Nanoscale Transistors: Physics and Technology

Unit 1: Transistor Fundamentals

Quick Review of Semiconductor Fundamentals

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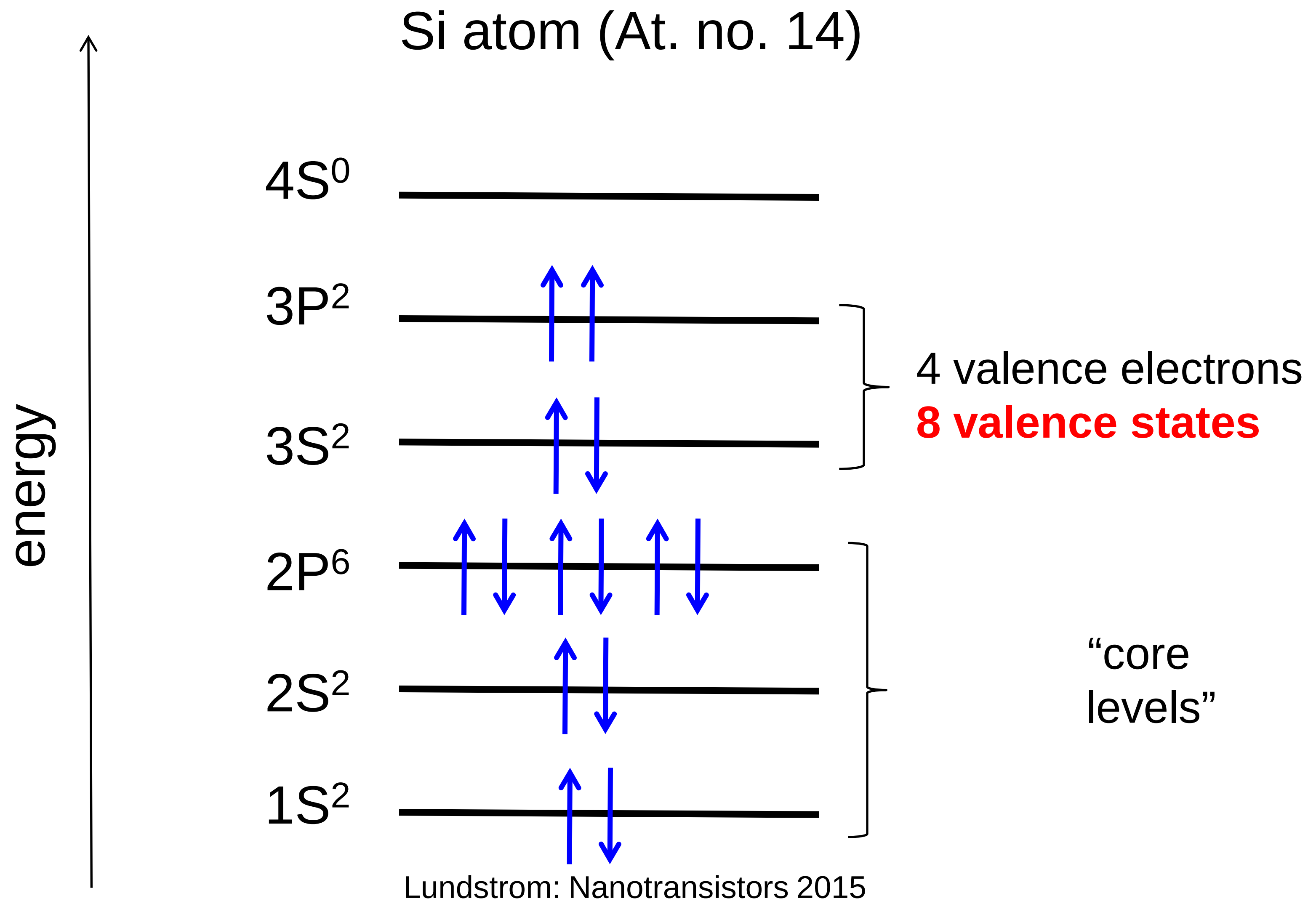
Essentials of semiconductor physics

1) Energy bands
2) Doping
3) Fermi function and Fermi level
4) Carrier densities
5) Drift-diffusion equation
6) Energy band diagrams
7) Quasi-Fermi levels

Silicon energy levels



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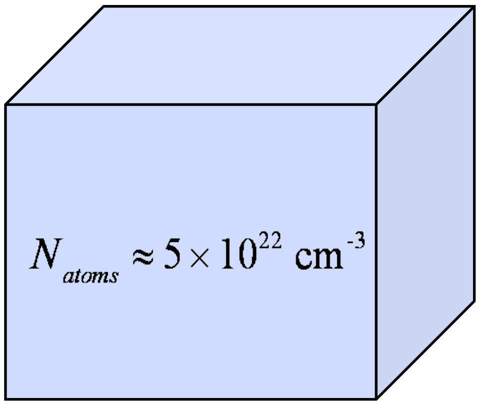
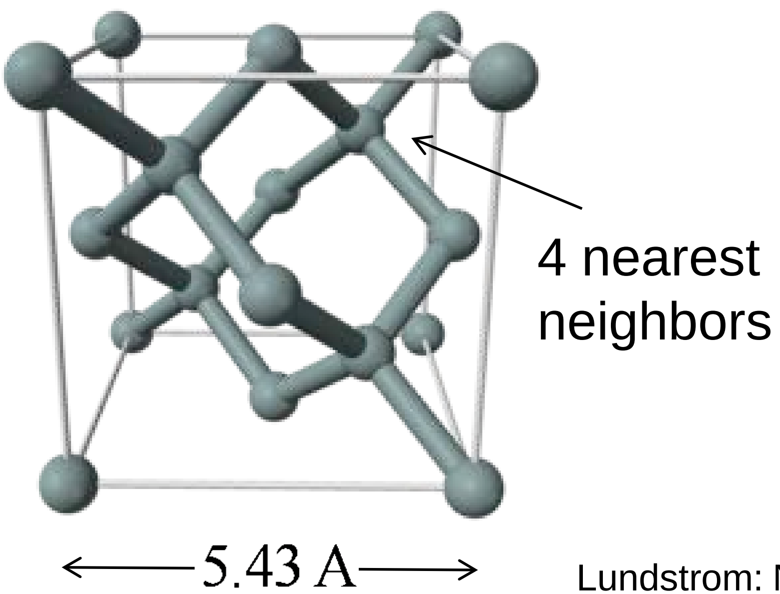
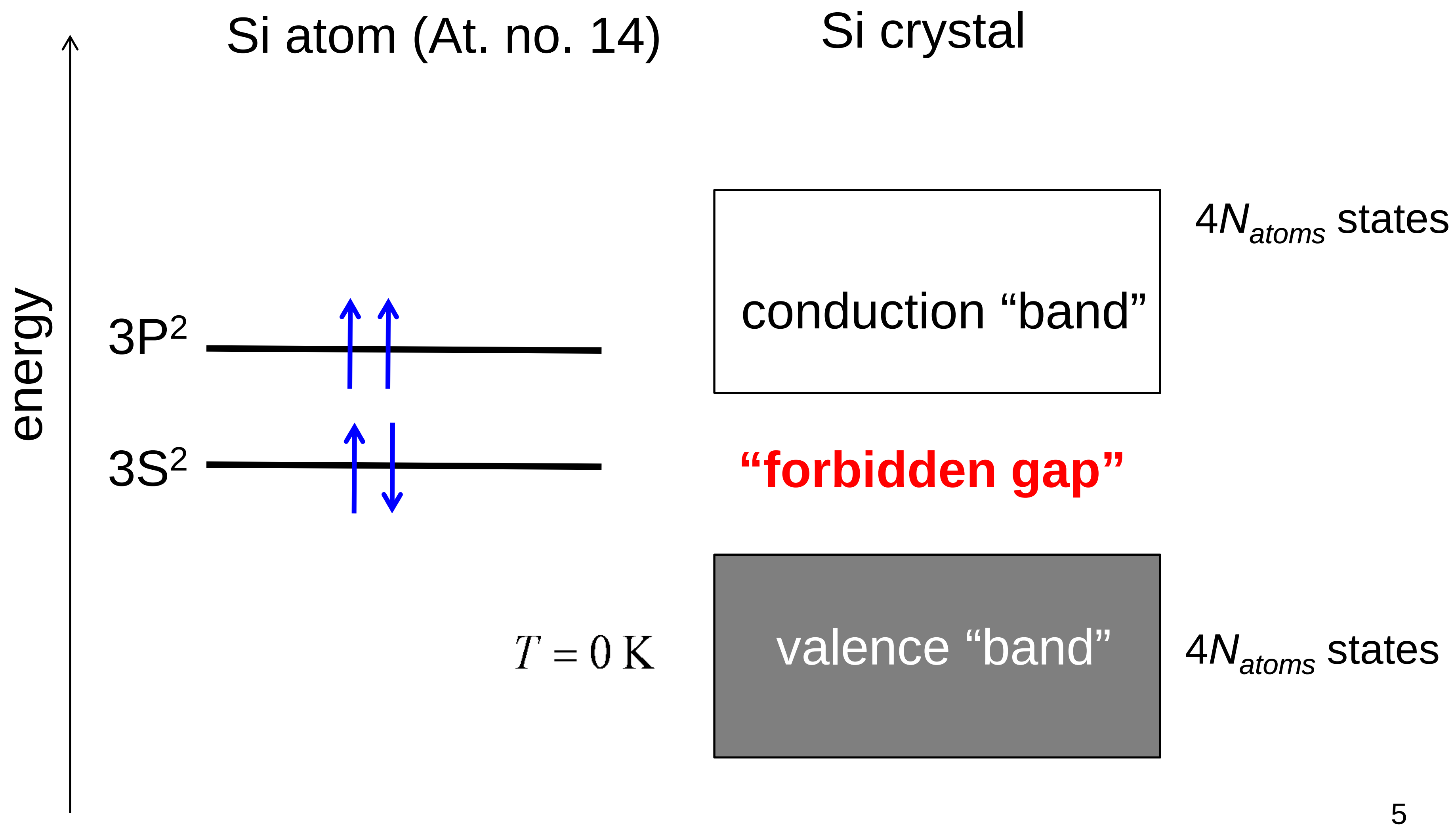
Silicon energy levels / energy bands





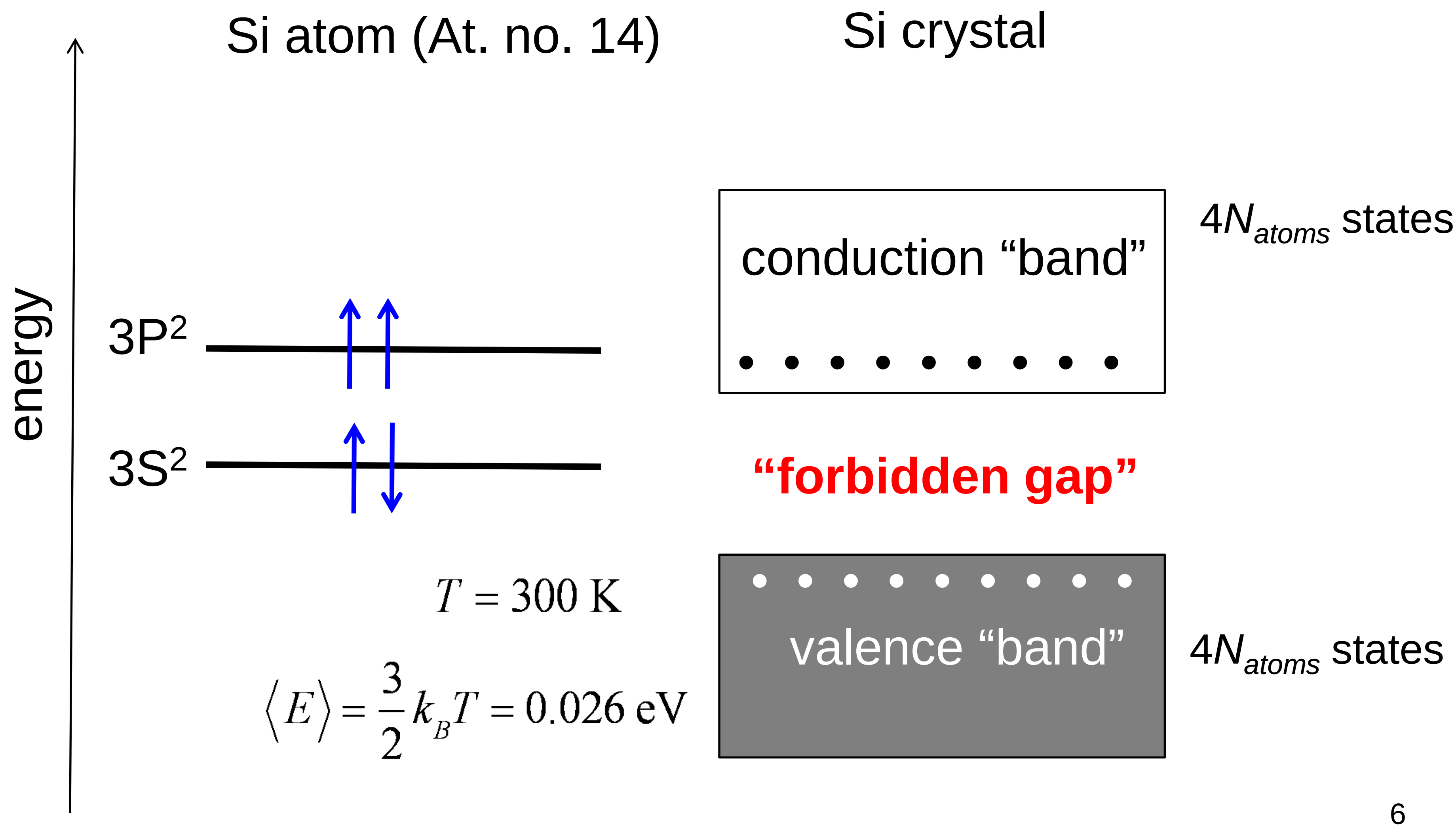
- Only the valence states are of interest to us.
- The 8 valence states give rise to $8N_{atoms}$ states per cm^3 in the solid.
- But the **interaction** of the electron wavefunctions alters the discrete energy levels of the isolated Si atoms.

Silicon energy levels → energy bands



$T = 0\ \mathrm{K}$

Silicon energy levels → energy bands

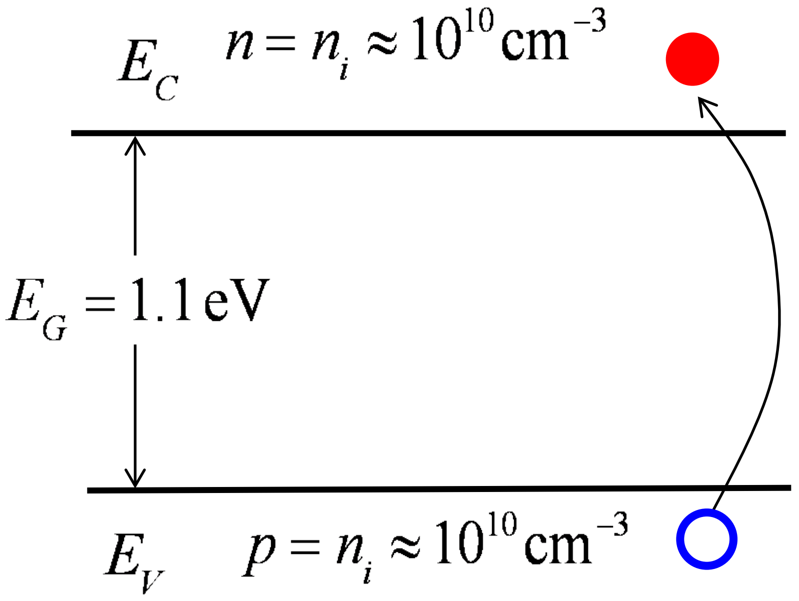


$$T = 300\ \mathrm{K}$$

$$\langle E \rangle = \frac{3}{2} k_B T = 0.026\ \mathrm{eV}$$

"Energy band diagrams"

Intrinsic Si



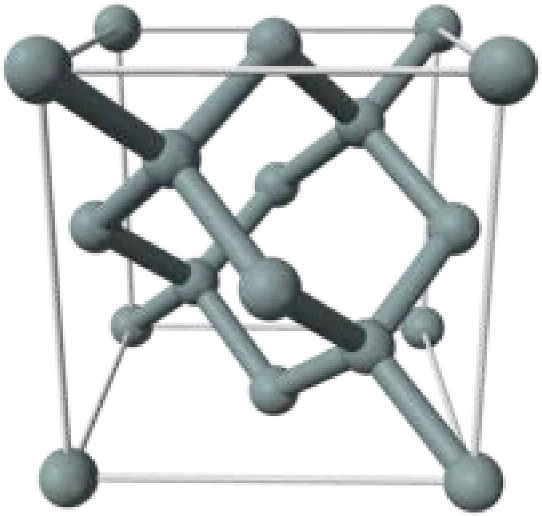
$$k_B T = 0.026\ \mathrm{eV} \quad (T = 300\ \mathrm{K})$$

$$P \sim e^{-E_G/k_B T}$$

$$n = p = n_i$$

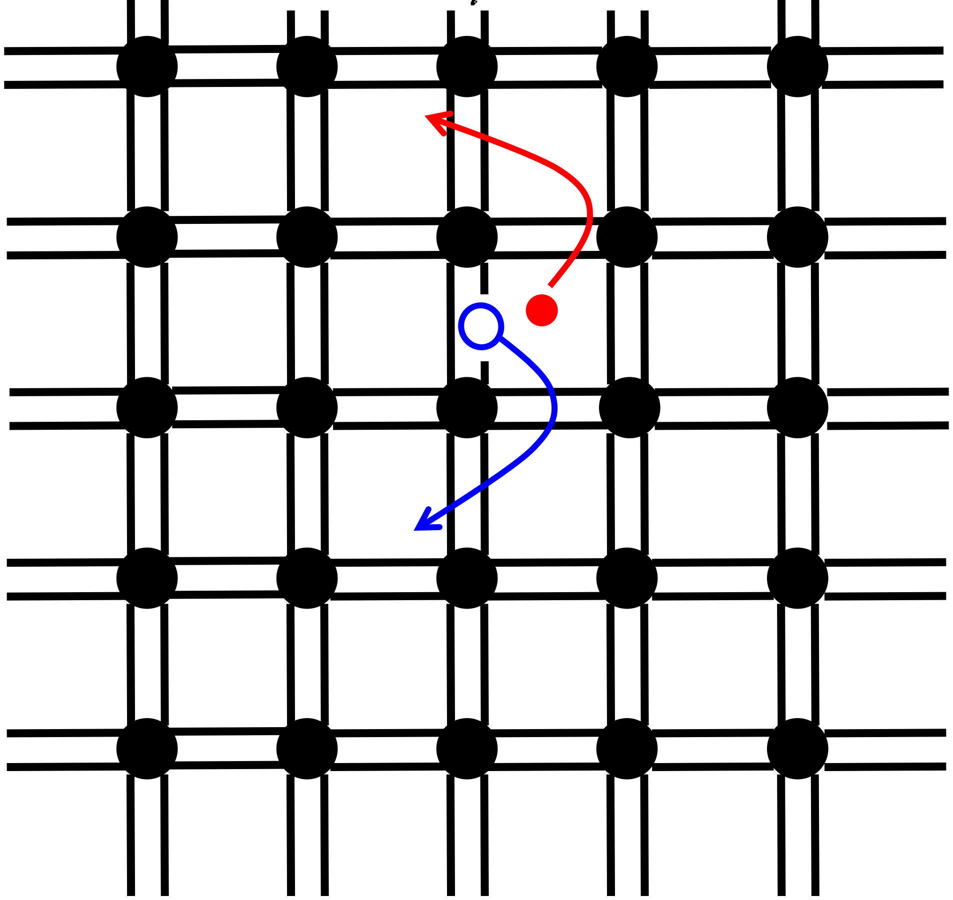
Intrinsic silicon

Si crystal structure



4 nearest neighbors

$$n = p = n_i \propto e^{-E_G/k_B T}$$



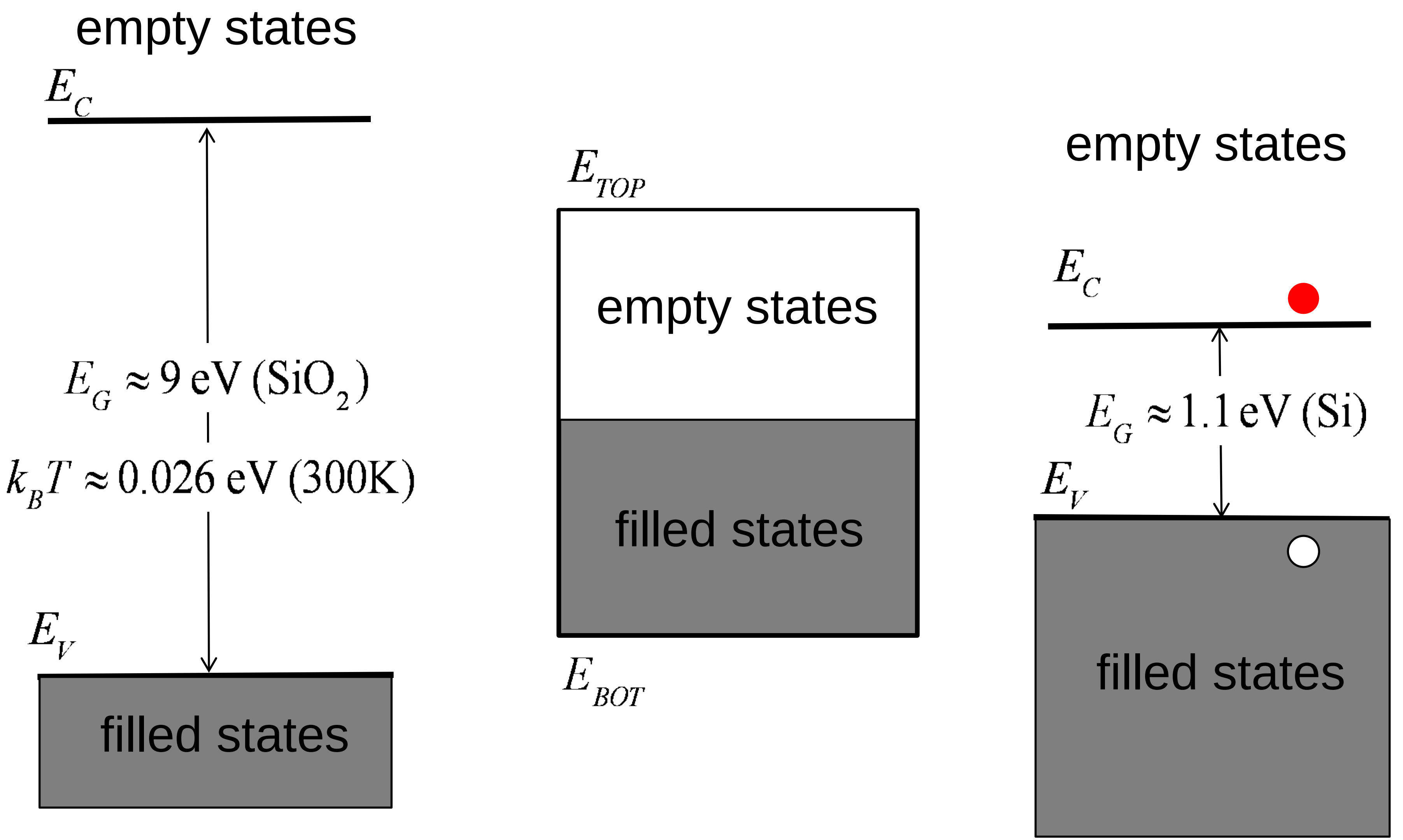
Metals insulators and semiconductors

insulators: don’t conduct electricity well
usually don’t conduct heat well

metals: conduct electricity (and heat) well.

semiconductors: in-between, **but**
their properties can be controlled

insulators metals semiconductors



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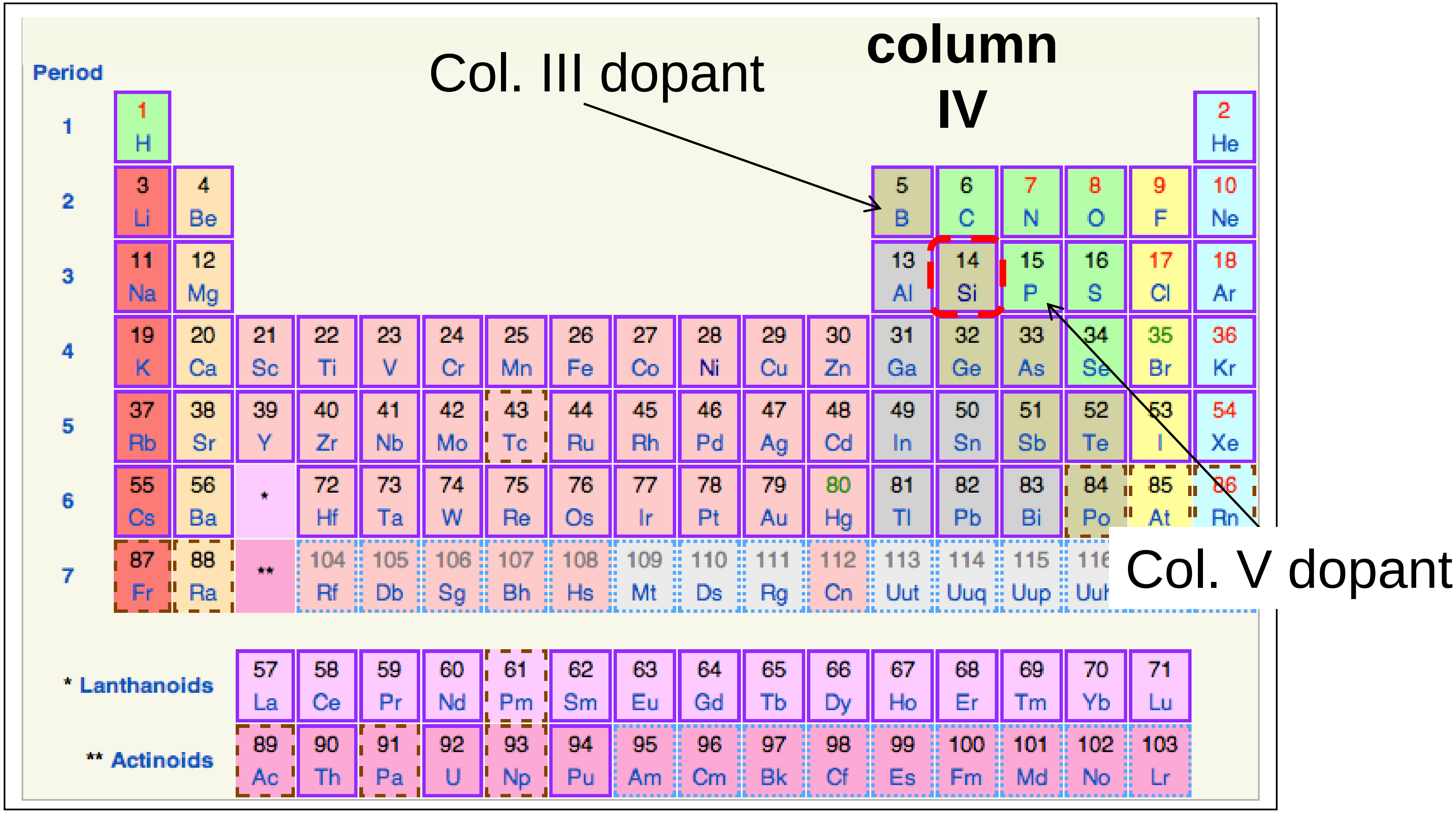
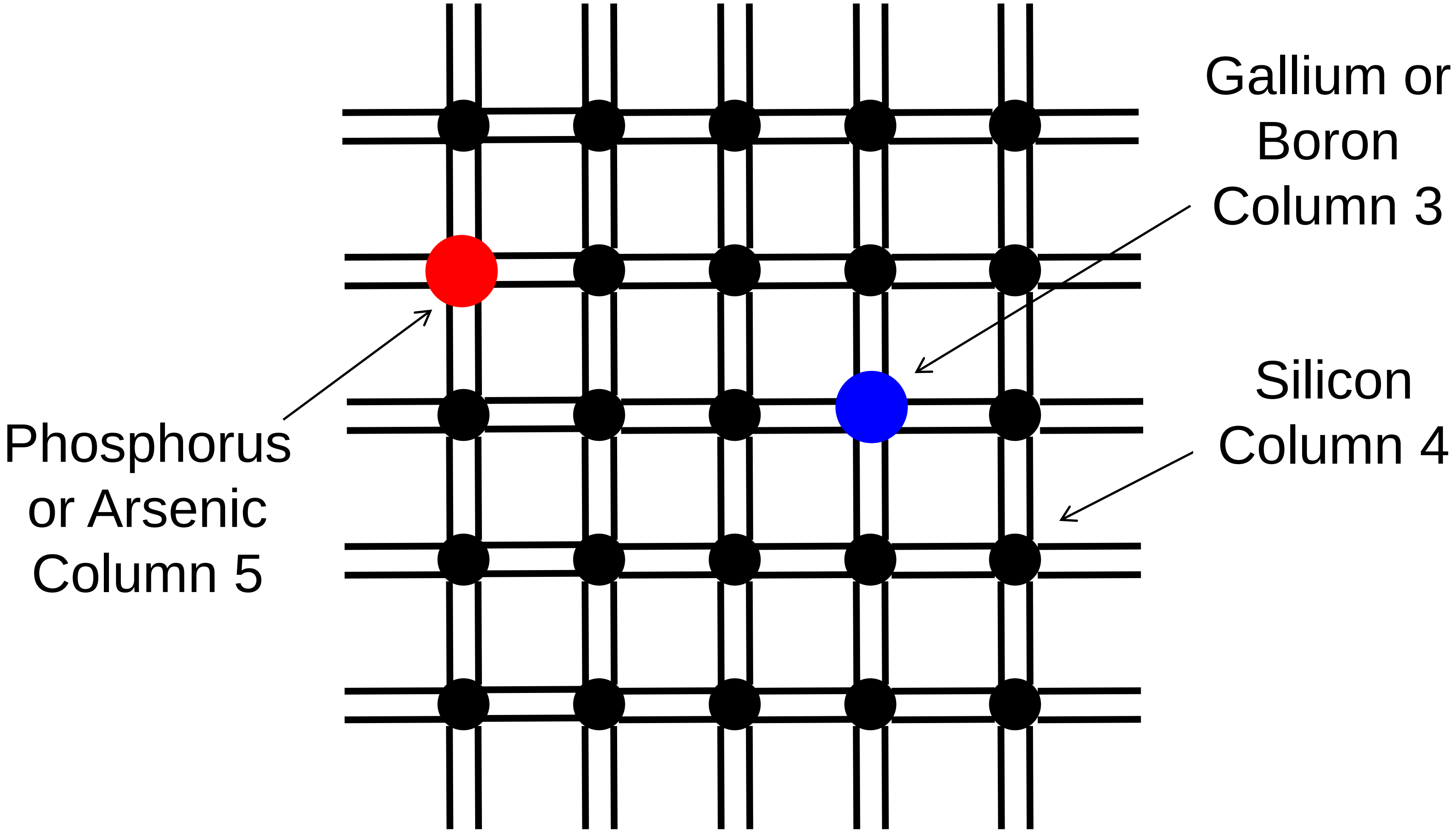
Covalent (column IV) semiconductors

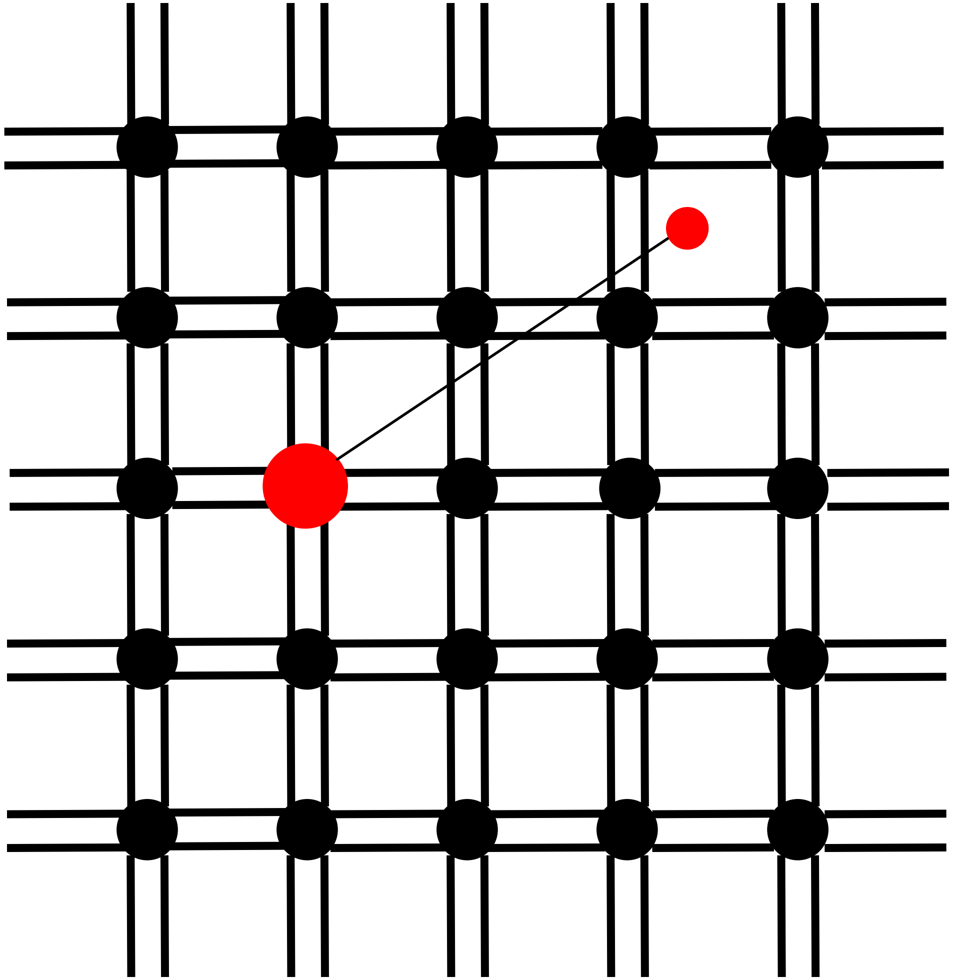


http://en.wikipedia.org/wiki/Periodic_table

Doping


Gallium or
Boron
Column 3
Silicon
Column 4
Phosphorus
or Arsenic
Column 5

N-type doping (donors)



$$E_H = -\frac{13.6}{n^2}\,\text{eV} \qquad n = 1,2,3,\ldots$$

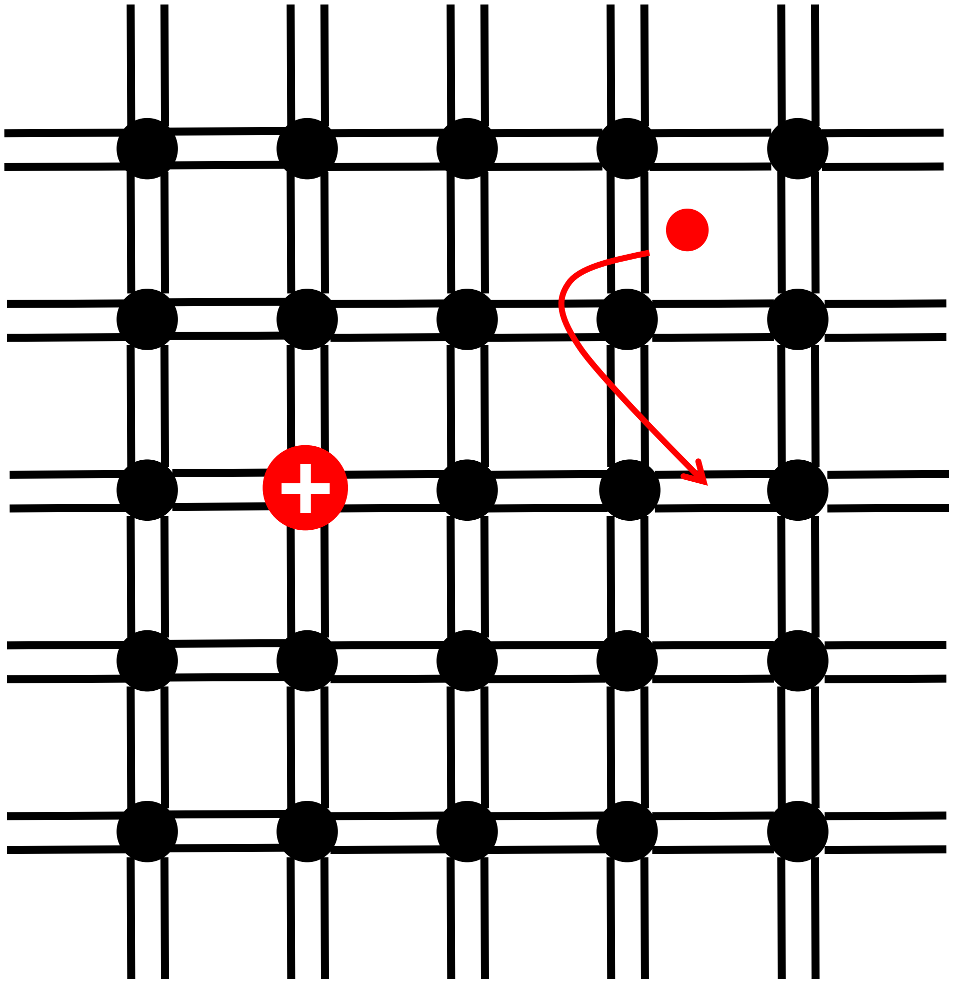
$$E_D = -\frac{m_0 q^4}{2\left(4\pi\varepsilon_S \hbar n\right)^2}\,\text{eV}$$

$$E_D < 0.05\,\text{eV}$$

Weakly bound

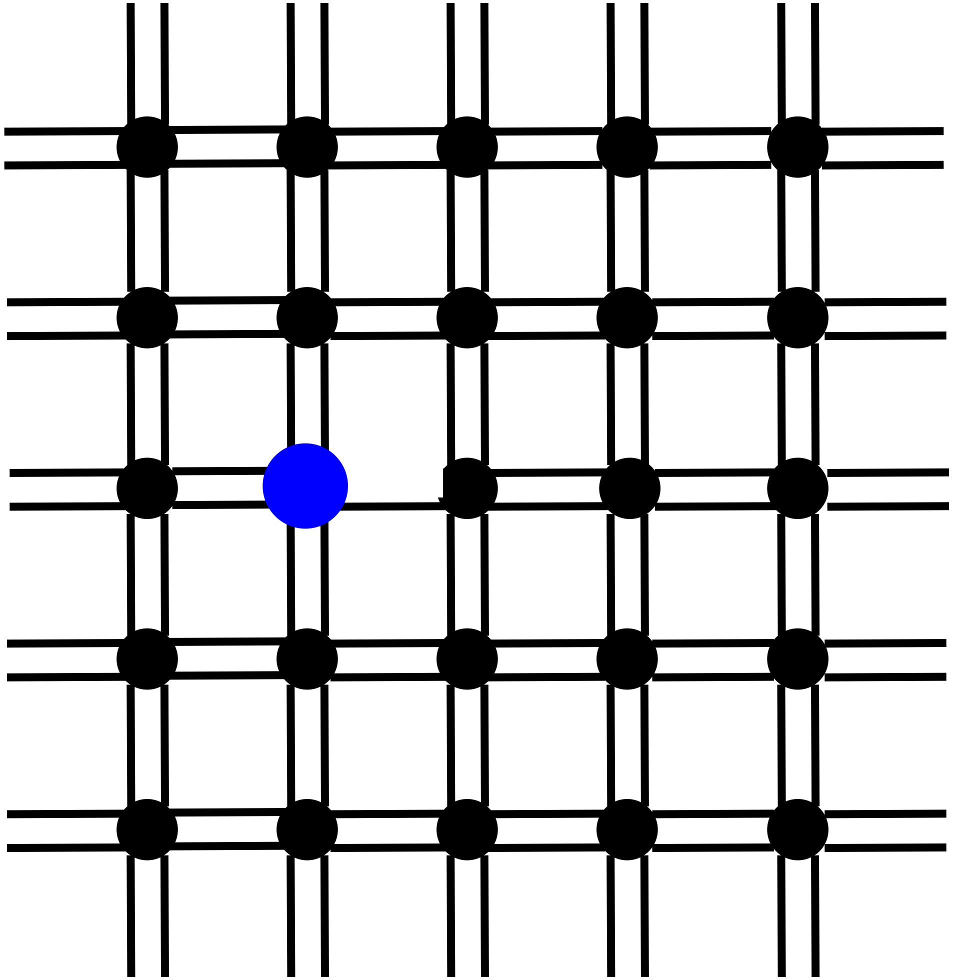
Easily broken at room temperature

Ionized donors

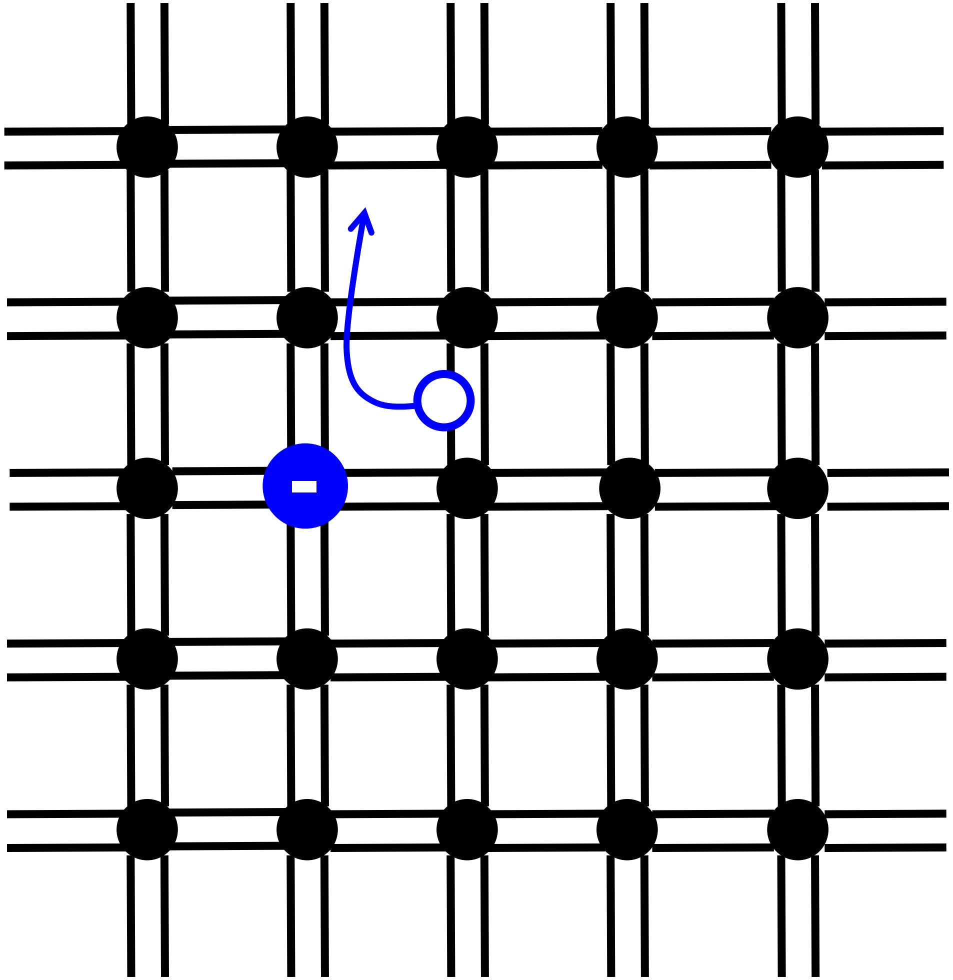


$$n = N_D^+ \approx N_D$$

P-type doping (acceptors)

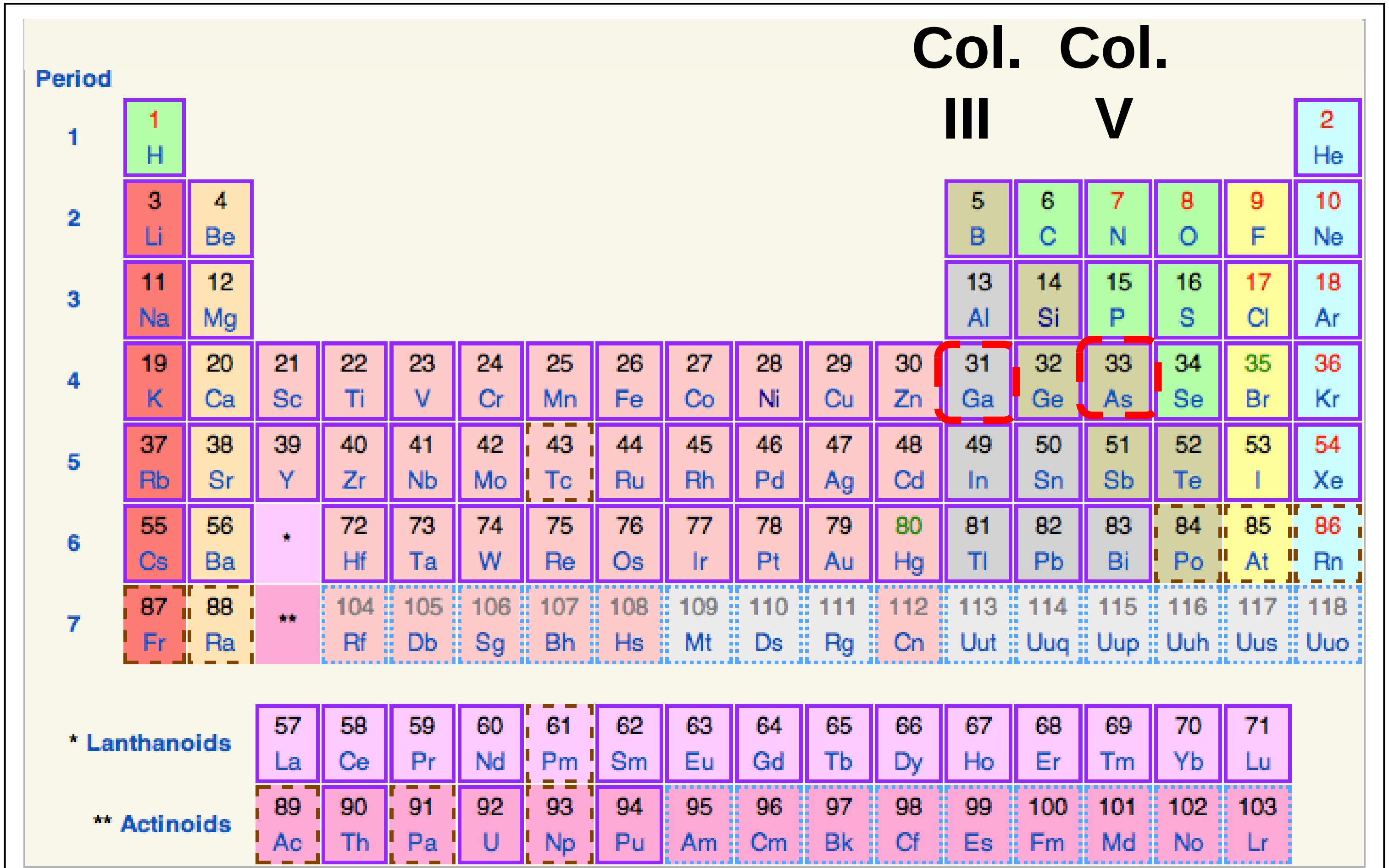


Ionized acceptors



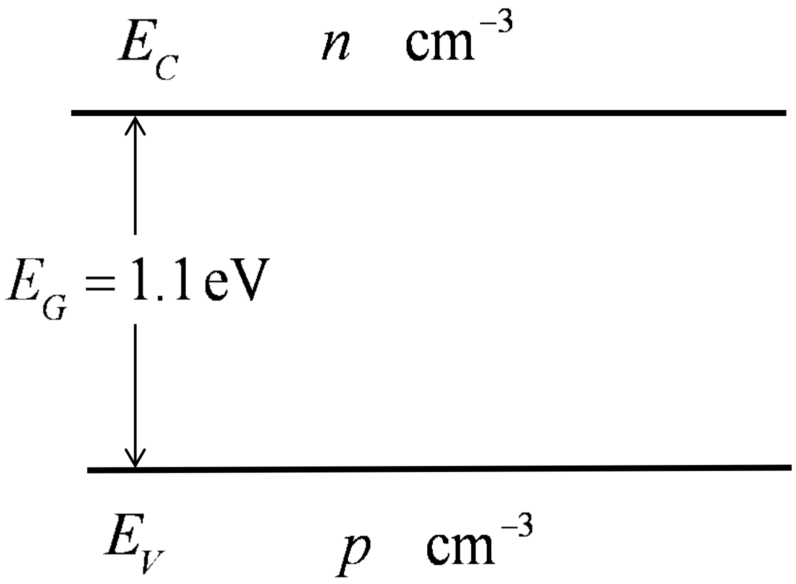
$$p = N_A^- \approx N_A$$

III-V semiconductors



http://en.wikipedia.org/wiki/Periodic_table

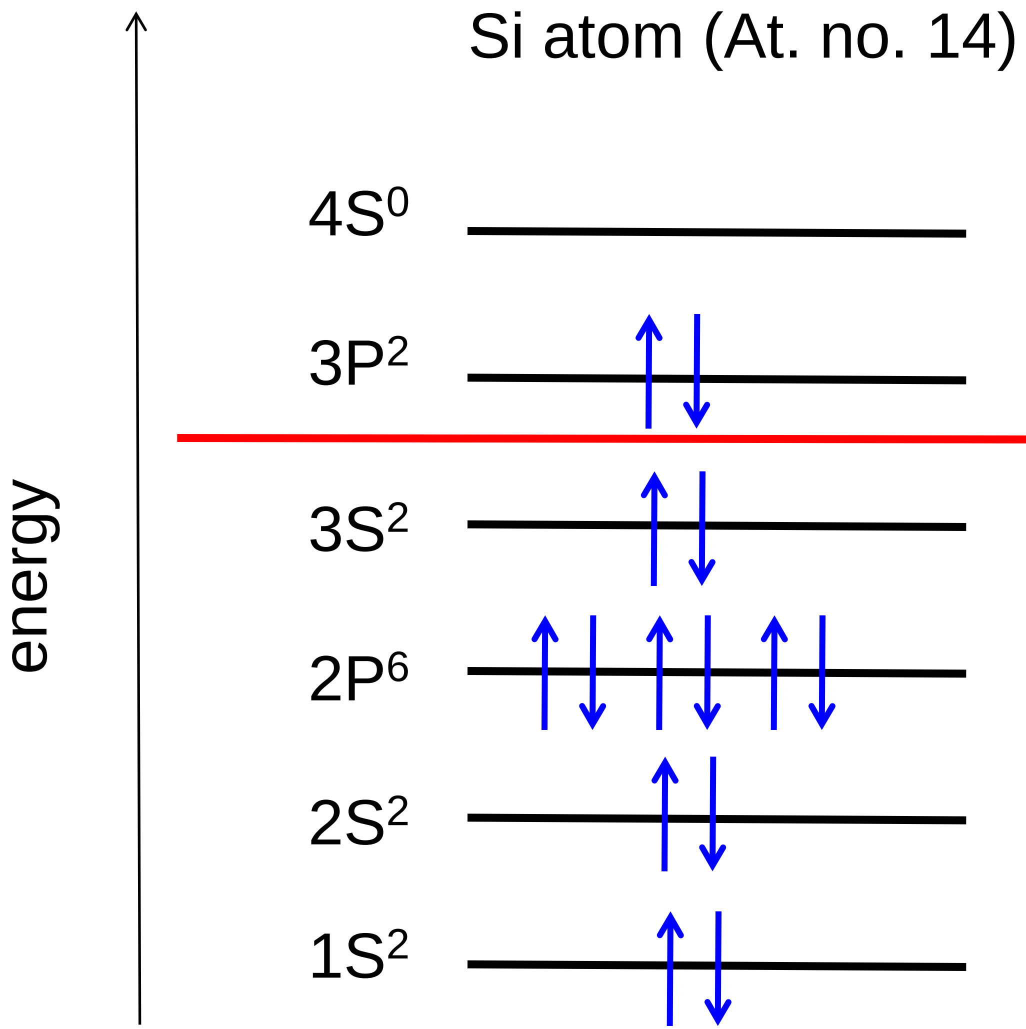
Filling states



What is the probability that states in the conduction band are **filled**?

What is the probability that states in the valence band are **empty**?

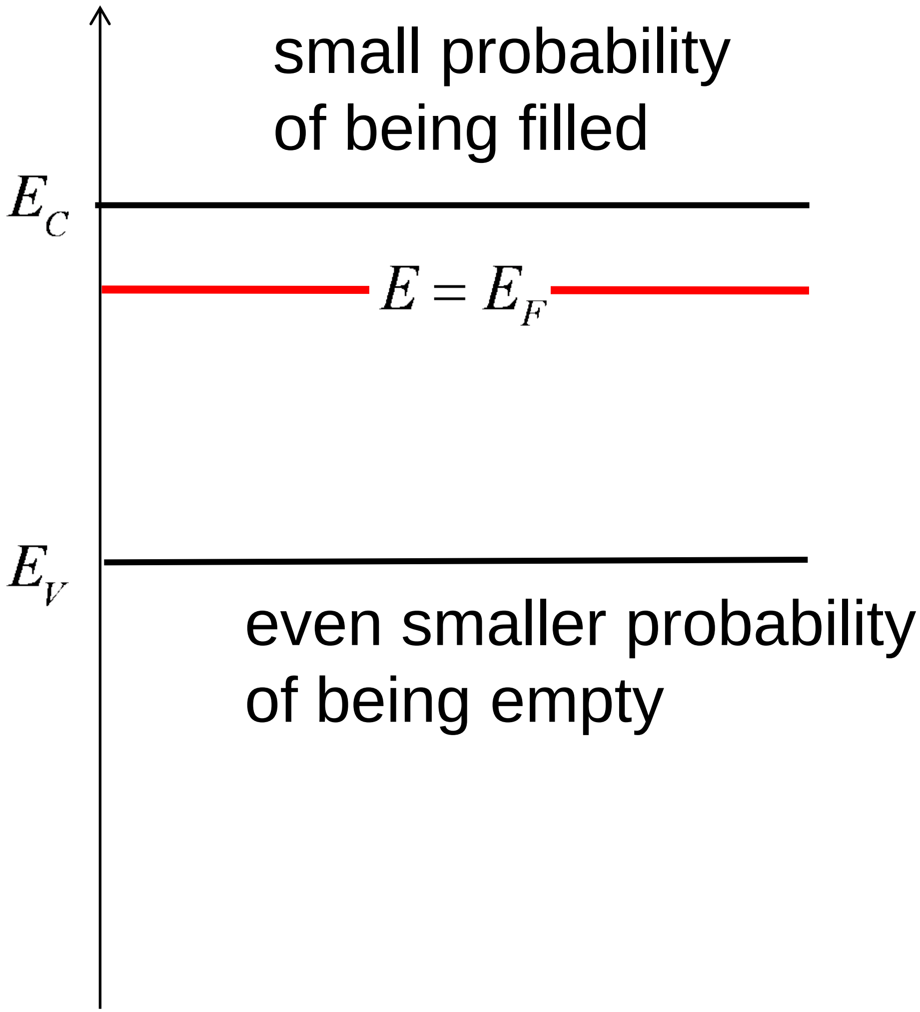
Occupation of states



States way above have very little probability of being occupied.

States below this energy have a high probability of being occupied.

Fermi level / Fermi function

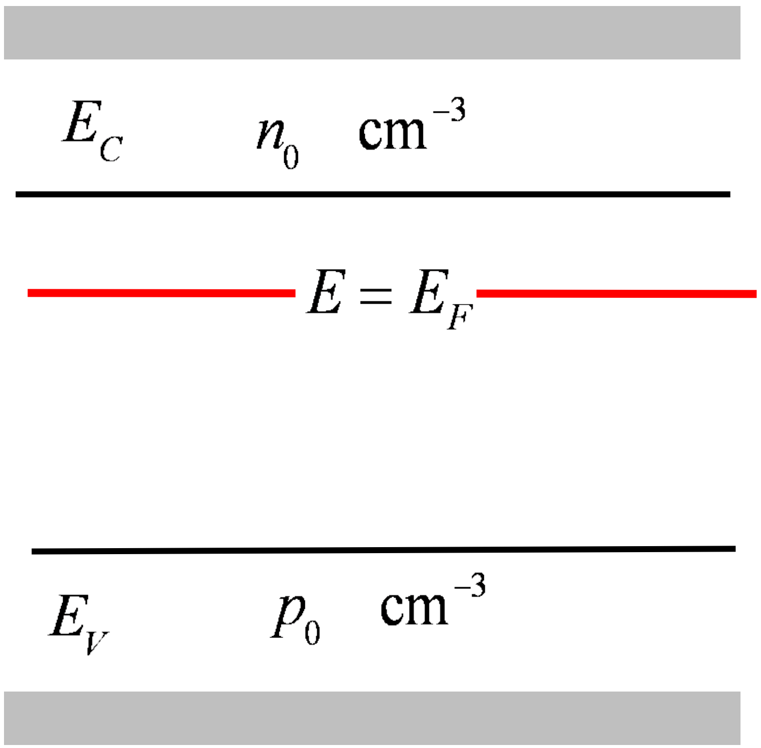


$$f_0(E) = \frac{1}{1 + e^{(E - E_F)/k_B T}}$$

(Fermi function)

(E_F, Fermi level or electrochemical potential)

Carrier densities and density-of-states



$$n_0(E)dE = D_C(E)dE\, f_0(E)$$

$$n_0 = \int_{E_C}^{E_C^{top}} n_0(E)dE = \int_{E_C}^{E_C^{top}} D_C(E) f_0(E, E_F)dE$$

$$p_0 = \int_{E_V^{bot}}^{E_V} p_0(E)dE = \int_{E_V^{bot}}^{E_V} D_V(E)\left[1 - f_0(E, E_F)\right]dE$$

$$p_0(E)dE = D_V(E)dE\left[1 - f_0(E)\right]$$

Carrier densities and Fermi level

$$n_0 = N_C \mathcal{F}_{1/2}\left\{\left(E_F - E_C\right)/k_B T\right\} \text{ cm}^{-3}$$

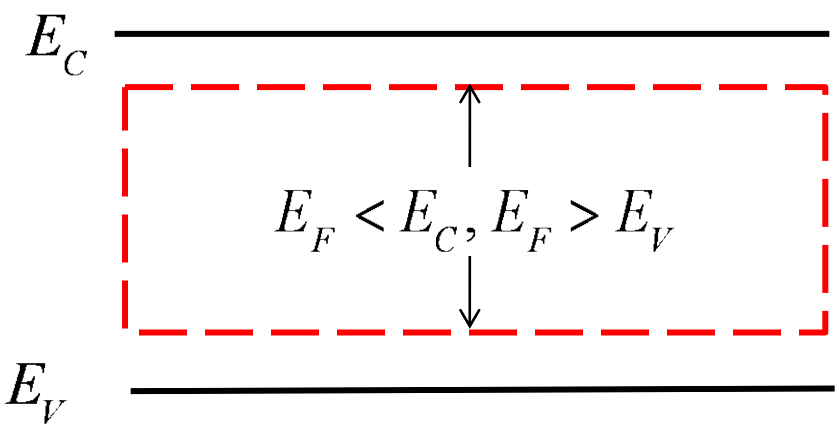
$$p_0 = N_V \mathcal{F}_{1/2}\left\{\left(E_V - E_F\right)/k_B T\right\} \text{ cm}^{-3}$$

N_C, N_V: "effective DOS"

Raseong Kim and Mark Lundstrom, "Notes on Fermi-Dirac Integrals, 3rd Ed.," https://nanohub.org/resources/5475.

Non-degenerate semiconductors

$$n_0 = N_C \mathcal{F}_{1/2}\left\{\left(E_F - E_C\right)/k_B T\right\} \text{cm}^{-3}$$



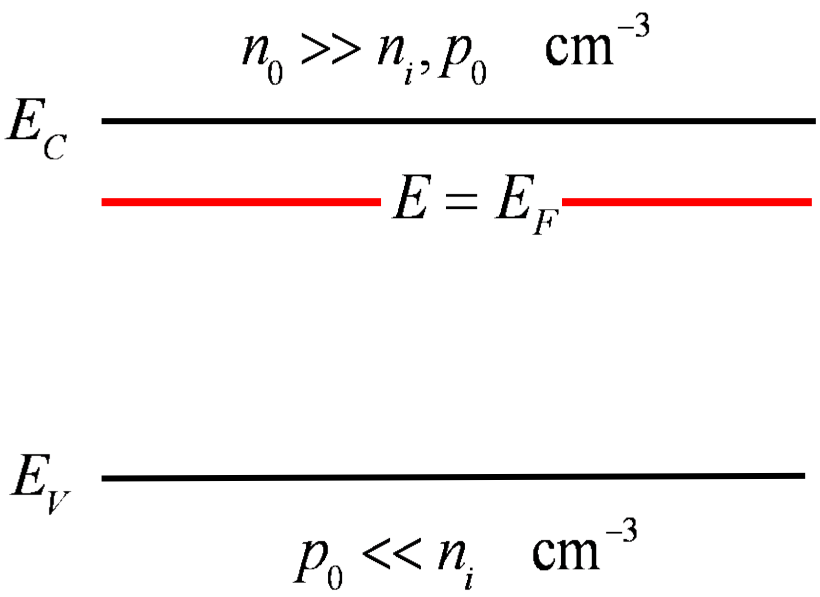
$$n_0 = N_C \exp\left\{\left(E_F - E_C\right)/k_B T\right\} \text{cm}^{-3}$$

$$p_0 = N_V \exp\left\{\left(E_V - E_F\right)/k_B T\right\} \text{cm}^{-3}$$

$$n_0 p_0 = n_i^2 = N_C N_V e^{-E_G/k_B T}$$

$$p_0 = N_V \mathcal{F}_{1/2}\left\{\left(E_V - E_F\right)/k_B T\right\} \text{cm}^{-3}$$

N-type semiconductor



$$n_0 = N_D \text{ cm}^{-3}$$

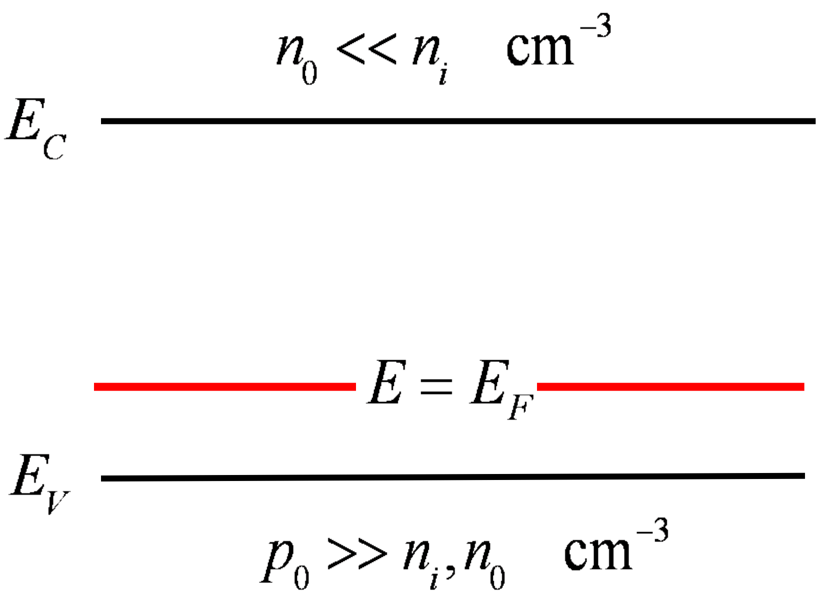
$$n_0 = N_C \exp\left\{\left(E_F - E_C\right)/k_B T\right\} \text{cm}^{-3}$$

$$p_0 = N_V \exp\left\{\left(E_V - E_F\right)/k_B T\right\} \text{cm}^{-3}$$

$$n_0 p_0 = n_i^2$$

$$p_0 = \frac{n_i^2}{n_0}$$

P-type semiconductor



$$p_0 = N_A \ \text{cm}^{-3}$$

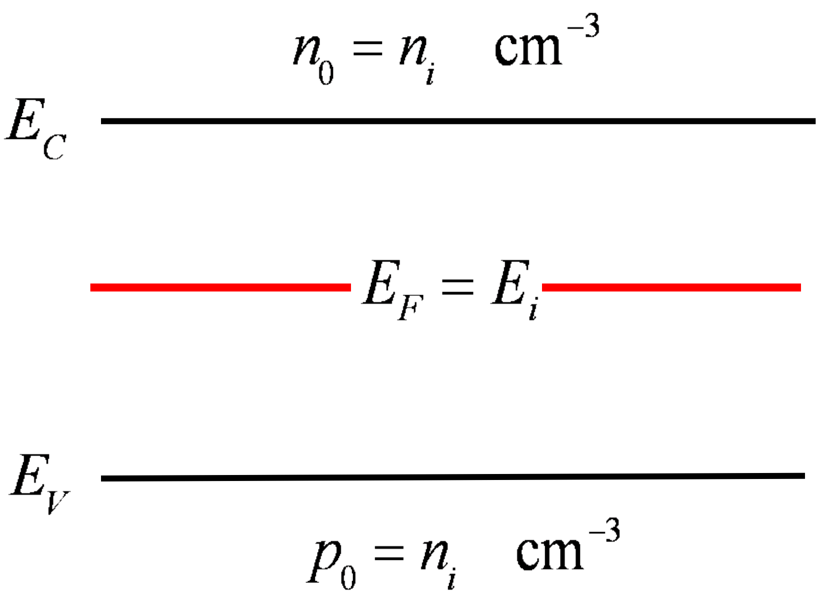
$$p_0 = N_V \exp\left\{\left(E_V - E_F\right)/k_B T\right\} \text{cm}^{-3}$$

$$n_0 = N_C \exp\left\{\left(E_F - E_C\right)/k_B T\right\} \text{cm}^{-3}$$

$$n_0 p_0 = n_i^2$$

$$n_0 = \frac{n_i^2}{p_0}$$

Intrinsic semiconductor



$$p_0 = n_0 = n_i \ \text{cm}^{-3}$$

Essentials of semiconductor physics

1) Energy bands
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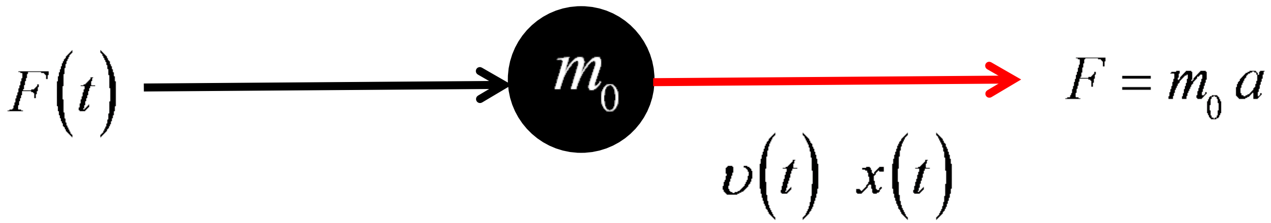
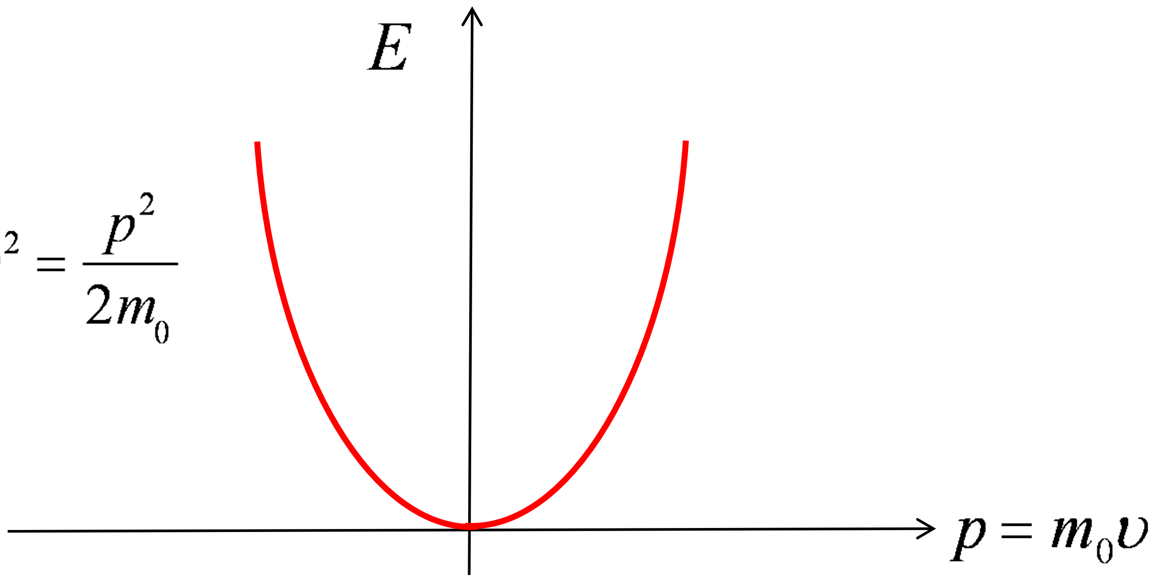
Classical particle: electron in free space



$$E = \frac{1}{2} m_0 \upsilon^2 = \frac{p^2}{2m_0}$$



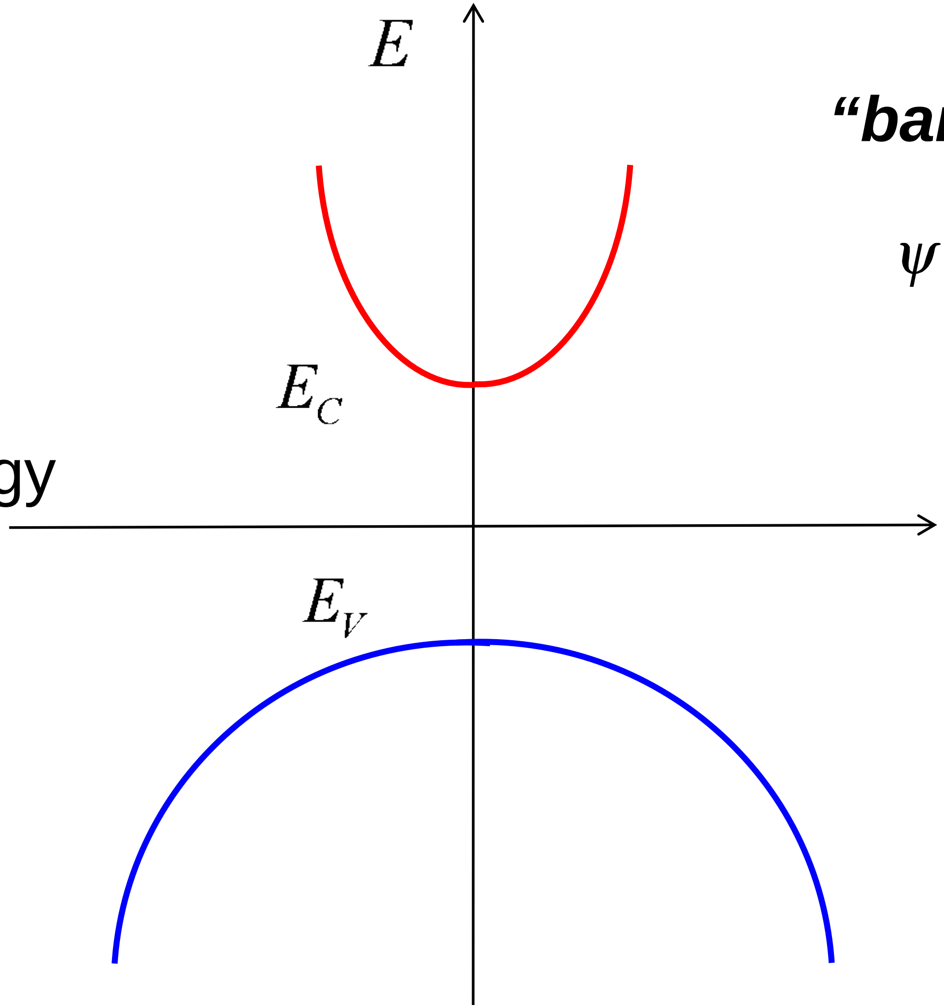
Energy and “crystal momentum”

$$E = E_C + \frac{p^2}{2m_n^*}$$

Electron energy

Hole energy

$$E = E_V - \frac{p^2}{2m_p^*}$$



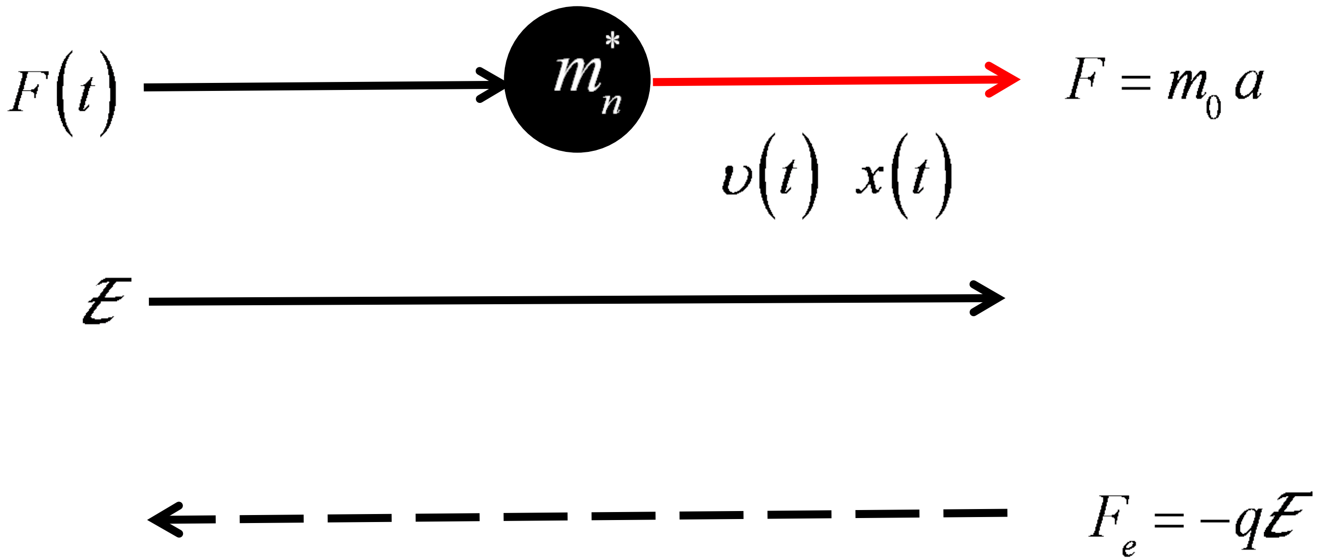
$$p = \hbar k$$

$$k = \frac{2\pi}{\lambda}$$

“band structure”

$$\psi(\vec{r}) = u_k(\vec{r})\, e^{i\vec{k}\cdot\vec{r}}$$

Drift current



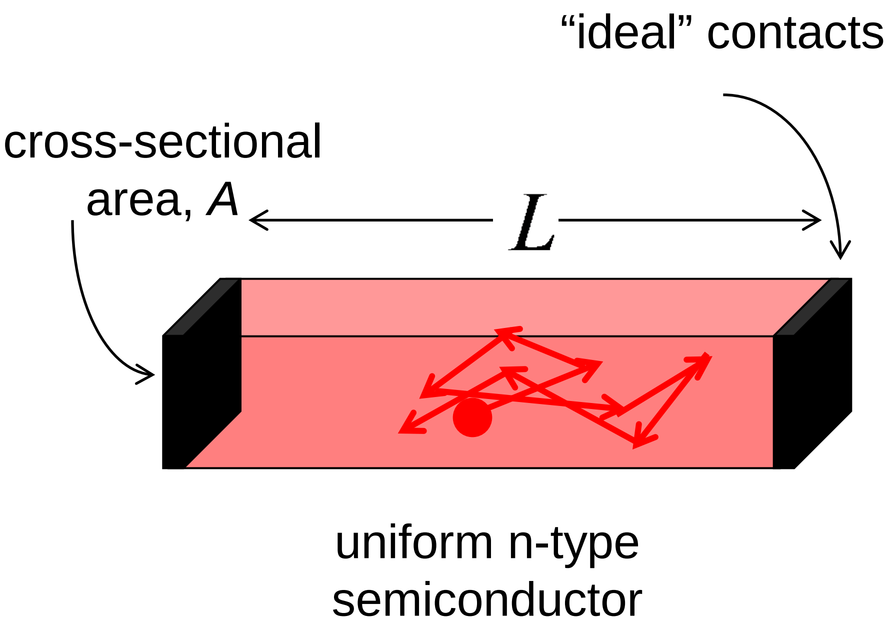
$$\upsilon_{dn} = -\mu_n \mathcal{E}$$

$$\mu_n = \left(\frac{q\tau}{m_n^*}\right) \text{cm}^2/\text{V-s}$$

$$J_n = -nq\upsilon_{dn} = +nq\mu_n \mathcal{E} \ \text{A/cm}^2$$

drift current

Thermal velocity



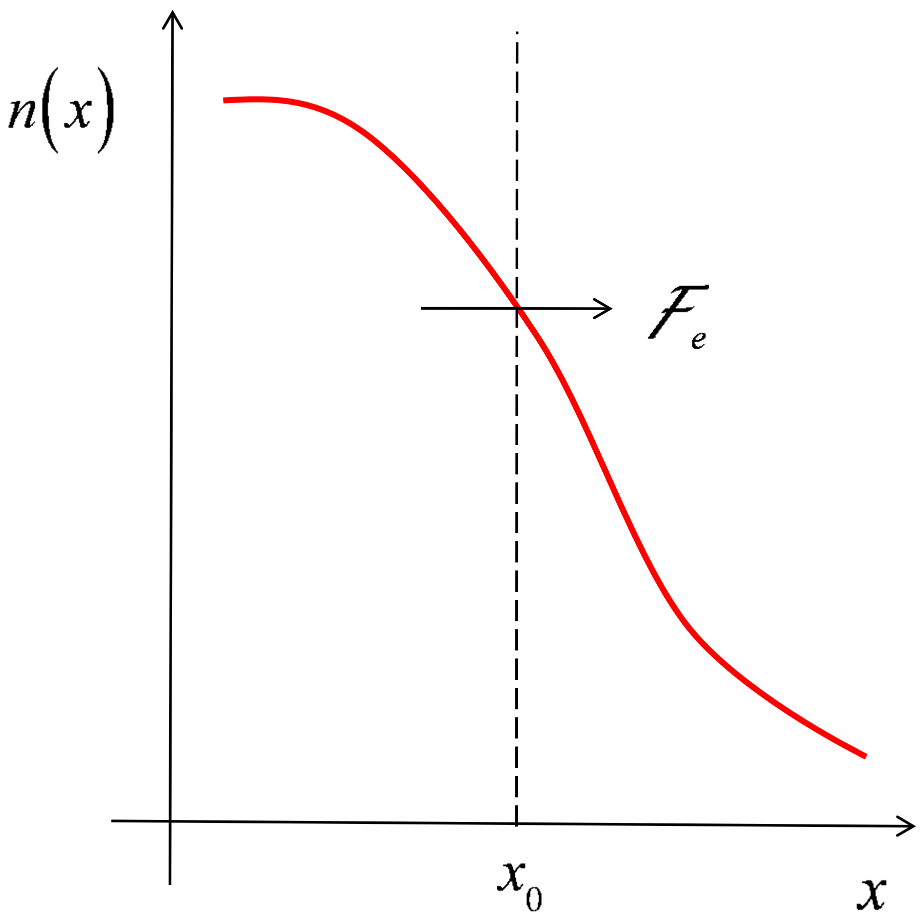
$$\langle KE \rangle = \frac{3}{2} k_B T$$

$$\langle KE \rangle = \frac{1}{2} m_n^* \langle \upsilon^2 \rangle$$

$$\sqrt{\langle \upsilon^2 \rangle} = \upsilon_{rms} = \sqrt{\frac{3k_B T}{m_n^*}}$$

$$\upsilon_{rms} \approx 10^7 \text{ cm/s}$$

Diffusion current



$$\mathcal{F}_e = -D\frac{dp}{dx} \qquad \frac{\#}{\text{cm}^2\text{-s}}$$

$$D \qquad \text{cm}^2/\text{s}$$

(diffusion coefficient)

(Adolph Fick, 1855)

$$J_n = -q\mathcal{F}_e = +qD_n\frac{dn}{dx} \text{ A/cm}^2$$

diffusion current

Drift-diffusion equation

$$J_{n-drift} = -nq\upsilon_{dn} = +nq\mu_n \mathcal{E} \text{ A/cm}^2$$

drift current

$$J_{n-diff} = -q\mathcal{F}_e = +qD_n \frac{dn}{dx} \text{ A/cm}^2$$

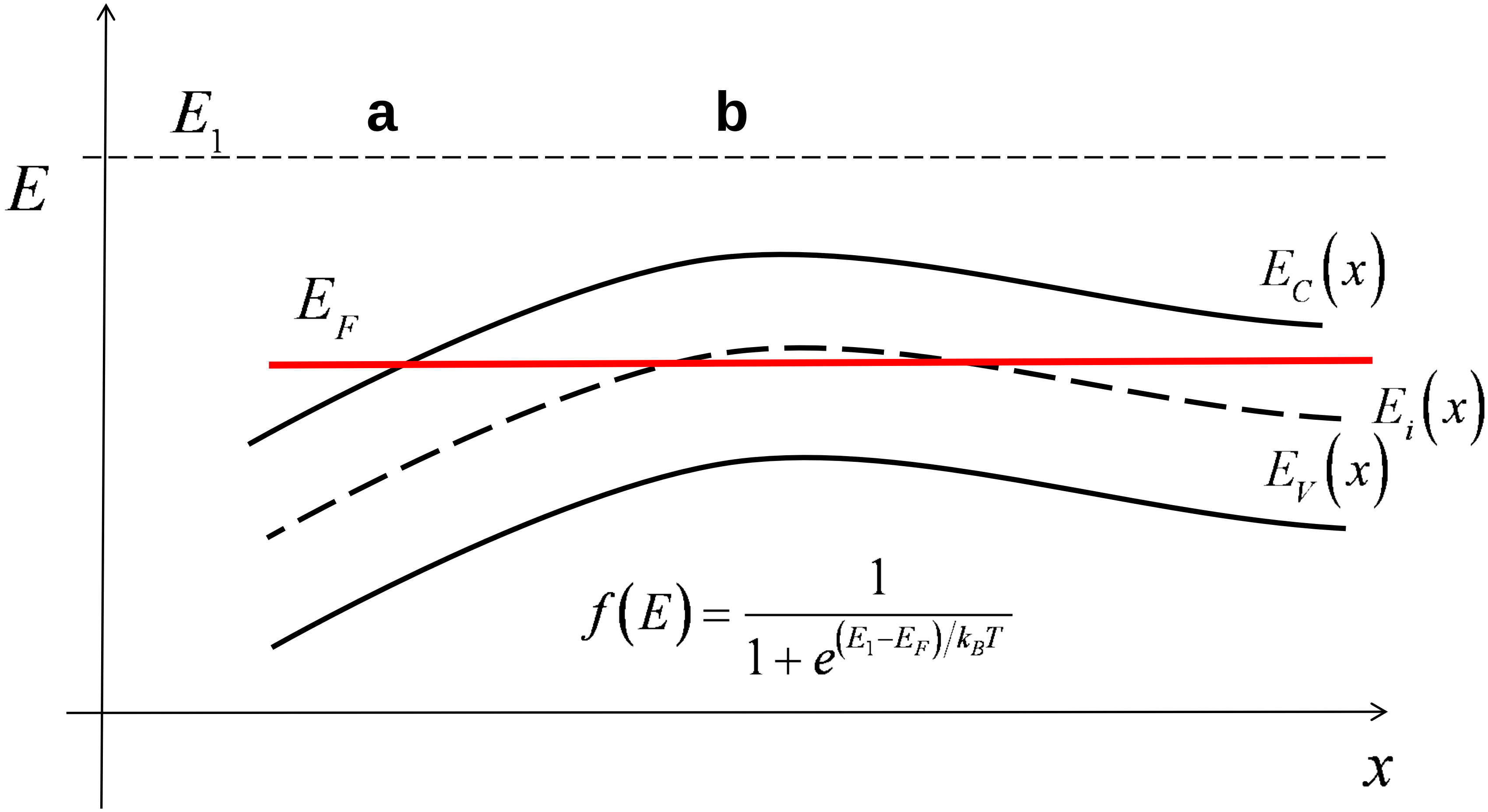
diffusion current

$$J_n = J_{n-drift} + J_{n-diff} = nq\mu_n \mathcal{E} + qD_n \frac{dn}{dx}$$

$$\frac{D_n}{\mu_n} = \frac{k_B T}{q}$$

(Einstein relation)

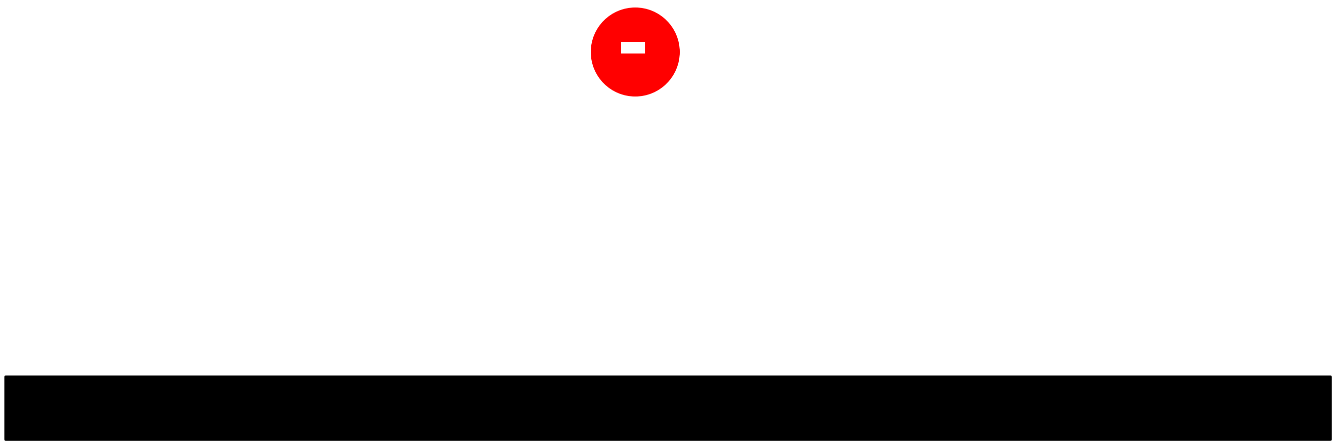
Energy bands can bend



But the Fermi level is constant (in equilibrium)!

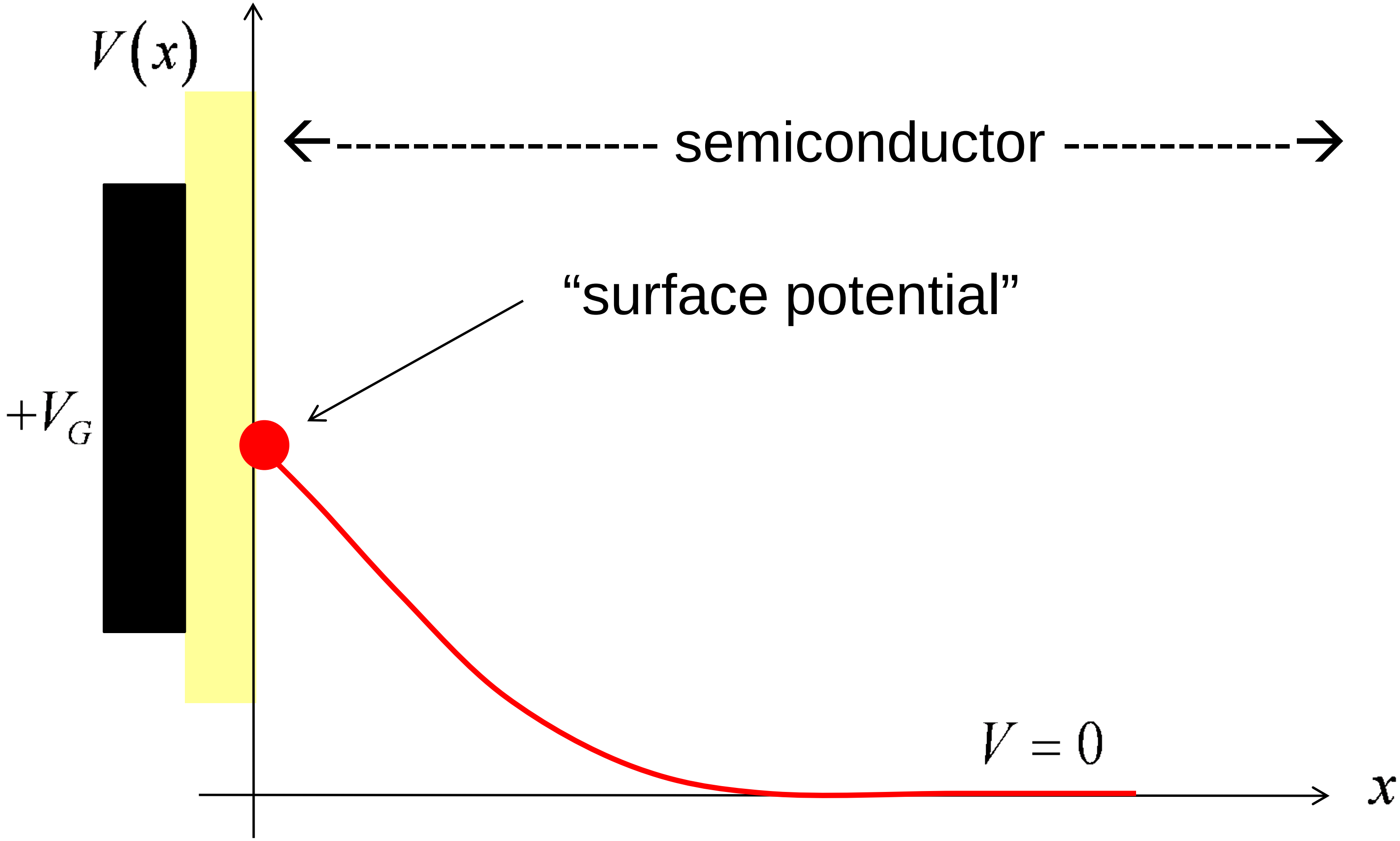
Voltage and electron potential energy

$$E = -qV$$

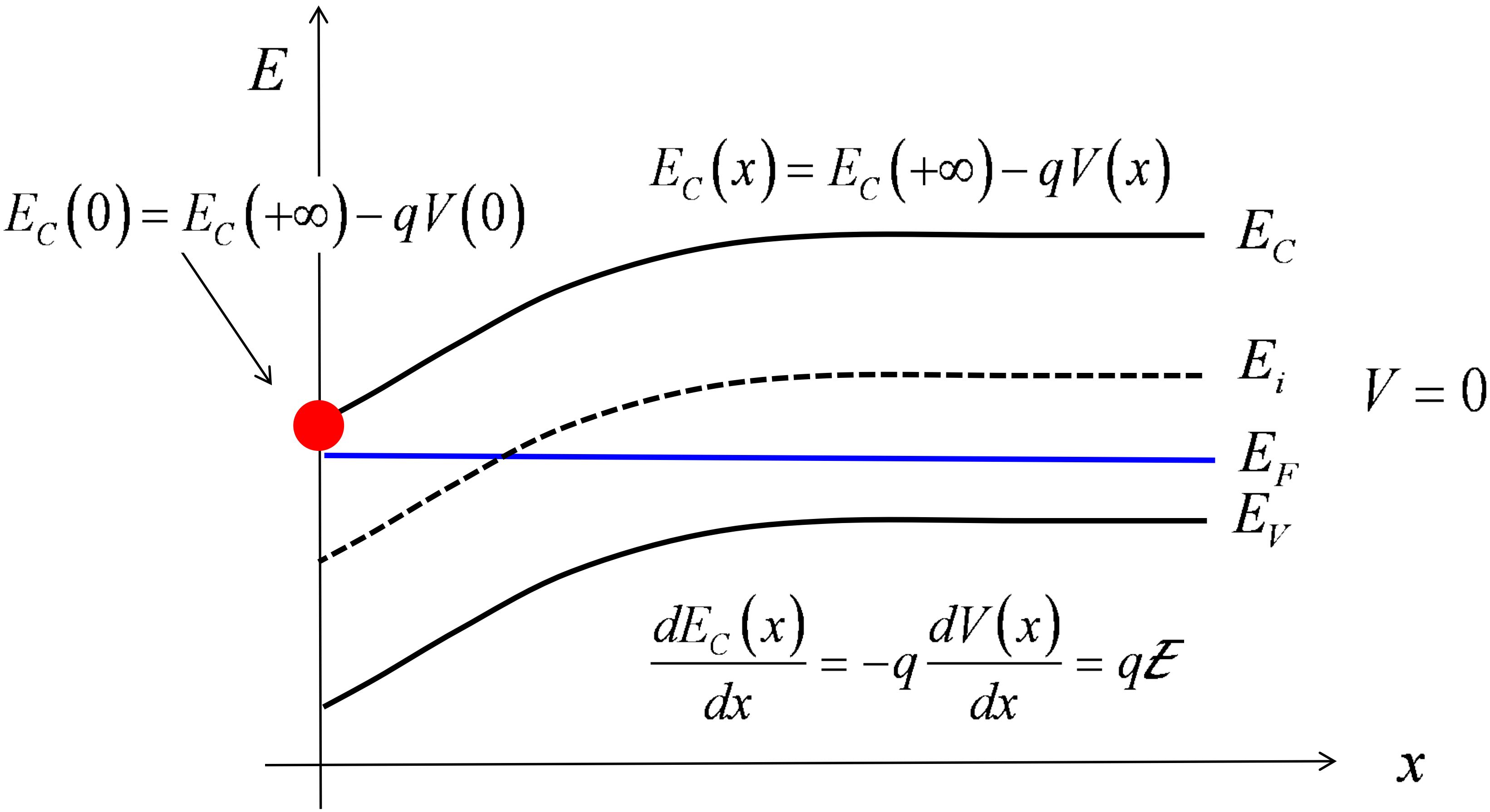


$+V$

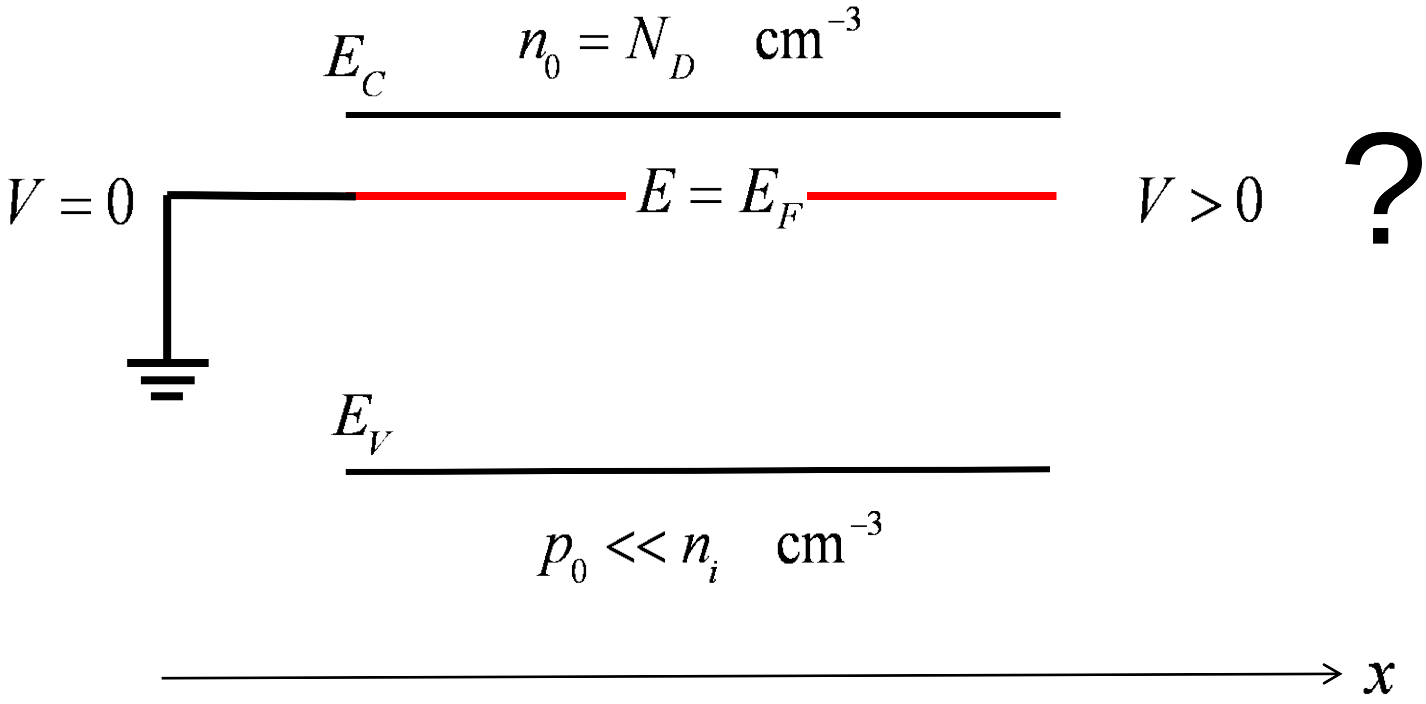
Example: electrostatic potential vs. position



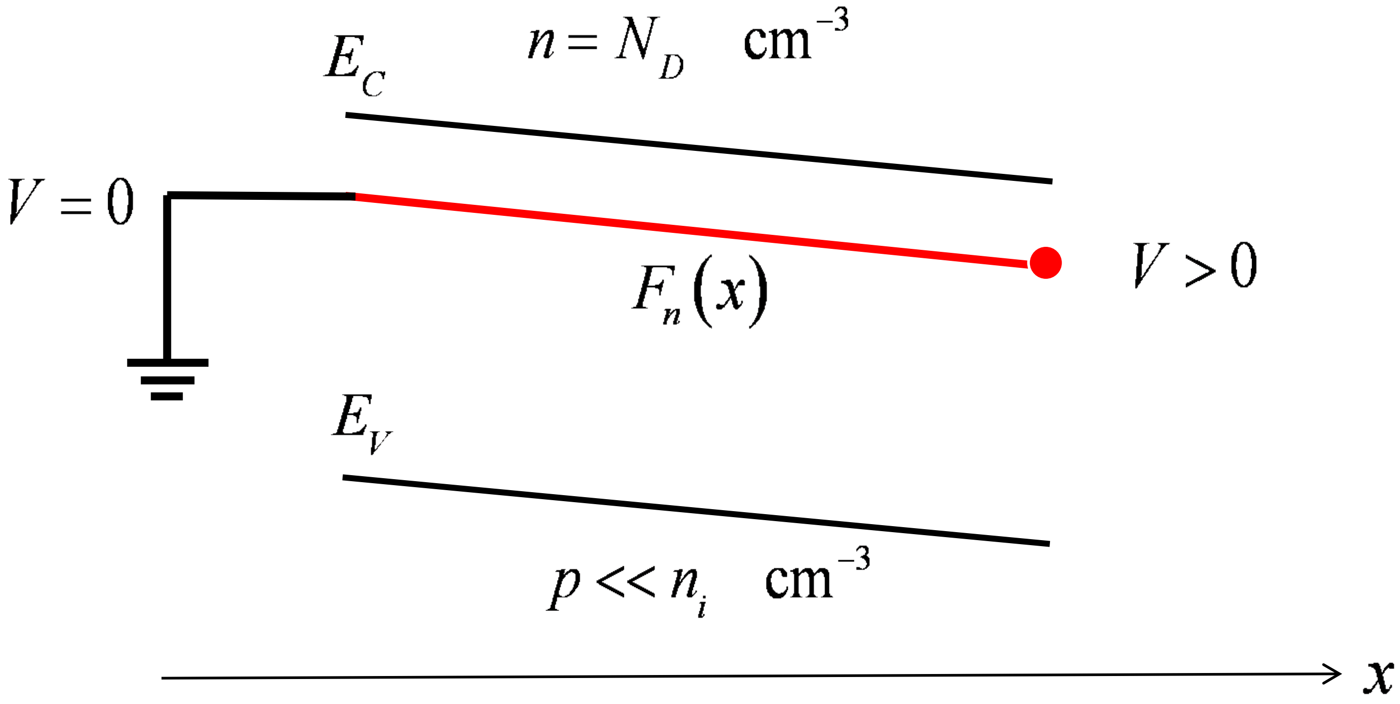
Bandbending in semiconductors



Quasi-Fermi levels



Quasi-Fermi levels



Carrier density and QFL's

Out of equilibrium, the Fermi level must be replaced by two QFL's – one for electrons and one for holes.

$$n_0 = N_C \exp\left\{\left(E_F - E_C\right)/k_B T\right\} \rightarrow n = N_C \exp\left\{\left(F_n - E_C\right)/k_B T\right\} \text{ cm}^{-3}$$

$$p_0 = N_V \exp\left\{\left(E_V - E_F\right)/k_B T\right\} \rightarrow p = N_V \exp\left\{\left(E_V - F_p\right)/k_B T\right\} \text{ cm}^{-3}$$

Current and QFL's

The Fermi level is constant in equilibrium.

The quasi-Fermi levels can be position dependent.

$$J_n = nq\mu_n \mathcal{E} + qD_n \frac{dn}{dx} \qquad J_n = n\mu_n \frac{dF_n}{dx}$$

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References: Basic semiconductor physics

Semiconductor Device Fundamentals., R.F. Pierret, Addison-Wesley, 1996.

Advanced Semiconductor Fundamentals, 2nd ed., R.F. Pierret, Prentice-Hall, 2002.

ECE 305: Semiconductor Devices:
https://nanohub.org/groups/ece305lundstrom

ECE 606: Solid State Devices
https://nanohub.org/groups/ece606lundstrom