Fundamentals of Nanotransistors
Unit 4 Homework SOLUTIONS

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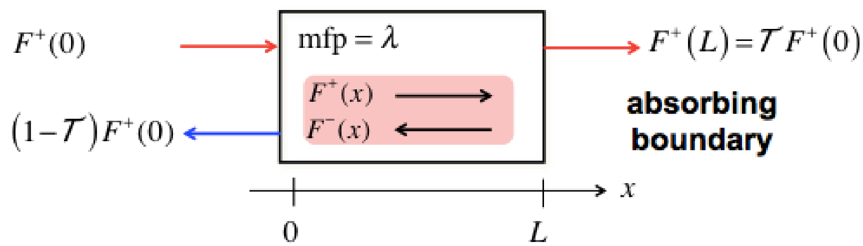
1) Consider the problem shown below, electron transport across a slab of arbitrart length, L. Assume that the positive and negative fluxes both travel at the unidirectional thermal velocity, υ_T cm/s. Assume 2D carriers, so that the particle fluxes have units of $(\text{cm-s})^{-1}$.



Answer the following questions.

1a) Derive an expression for the carrier densities at $x=0$, $n_S(0)$ and at $x=L$, $n_S(L)$ cm^{-2} in terms of the injected flux, $F^+(0)$ cm^{-2}s^{-1}.

Solution:

$$n_S(0)=n_S^+(0)+n_S^-(0)=\frac{F^+(0)}{\upsilon_T}+\frac{(1-\mathcal{T})F^+(0)}{\upsilon_T}$$

$$\boxed{n_S(0)=\frac{(2-\mathcal{T})F^+(0)}{\upsilon_T}}$$

$$n_S(L)=n_S^+(L)+n_S^-(0)=\frac{\mathcal{T}F^+(0)}{\upsilon_T}+0$$

$$\boxed{n_S(L)=\frac{\mathcal{T}F^+(0)}{\upsilon_T}}$$

1b) Compute the ratio of the net flux $F=F^+-F^-$, to the gradient of the particle density, dn_S/dx and provide a physical explanation for the result.

Unit 4 Homework SOLUTIONS (continued)

Solution:

$$F = F^+(0) - (1-\mathcal{T})F^+(0) = \mathcal{T}F^+(0) \qquad (*)$$

Using:

$$n_S(0) = \frac{(2-\mathcal{T})F^+(0)}{\upsilon_T}$$

$$n_S(L) = \frac{\mathcal{T}F^+(0)}{\upsilon_T}$$

we find:

$$\frac{dn_S}{dx} = -\frac{n_S(0) - n_S(L)}{L} = -\frac{F^+(0)}{\upsilon_T}\frac{2(1-\mathcal{T})}{L} \qquad (**)$$

(Note that we can readily show that $n_S(x)$ is linear inside the slab.)

Dividing (*) by (**), we find

$$\frac{F}{dn_S/dx} = -\frac{\mathcal{T}}{(1-\mathcal{T})}\frac{L\upsilon_T}{2}$$

Using $\mathcal{T} = \dfrac{\lambda}{\lambda + L}$, we find

$$\boxed{\frac{F}{dn_S/dx} = -\frac{\upsilon_T \lambda}{2}}$$

Explanation:

Define a quantity, $D = \upsilon_T \lambda/2$ and note that the units are cm^2/s – the units of a diffusion coefficient. With this definition, the net flux can be written as:

$$F = -D\, dn_S/dx,$$

which we recognize as Fick's Law of diffusion. Note that we have made no assumption about the length of the slab, *L*. It doesn't matter whether the slab is much longer than the mfp for backscattering or much shorter, Fick's Law holds. This is, in fact, true, but not widely understood.

1c) Consider a slab with a mfp for backscattering of λ and an arbitrary length, L. What is the maximum magnitude of the gradient of the particle density, dn_S/dx as the length is varied from very short to very long?

Solution:

From the answer to part 1b), we have

$$\frac{dn_S}{dx} = -\frac{n_S(0) - n_S(L)}{L} = -\frac{F^+(0)}{\upsilon_T}\frac{2(1-\mathcal{T})}{L}$$

Using $\mathcal{T} = \dfrac{\lambda}{\lambda + L}$, we find

$$\frac{dn_S}{dx} = -\frac{2F^+(0)}{\upsilon_T}\frac{1}{\lambda + L} \qquad (*)$$

In the diffusive limit, $L \gg \lambda$ and $n(0) \approx 2n^+(0) = 2F^+(0)/\upsilon_T$, so from (*), we find

$$\left|\frac{dn_S}{dx}\right| \approx \frac{n(0)}{L}$$

In the ballistic limit, $L \ll \lambda$ and $n(0) = n^+(0) = F^+(0)/\upsilon_T$, so from (*), we find

$$\left|\frac{dn_S}{dx}\right| = \frac{n(0)}{\lambda/2}.$$

We conclude that

$$\boxed{\frac{n(0)}{L} < \left|\frac{dn_S}{dx}\right| < \frac{n(0)}{\lambda/2}}$$

It is often thought that for thin slabs, the concentration gradient will be large, and Fick's Law will break down, but this analysis shows that as the slab becomes much thinner than a mfp, the magnitude of the concentration gradient approaches an upper limit that is determined by the mfp. This helps explain why Fick's Law works from the ballistic to diffusive limit for this problem.

2) Consider a Si N-MOSFET with a density of mobile electrons of $n_S = 10^{13}$ cm^{-2}. Assume Maxwell-Boltzmann statistics and room temperature. Assume that $\mathcal{T} = 0.7$ and that V_{DS} is large. What fraction of the charge at the virtual source (top of the energy barrier) is due to electrons with negative velocities?

Solution:

From Unit 4, Lecture 4, we found

$$F^+(0) = \frac{F^+_{BALL}(0)}{(2-\mathcal{T})},$$

so

$$F^+/F^+_{BALL} = \frac{1}{(2-\mathcal{T})} = \frac{1}{2-0.7} = 0.77 \qquad (*)$$

In the ballistic case, all of the electrons have positive velocities, so

$$Q_n(0) = qn_S^+(0) = qF^+_{BALL}(0)/\upsilon_T$$

In the second case, some of the mobile charge is due to electrons with negative velocities:

$$Q_n(0) = q\left[n_S^+(0) + n_S^-(0)\right],$$

where

$$n_S^-(0) = (1-\mathcal{T})F^+(0)/\upsilon_T$$

In a well-designed MOSFET, the mobile charge is the same in both cases, so

$$\frac{qn_S^-(0)}{Q_n(0)} = \frac{q(1-\mathcal{T})F^+(0)/\upsilon_T}{qF^+_{BALL}(0)/\upsilon_T} = (1-\mathcal{T})\frac{F^+(0)}{F^+_{BALL}(0)}$$

Using (*), we find

$$\frac{qn_S^-(0)}{Q_n(0)} = (1-\mathcal{T})\frac{F^+(0)}{F^+_{BALL}(0)} = (1-0.7)0.77 = 0.23$$

$$\boxed{\frac{qn_S^-(0)}{Q_n(0)} = 0.23}$$

3) Consider the following numbers typical of an L = 30 nm, silicon ETSOI (Extremely Thin SOI) MOSFET at room temperature:

$$C_{inv} = 2\times 10^{-6}\ \text{F/cm}^2$$
$$I_{ON} = 900\ \mu\text{A}/\mu\text{m}$$
$$V_T = 0.35\ \text{V at } V_{DD} = 1.0\ \text{V}$$
$$\mu_n = 225\ \text{cm}^2/\text{V-s}$$
$$R_{SD} = R_S + R_D = 270\ \Omega-\mu\text{m}$$

3a) Assume that V_{GS} = V_{DS} = 1.0 V and estimate the average velocity at the top of the barrier, $\upsilon_{inj} = \langle \upsilon(0) \rangle$, under on-current conditions.

Unit 4 Homework SOLUTIONS (continued)

Solution:

$$I_{DS} = W|Q_n|\langle \upsilon(0)\rangle$$

The device is 1 micrometer wide, so

$$900\times 10^{-6} = 10^{-4}|Q_n|\langle \upsilon(0)\rangle$$

$$|Q_n| = C_{inv}\left(V_{GS} - I_D R_S - V_T\right)$$

$$|Q_n| = 2\times 10^{-6}\left(1-900\times 10^{-6}\times 135-0.35\right) = 1.06\times 10^{-6} \text{ C/cm}^2$$

$$\langle \upsilon(0)\rangle = \frac{900\times 10^{-6}}{10^{-4}\left(1.06\times 10^{-6}\right)} = 0.85\times 10^{7} \text{ cm/s}$$

$$\boxed{\langle \upsilon(0)\rangle = 0.85\times 10^{7}}$$

3b) Determine how close to the ballistic limit this MOSFET operates under on-current conditions. (You may assume non-degenerate carrier statistics to keep the math simple.)

Solution:

$$I_{ON} = W|Q_n(0)|\langle \upsilon(0)\rangle = W|Q_n(0)|\upsilon_{inj}$$

$$I_{ON}^{ball} = W|Q_n(0)|\langle \upsilon^{ball}(0)\rangle = W|Q_n(0)|\upsilon_{inj}^{ball}$$

The ballistic on-current ratio is:

$$B_{SAT} = \frac{I_{ON}}{I_{ON}^{ball}} = \frac{\upsilon_{inj}}{\upsilon_{inj}^{ball}}$$

For non-degenerate conditions:

$$\upsilon_{inj}^{ball} = \upsilon_T = \sqrt{\frac{2k_BT}{\pi m^*}} = 1.23\times 10^{7}\ \text{cm/s}$$ (See Unit 3 HW, problem 1)

$$B_{SAT} = \frac{0.85}{1.23} = 0.69$$

$$\boxed{B_{SAT} = \frac{I_{ON}}{I_{ON}^{BALL}} = 0.69}$$

Unit 4 Homework SOLUTIONS (continued)

3c) Determine the transmission under on-current conditions, $\mathcal{T}_{SAT}$.

Solution:

$$B_{SAT} = \frac{\mathcal{T}_{SAT}}{2-\mathcal{T}_{SAT}}$$

Solve for $\mathcal{T}_{SAT}$:

$$\mathcal{T}_{SAT} = \frac{2B_{SAT}}{B_{SAT}+1} = \frac{2\times 0.69}{1.69} = 0.82$$

$$\boxed{\mathcal{T}_{SAT} = 0.82}$$

3d) Determine the length of the critical region under on-current conditions. Express your answer in terms of the channel length, i.e. as ℓ/L .

Solution:

We can begin with:

$$\mathcal{T}_{SAT} = \frac{\lambda}{\lambda+\ell} = \frac{1}{1+\ell/\lambda}$$

Solve for the critical length:

$$\ell = \lambda\left(1/\mathcal{T}_{SAT}-1\right) = \lambda\left(1/0.82-1\right) = 0.22\lambda \qquad (*)$$

We need to find the mean-free-path, λ, which can be found from the given mobility:

$$\mu_n = \frac{\upsilon_T\lambda}{2\left(k_B T/q\right)}$$ (assuming non-degenerate carrier statistics).

$$\lambda = \frac{2\left(k_B T/q\right)\mu_n}{\upsilon_T} = \frac{2\times 0.026\times 225}{1.23}\ \text{nm} = 9.5\ \text{nm}$$

Now we can determine the critical length from (*)

$$\ell = 0.22\lambda = 0.22\times 9.5 = 2.1\ \text{nm}$$

$$\ell/L = \frac{2.1\ \text{nm}}{30\ \text{nm}} = 0.07$$

$$\boxed{\ell/L = 0.07}$$

As expected, the critical length for backscattering under high drain bias is only a small fraction of the channel length.

Unit 4 Homework SOLUTIONS (continued)

4) Consider an L = 22 nm silicon MOSFET with $\mu_n = 250$ cm²/V-s. What is the apparent mobility for this device? Assume room temperature, $m^* = 0.19 m_0$ and Maxwell-Boltzmann statistics.

Solution:

$$\mu_B = \frac{\upsilon_T L}{2\left(k_B T/q\right)} = \frac{1.2\times10^7 \times 22\times10^{-7}}{2\times0.026} = 508 \text{ cm}^2/\text{V-s}$$

$$\frac{1}{\mu_{app}} = \frac{1}{\mu_n} + \frac{1}{\mu_B} = \frac{1}{250} + \frac{1}{508} = 0.006 \rightarrow \mu_{app} = 168 \text{ cm}^2/\text{V-s}$$

$$\boxed{\mu_{app} = 168\ \text{cm}^2/\text{V-s}}$$

The fact that the apparent mobility is less than the scattering-limited mobility indicates that this MOSFET operates in the quasi-ballistic regime.

5) Consider an L = 22 nm III-V (InGaAs) HEMT with an Indium-rich channel with $\mu_n = 10{,}000$ cm²/V-s. What is the apparent mobility for this device? Assume room temperature, $m^* = 0.08 m_0$ and Maxwell-Boltzmann statistics.

Solution:

$$\mu_B = \frac{\upsilon_T L}{2\left(k_B T/q\right)}$$

$$\upsilon_T = \sqrt{\frac{2k_B T}{\pi m^*}} = \sqrt{\frac{2\times1.38\times10^{-23}\times300}{3.14\times0.08\times9.11\times10^{-31}}} = 1.9\times10^5 \text{ m/s}$$

$$\mu_B = \frac{\upsilon_T L}{2\left(k_B T/q\right)} = \frac{1.9\times10^7 \times 22\times10^{-7}}{2\times0.026} = 804 \text{ cm}^2/\text{V-s}$$

$$\frac{1}{\mu_{app}} = \frac{1}{\mu_n} + \frac{1}{\mu_B} = \frac{1}{10{,}000} + \frac{1}{804} = 0.0013 \rightarrow \mu_{app} = 744 \text{ cm}^2/\text{V-s}$$

$$\boxed{\mu_{app} = 744\ \text{cm}^2/\text{V-s}}$$

The fact that the apparent mobility is close to the ballistic mobility indicates that this MOSFET would operate very close to the ballistic limit.

Unit 4 Homework SOLUTIONS (continued)

6) The number of channels in the Fermi window is an important parameter. For T = 0 K, it is $M(E_F)$ and for T > 0 K, it is $\langle M_{2D} \rangle$. Assume a Si MOSFET under high gate bias with $n_S = 10^{13}$ cm^{-2}. Also assume room temperature, an effective mass of $m^* = m_t^* = 0.19 m_0$, a valley degeneracy of 2, and operation at a small drain to source voltage. Answer the following questions.

6a) Compare $\langle M_{2D} \rangle$ at room temperature to its value at T= 0 K assuming that $n_S = 10^{13}$ cm^{-2} in both cases. You should evaluate $\langle M_{2D} \rangle$ assuming **Fermi-Dirac carrier statistics**.

Solution:
First, we need to find the Fermi level at T = 0 K.

$$n_S = \int_{E_C}^{\infty} D_{2D}(E) f_0(E) dE$$ (low V_{DS}, so we include both + and – velocities in the DOS)

$$n_S = \int_{E_C}^{E_F} D_{2D}(E)(1) dE = \frac{g_V m^*}{\pi\hbar^2} \int_{E_C}^{E_F} dE = \frac{g_V m^*}{\pi\hbar^2}(E_F - E_C)$$

$$(E_F - E_C) = \frac{n_S}{\left(\frac{g_V m^*}{\pi\hbar^2}\right)}$$

Putting in numbers:

$$\left(\frac{g_V m^*}{\pi\hbar^2}\right) = 9.91 \times 10^{36} \text{ /J-m}^2 = 9.91 \times 10^{32} \text{ /J-cm}^2$$

$$\frac{(E_F - E_C)}{q} = \frac{10^{13}}{1.6 \times 10^{-19}(9.91 \times 10^{32})} = 6.26 \times 10^{-2} \text{ V}$$

Now use:

$$M_{2D}(E_F) = \frac{g_V \sqrt{2m^*(E_F - E_C)}}{\pi\hbar} = \frac{2\sqrt{2 \times 0.19 \times 9.11 \times 10^{-31} \times 6.26 \times 10^{-2} \times 1.6 \times 10^{-19}}}{\pi(1.055 \times 10^{-34})}$$

$$M_{2D}(E_F) = 3.56 \times 10^8 \text{ m}^{-1}$$

Next, we compute $\langle M_{2D} \rangle$ at room temperature.

$$\langle M_{2D} \rangle = \frac{h}{4}\left(\frac{g_V m_t^*}{\pi\hbar^2}\right)\upsilon_T \mathcal{F}_{-1/2}(\eta_F) \qquad (*)$$

Unit 4 Homework SOLUTIONS (continued)

We first need to compute η_F, the normalized Fermi level at room temperature. Begin with the relation of the 2D carrier density to the Fermi level:

$$n_S = N_{2D}\mathcal{F}_0(\eta_F)$$

$$\mathcal{F}_0(\eta_F) = \ln\left(1+e^{\eta_F}\right) = \frac{n_S}{N_{2D}}$$

$$\eta_F = \ln\left(e^{n_S/N_{2D}} - 1\right) = \frac{E_F - E_C}{k_B T}$$

Compute the effective density of states:

$$N_{2D} = \left(\frac{g_V m_t^* k_B T}{\pi\hbar^2}\right) = 4.11\times 10^{12} \text{ cm}^{-2}$$

Solve for the Fermi level:

$$\eta_F = \ln\left(e^{10/4.13} - 1\right) = 2.34$$

Returning to (*)

$$\langle M_{2D} \rangle = \frac{h}{4}\left(\frac{g_V m_t^*}{\pi\hbar^2}\right)\upsilon_T \mathcal{F}_{-1/2}(\eta_F)$$

We have computed $\upsilon_T = \sqrt{2k_B T/(\pi m^*)}$ for these parameters before, the result is

$\upsilon_T = 1.2\times 10^7$ cm/s

$$\langle M_{2D} \rangle = \frac{6.64\times 10^{-34}}{4}\left(9.91\times 10^{36}\right)\left(1.2\times 10^5\right)\mathcal{F}_{-1/2}(\eta_F) \quad \text{(Be careful to use MKS units!)}$$

$$= 1.97\times 10^8 \mathcal{F}_{-1/2}(2.34)$$

Note that an iPhone app is available to compute Fermi-Dirac integrals. An online tool is also available at https://nanohub.org/resources/fdical.

$$\mathcal{F}_{-1/2}(2.34) = 1.60$$

$$\langle M_{2D} \rangle = 3.16\times 10^8 \text{ m}^{-1}$$

$$\boxed{\frac{\langle M_{2D} \rangle}{M_{2D}(E_F)} = 0.89}$$

We conclude that the assumption of complete degeneracy (i.e. $f_0 = 1$ for $E < E_F$ and $f_0 = 0$ for $E > E_F$) is not particularly accurate – even for large gate voltages where n_S is large..

Unit 4 Homework SOLUTIONS (continued)

6b) Evaluate $\langle M_{2D} \rangle$ at room temperature assuming **non-degenerate carrier statistics**. Compare the result to that obtained with Fermi-Dirac statistics in question 6a).

Solution:

For non-degenerate semiconductors, Fermi-Dirac integrals become exponentials. From 6a):

$$n_S = N_{2D}\mathcal{F}_0(\eta_F) \rightarrow n_S = N_{2D}e^{\eta_F}$$

$$\langle M_{2D} \rangle = \frac{h}{4}\left(\frac{g_V m_t^*}{\pi\hbar^2}\right)\upsilon_T \mathcal{F}_{-1/2}(\eta_F) \rightarrow \frac{h}{4}\left(\frac{g_V m_t^*}{\pi\hbar^2}\right)\upsilon_T e^{\eta_F}$$

Using the first equation, the second equation becomes

$$\langle M_{2D} \rangle = \frac{h}{4}\left(\frac{g_V m_t^*}{\pi\hbar^2}\right)\upsilon_T \frac{n_S}{N_{2D}}$$

Using the results of 6a),

$$\langle M_{2D} \rangle = 1.97\times10^8\left(\frac{n_S}{N_{2D}}\right) = 1.97\times10^8\left(\frac{10^{13}}{4.13\times10^{12}}\right) = 4.79\times10^8 \text{ m}^{-1}$$

$$\frac{\langle M_{2D} \rangle_{MB}}{\langle M_{2D} \rangle_{FD}} = \frac{4.79}{3.16} = 1.52$$

$$\boxed{\frac{\langle M_{2D} \rangle_{MB}}{\langle M_{2D} \rangle_{FD}} = 1.52}$$

We conclude that while the use of nondegenerate carrier statistics (Maxwell-Boltzmann) simplifies the mathematics, it is not particularly accurate under high gate voltage.

6c) Consider a very wide MOSFET with $W = 0.1\ \mu\text{m}$. How many channels are there in the Fermi window when $n_S = 10^{13}$ cm^{-2}?

Solution:

$$\langle M \rangle = \langle M_{2D} \rangle_{FD} W = 3.16\times10^8\times10^{-7} = 31.6$$

$$\boxed{\langle M \rangle = 31}$$

For a MOSFET this wide and with such a high carrier density, we would not expect to clearly observe quantized observe quantized conductance, but the number of channels is getting small enough that it is becoming countable.