Fundamentals of Nanotransistors

Unit 2: MOS Electrostatics

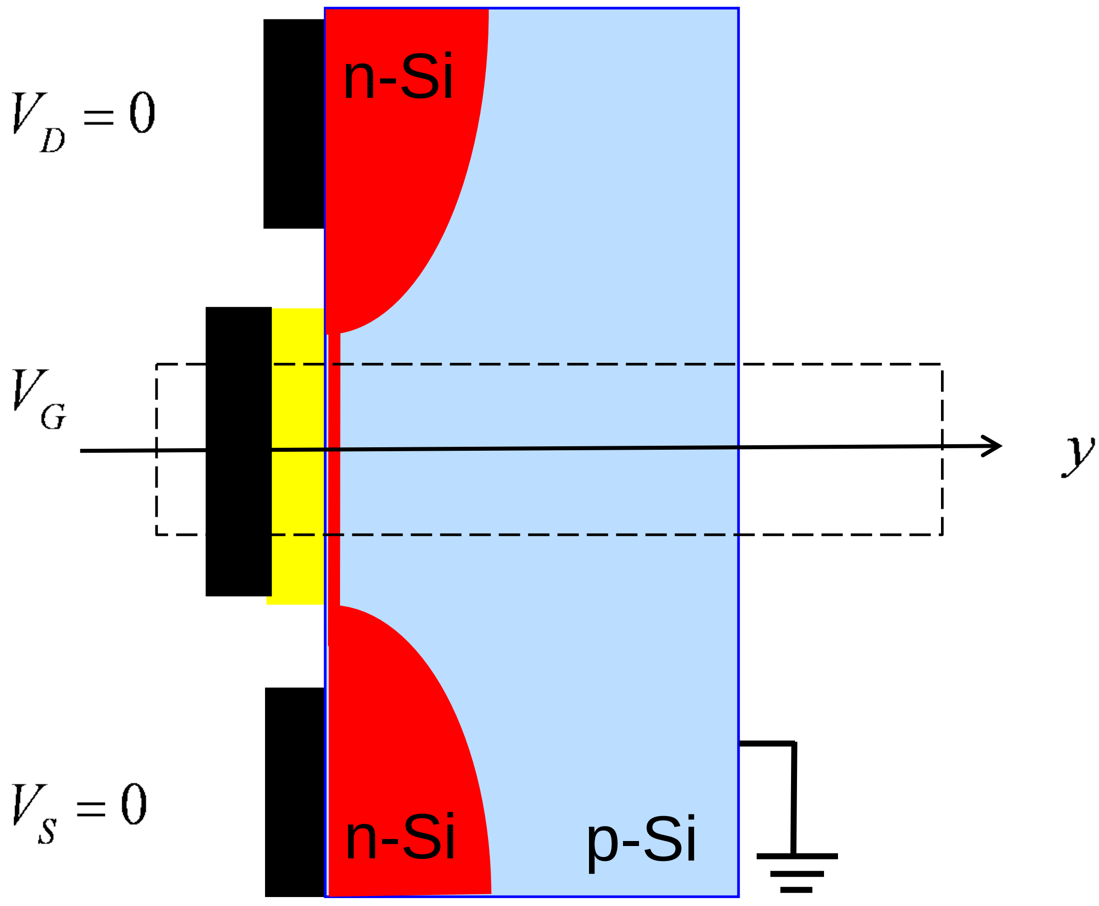
Lecture 2.2: Depletion Approximation

Mark Lundstrom

lundstro@purdue.edu
Electrical and Computer Engineering
Birck Nanotechnology Center
Purdue University, West Lafayette, Indiana USA

PURDUE UNIVERSITY

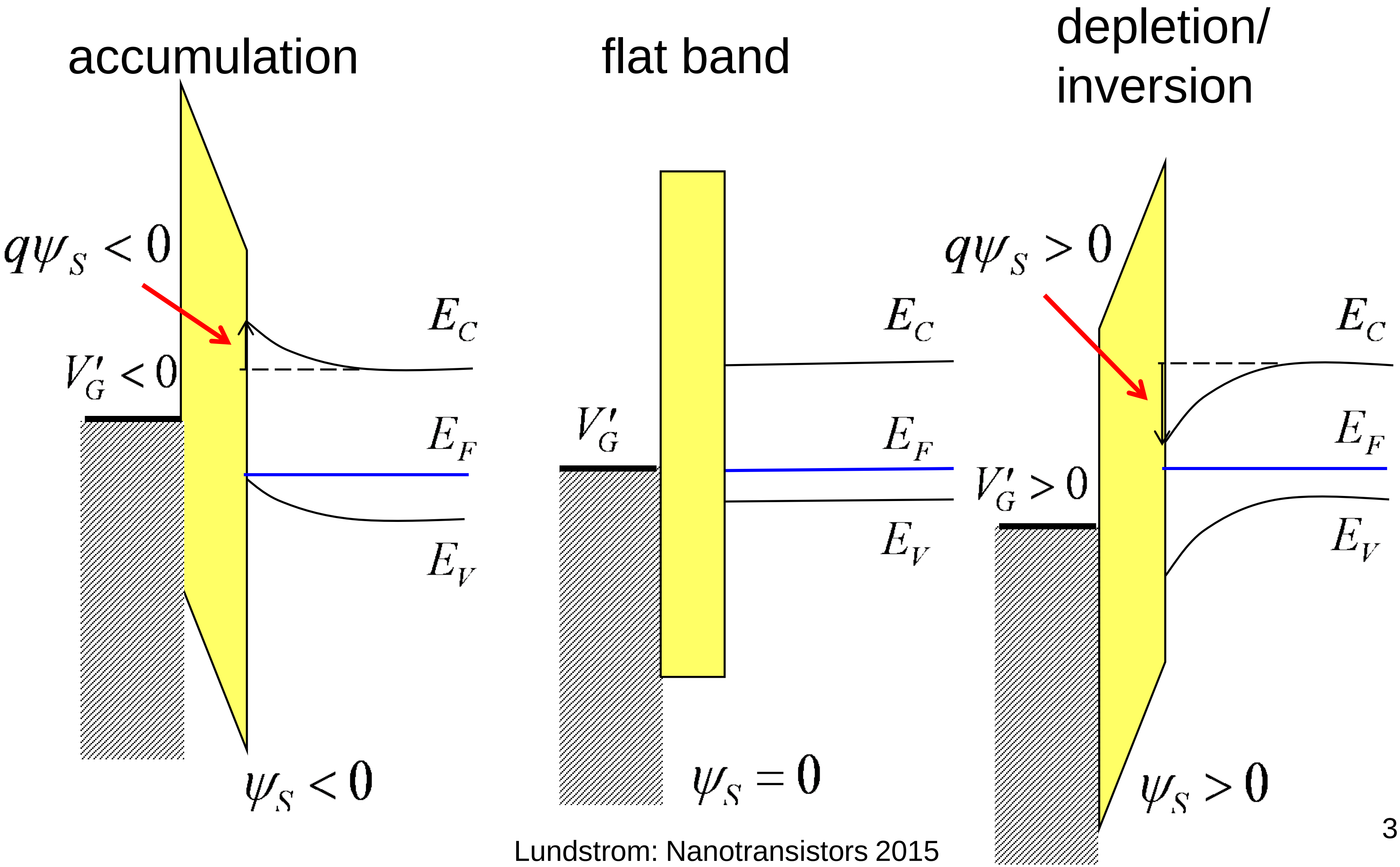
MOS capacitor



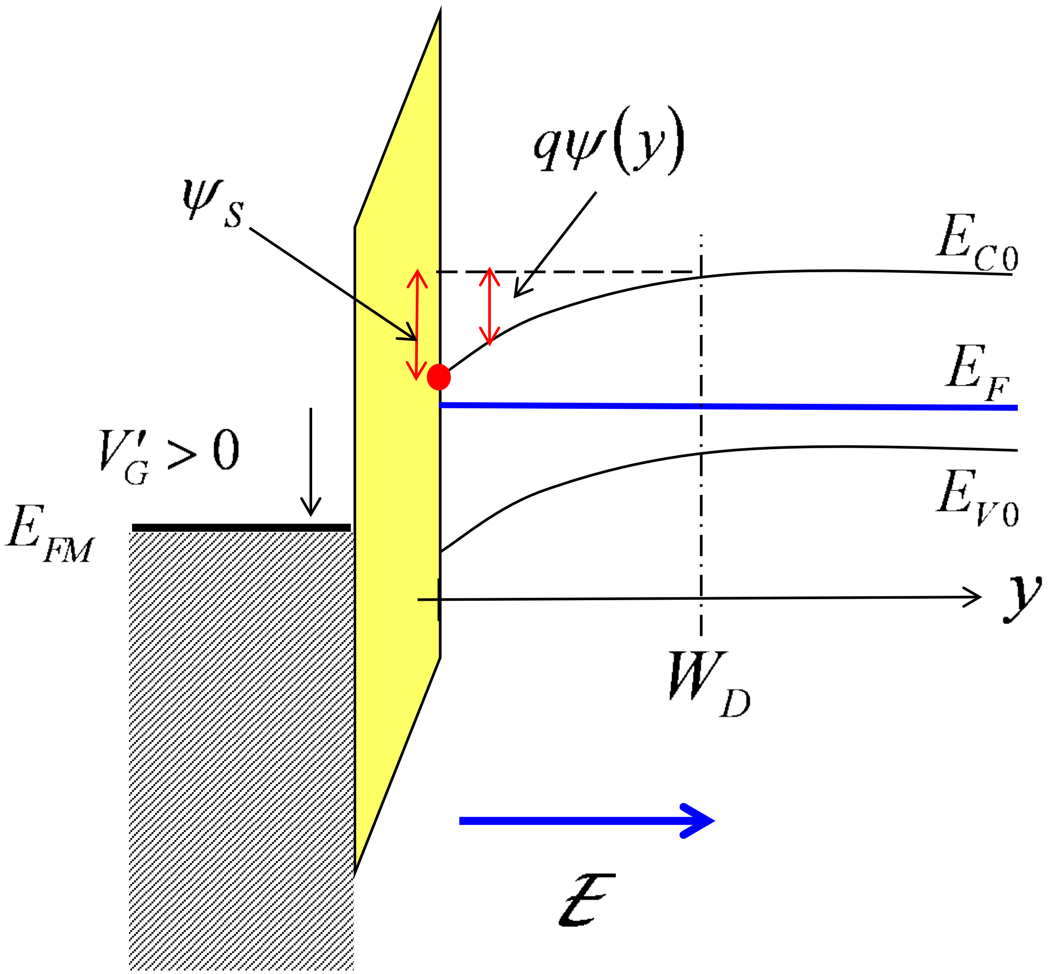
$$E_C(y) = E_{C0} - q\psi(y)$$

$$E_V(y) = E_{V0} - q\psi(y)$$

1D MOS electrostatics



Depletion



$$0 < \psi_S < 2\psi_B$$

$$\rho(y) = q\left[p(y) - n(y) - N_A^-\right]$$

$y < W_D$:

$$p(y) << N_A^-$$

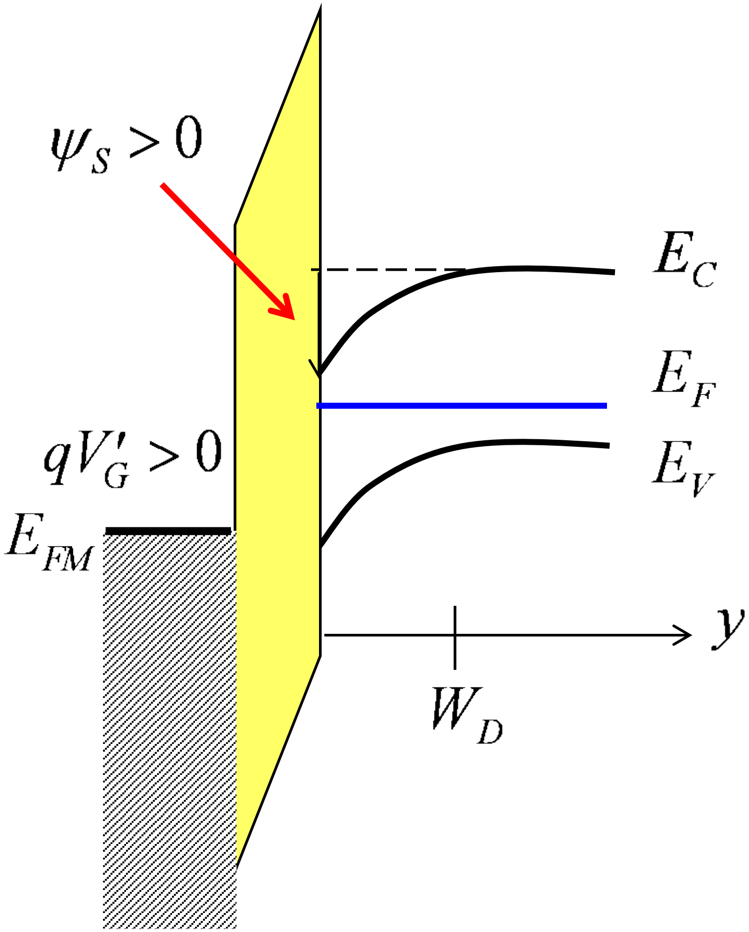
$$n(y) << N_A^-$$

$$\rho(y) \approx -qN_A$$

$y \geq W_D$:

$$\rho(y) \approx 0$$

Poisson equation

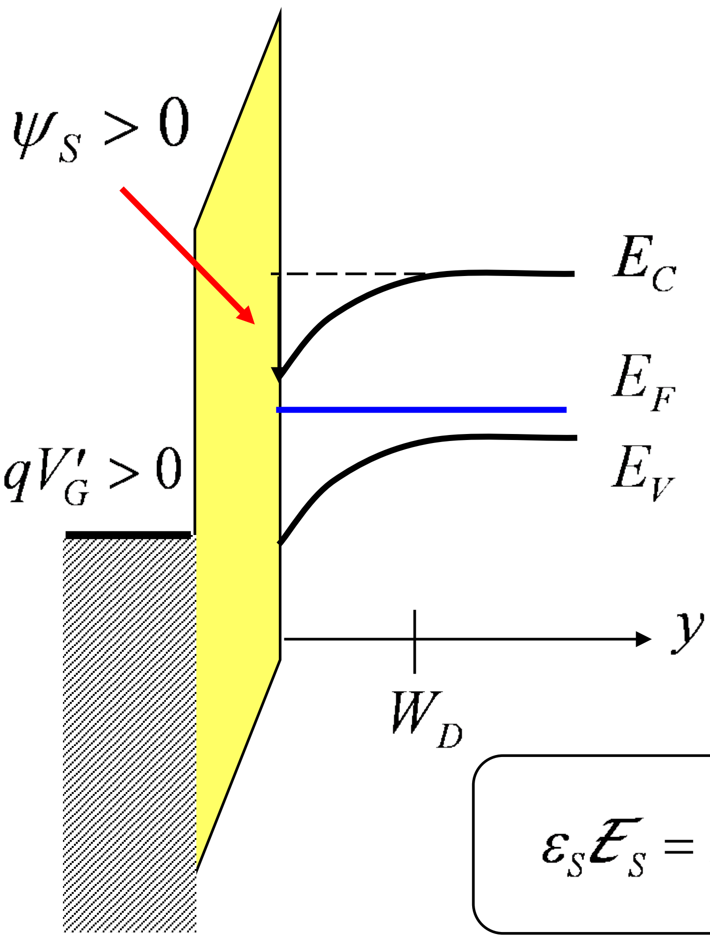


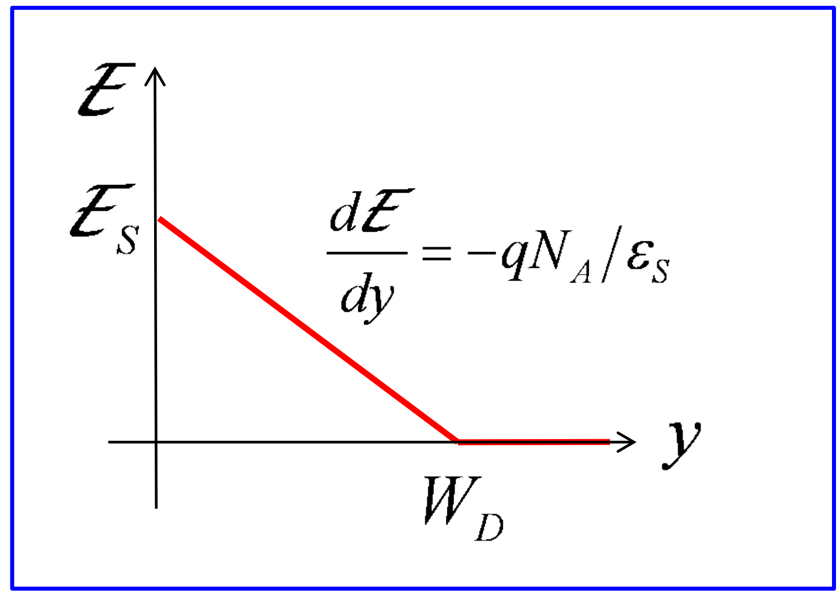
$$\frac{dD(y)}{dy} = \rho(y)$$

$$\frac{d\mathcal{E}}{dy} = \frac{\rho(y)}{\varepsilon_S} = -\frac{qN_A}{\varepsilon_S}$$

$$\mathcal{E}(y) = -\frac{d\psi(y)}{dy}$$

Electric field



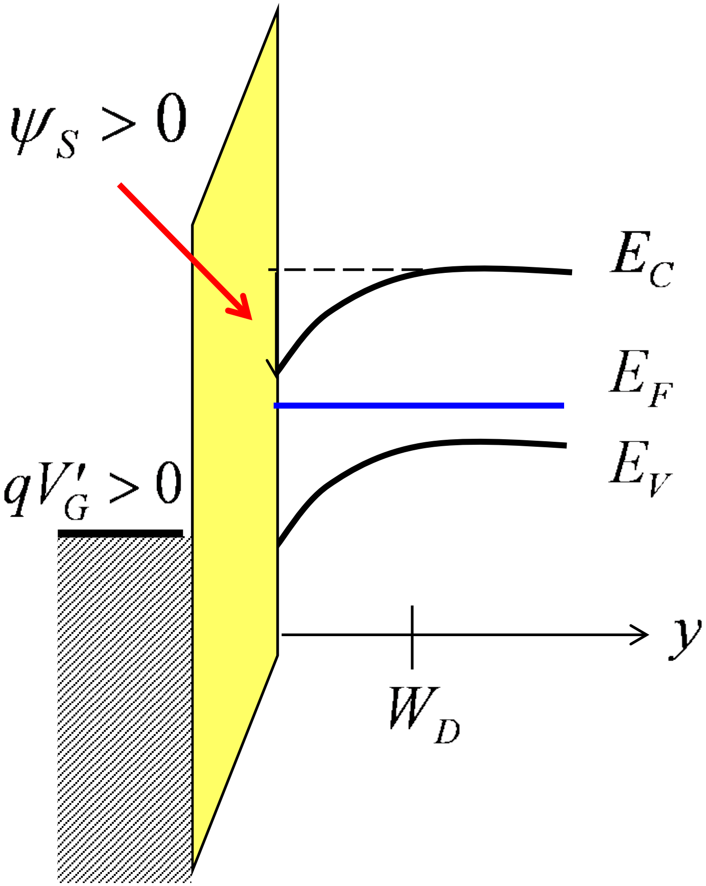


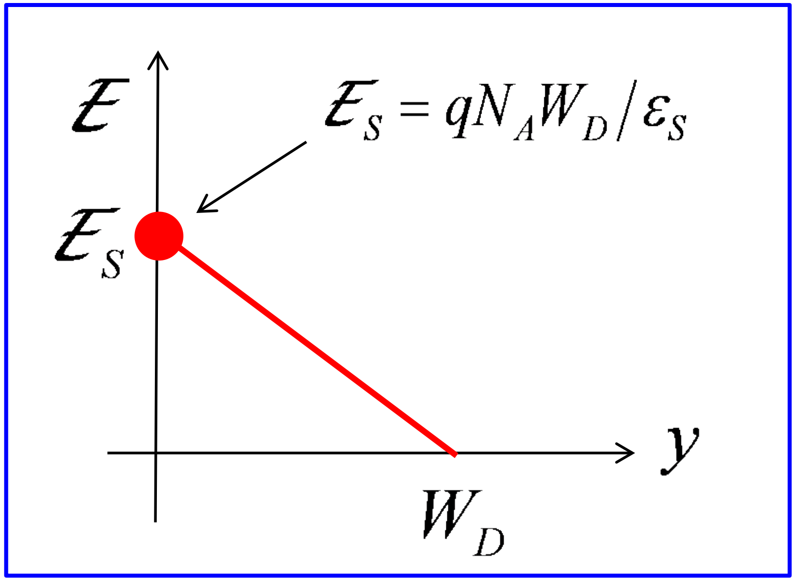
$$\varepsilon_S \mathcal{E}_S = D_S = -Q_S$$

$$\mathcal{E}(y) = \frac{qN_A}{\varepsilon_S}(W_D - y)$$

$$\mathcal{E}_S = \frac{qN_A W_D}{\varepsilon_S} = \frac{-Q_S}{\varepsilon_S}$$

Electrostatic potential



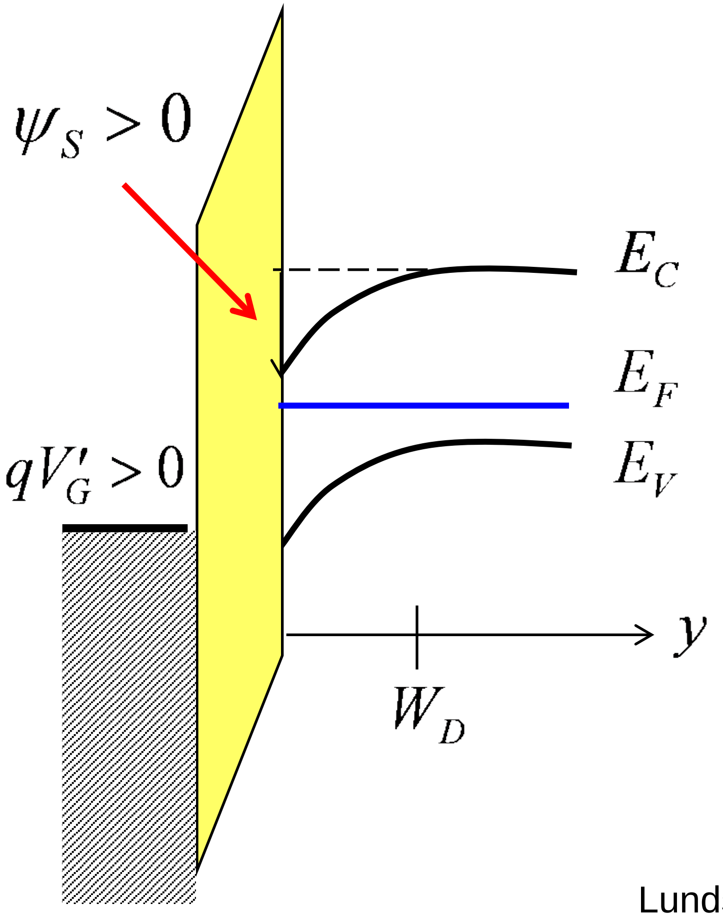


$$\psi(y) = -\int \mathcal{E}(y)\,dy$$

$$\psi_S = \frac{1}{2}\mathcal{E}_S W_D$$

$$W_D = \sqrt{2\varepsilon_S \psi_S / qN_A}$$

Depletion charge per cm^2



$$W_D = \sqrt{2\varepsilon_S \psi_S / qN_A}$$

$$Q_D = -qN_A W_D = -\sqrt{2qN_A \varepsilon_S \psi_S} \ \ \text{C/cm}^2$$

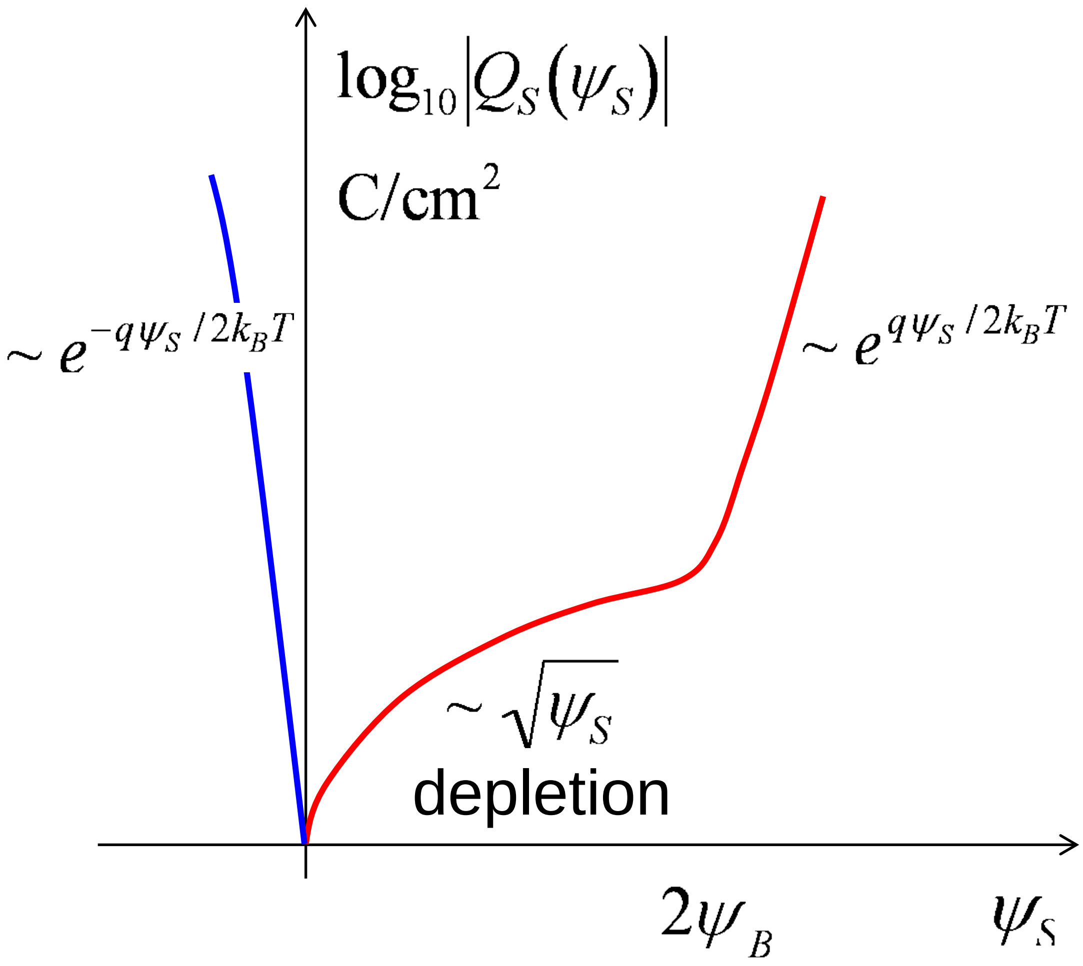
(depletion charge)

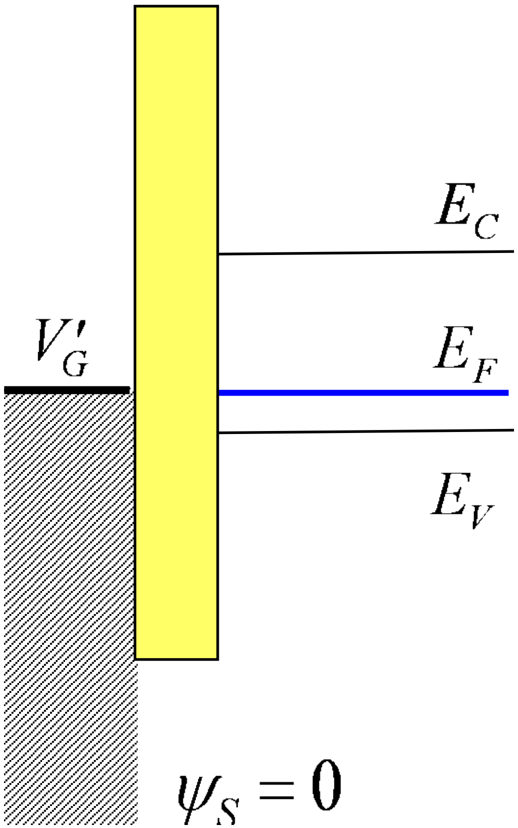
$$Q_S = Q_D + Q_n \ \ \text{C/cm}^2$$

(total charge in semiconductor)

$$|Q_S| \approx |Q_D| \sim \sqrt{\psi_S}$$

MOS electrostatics





Example

P-type Si doped at: $N_A = 10^{18}\ \text{cm}^{-3}$

$T = 300\ \text{K}$ $\qquad n_i(300\ \text{K}) = 10^{10}\ \text{cm}^{-3}$ $\qquad \kappa_{Si} = 11.8$

$\psi_S = 0.5\ \text{V}$ $\qquad k_B T/q = 0.026\ \text{V}$

Find:

i) the electric field at the surface

ii) the width of the depletion layer

Example

P-type Si doped at: $N_A = 10^{18}\ \text{cm}^{-3}$

$T = 300\ \text{K}$ $\quad n_i(300\ \text{K}) = 10^{10}\ \text{cm}^{-3}$

$\psi_S = 0.5\ \text{V}$

1) Check to see if we are in depletion or inversion.

$$\psi_S < 2\psi_B ?$$

$$\psi_B = \frac{k_B T}{q}\ln\left(\frac{N_A}{n_i}\right) = 0.026\ln\left(\frac{10^{18}}{10^{10}}\right) = 0.48\ \text{V}$$

$\psi_S < 2\psi_B ?$

$0.5 < 0.96\ \text{V}$

depletion ✓

Depletion layer thickness

P-type Si doped at: $N_A = 10^{18}\ \text{cm}^{-3}$

$T = 300\ \text{K}$

$n_i(300\ \text{K}) = 10^{10}\ \text{cm}^{-3}$

$\kappa_{Si} = 11.8$

$\psi_S = 0.5\ \text{V}$

$k_B T/q = 0.026\ \text{V}$

$$W_D = \sqrt{2\varepsilon_S \psi_S / qN_A}$$

$$W_D = \sqrt{2(11.8)\left(8.854\times10^{-12}\right)(0.5)\Big/\left[\left(1.6\times10^{-19}\right)10^{24}\right]}$$

$$W_D = 25.6\ \text{nm}$$

$$W_D = 25.6\times10^{-9}\ \text{m}$$

Electric field at the surface

P-type Si doped at: $N_A = 10^{18}\ \text{cm}^{-3}$

$T = 300\ \text{K}$

$n_i(300\ \text{K}) = 10^{10}\ \text{cm}^{-3}$

$\kappa_{Si} = 11.8$

$\psi_S = 0.5\ \text{V}$

$k_B T/q = 0.026\ \text{V}$

$$\psi_S = \frac{1}{2}\mathcal{E}_S W_D$$

$$\mathcal{E}_S = \frac{2\psi_S}{W_D} = \frac{2(0.5)}{25.6\times10^{-9}}$$

$$\mathcal{E}_S = 3.9\times10^{5}\ \frac{\text{V}}{\text{cm}}$$

$$\mathcal{E}_S = \frac{2\psi_S}{W_D}$$

$$\mathcal{E}_S = 3.9\times10^{7}\ \frac{\text{V}}{\text{m}}$$

$$\mathcal{E}_S = 390\ \frac{\text{kV}}{\text{cm}}$$

Lecture 2.2 Wrap-up

Given a surface potential, we can compute the electric field and depletion layer thickness (if we are in depletion), but what gate voltage produced this surface potential?

That is the subject of the next lecture.