example

$N_A = 10^{18}\ \text{cm}^{-3}$ $\quad W_D = 25\ \text{nm}$ $\quad \kappa_{ox} = 3.9$

$\psi_S = 0.5\ \ \text{V}$ $\quad t_{ox} = 2\ \text{nm}$ $\quad \kappa_{Si} = 11.8$

$$\psi_S = \frac{V_G'}{m}$$

$$C_D(\psi_S) = \frac{\varepsilon_S}{W_D(\psi_S)}$$

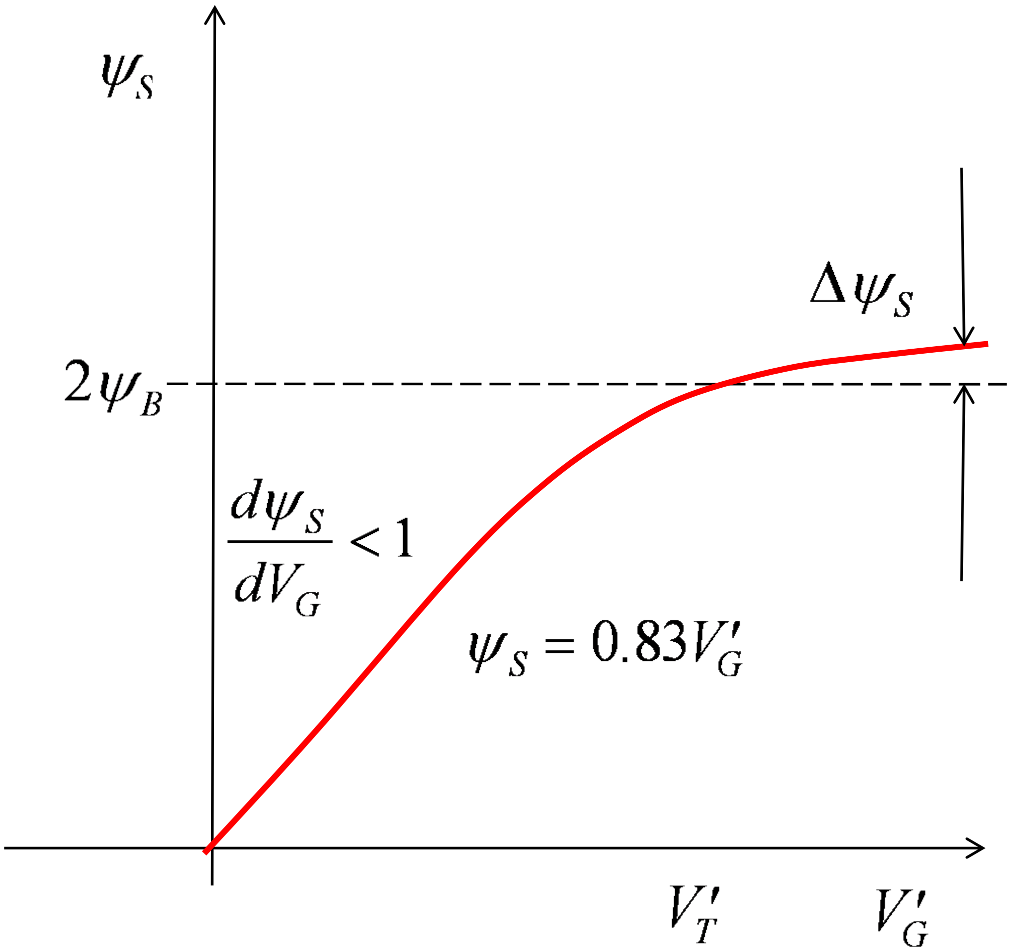
$$m = 1 + C_D / C_{ox}$$

$$m = 1 + \frac{\varepsilon_{Si}}{\varepsilon_{ox}} \frac{t_{ox}}{W_D} \qquad C_{ox} = \frac{\varepsilon_{ox}}{t_{ox}}$$

$$m = 1 + \left(\frac{12}{4}\right)\left(\frac{2}{25}\right) \approx 1.2$$

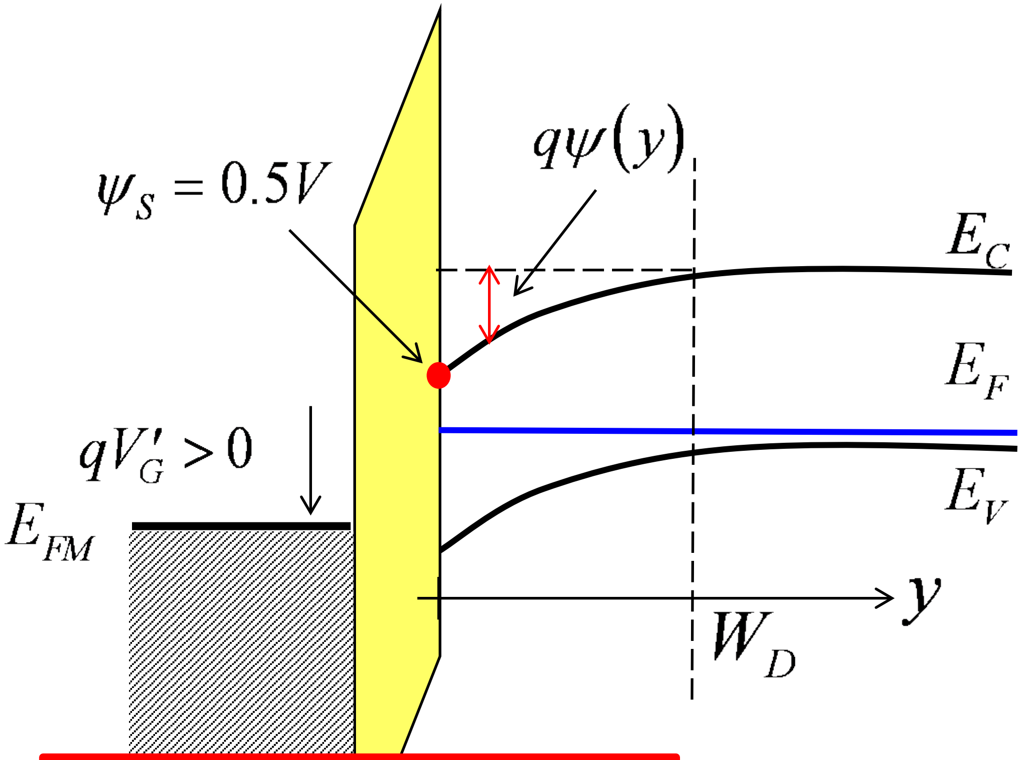
$$\psi_S = \frac{V_G'}{m} = 0.83 V_G'$$

Surface potential vs. **gate voltage**



$$V_G' = -\frac{Q_S(\psi_S)}{C_{ox}} + \psi_S$$

Recall: depletion example



$$W_D = \sqrt{2\varepsilon_S \psi_S / qN_A}$$

$$Q_S \approx Q_D = -qN_A W_D$$

$$\mathcal{E}_S = \frac{-Q_D}{\varepsilon_{Si}}$$

What gate voltage produced this surface potential?

$$W_D = 25 \text{ nm}$$

$$Q_D = -4.1 \times 10^{-7} \text{ C/cm}^2$$

$$Q_D/(-q) = 2.5 \times 10^{12} \text{ \#/cm}^2$$

$$\mathcal{E}_S = 3.9 \times 10^5 \text{ V/cm}$$

Gate voltage

$$V_G' = -\frac{Q_S(\psi_S)}{C_{ox}} + \psi_S$$

$$V_G' = -\frac{Q_D(\psi_S)}{C_{ox}} + \psi_S$$

$$V_G' = \frac{4.1 \times 10^{-7}}{1.73 \times 10^{-6}} + 0.5$$

$$= 0.24 + 0.5$$

$$= 0.75 \text{ V}$$ ✓

$$W_D = 25 \text{ nm}$$

$$Q_D = -4.1 \times 10^{-7} \text{ C/cm}^2$$

$$Q_D/(-q) = 2.5 \times 10^{12} \ \#/\text{cm}^2$$

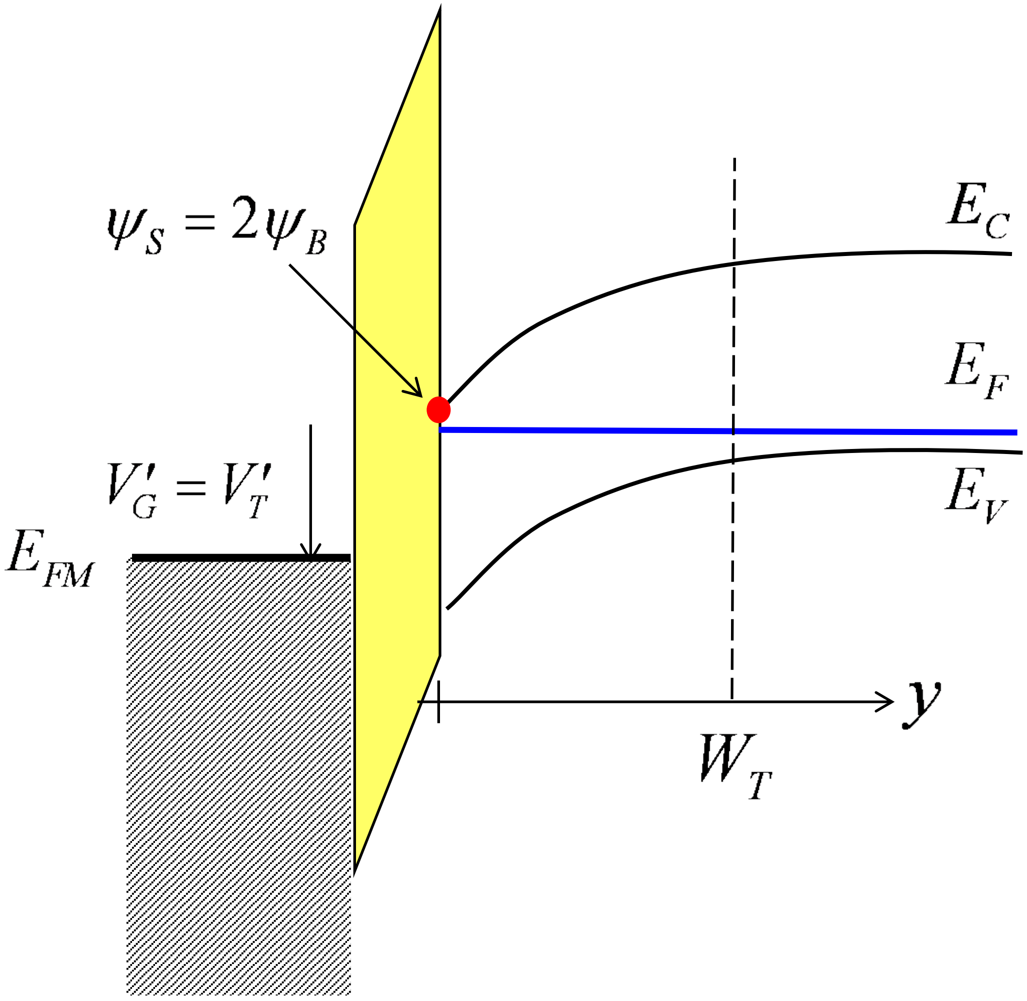
$$\mathcal{E}_S = 3.9 \times 10^5 \text{ V/cm}$$

$$C_{ox} = \frac{\varepsilon_{ox}}{t_{ox}}$$

$$= \frac{(3.9)(8.854 \times 10^{-12})}{2 \times 10^{-9}}$$

$$= 1.73 \times 10^{-2} \text{ F/m}^2$$

Onset of inversion



$$\psi_S = 2\psi_B$$

$$\psi_B = \frac{k_B T}{q} \ln\left(\frac{N_A}{n_i}\right)$$

$$N_A = 10^{18}\ \text{cm}^{-3}$$

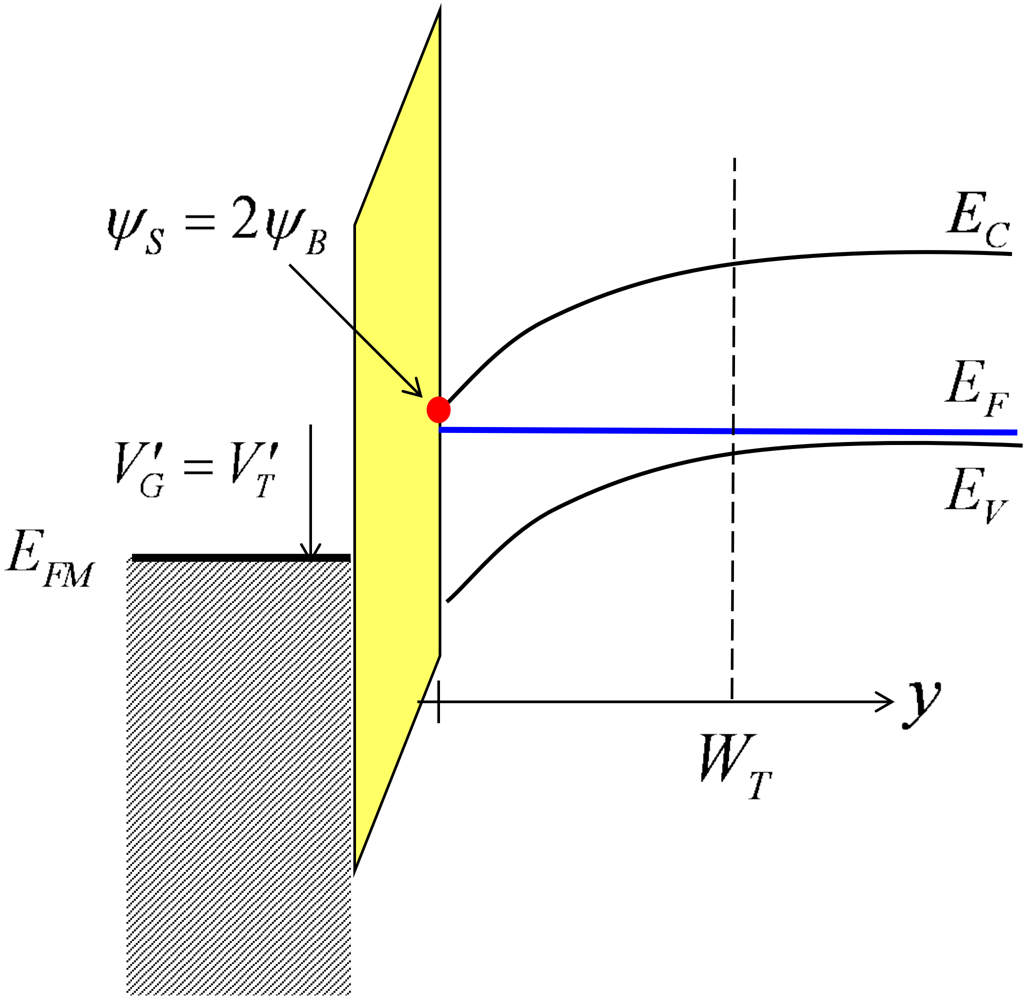
$$n_i = 10^{10}\ \text{cm}^{-3}$$

$$\psi_B = 0.48\ \text{V}$$

$$2\psi_B = 0.96\ \text{V}$$

threshold voltage

Onset of inversion



$$W_T = \sqrt{2\varepsilon_S (2\psi_B)/qN_A}$$

$$Q_S \approx Q_D = -qN_A W_T$$

$$\mathcal{E}_S = \frac{-Q_D}{\varepsilon_{Si}}$$

$$W_T = 35 \text{ nm}$$

$$Q_D = -5.6 \times 10^{-7} \text{ C/cm}^2$$

$$Q_D/q = 3.5 \times 10^{12} \ \#/\text{cm}^2$$

$$\mathcal{E}_S = 5.4 \times 10^{5} \text{ V/cm}$$

Threshold voltage

$$V_G' = -\frac{Q_S(\psi_S)}{C_{ox}} + \psi_S$$

$$V_T' = -\frac{Q_D(2\psi_B)}{C_{ox}} + \psi_S$$

$$V_T' = \frac{5.6 \times 10^{-7}}{1.73 \times 10^{-6}} + 0.96$$
$$= 0.32 + 0.96$$
$$= 1.28 \text{ V} \quad ✓$$

$$2\psi_B = 0.96 \text{ V}$$

$$W_D = 35 \text{ nm}$$

$$Q_D = -5.6 \times 10^{-7} \text{ C/cm}^2$$

$$C_{ox} = 1.73 \times 10^{-2} \text{ F/m}^2$$

Wrap up

1) The gate voltage induces charge in the semiconductor by bending the bands.

2) There is a simple (exact) relation between the gate voltage and the surface potential, but must be solved numerically.

$$V_G' = -\frac{Q_S(\psi_S)}{C_{ox}} + \psi_S$$

3) There is an approximate relation between gate voltage and surface potential that works well in depletion.

$$V_G' = -\frac{Q_D(\psi_S)}{C_{ox}} + \psi_S \qquad \psi_S = \frac{V_G}{m}$$

4) **Next:** Discuss realistic MOS-Capacitors