Fundamentals of Nanotransistors
Unit 3 Homework

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In the Unit 3 lectures, I largely used Maxwell-Boltzmann (nondegenerate) statistics for carriers, because it simplified the calculations but still illustrated the key points. In practice, one would need to include Fermi-Dirac statistics, and that complicates things by bringing in Fermi-Dirac integrals, but the fully degenerate (T = 0 K) case is just as easy to solve as the nondegenerate case, and it is also illustrative of more general principles. In this homework assignment, you will re-do many of the calculations done in the lectures, but this time assuming T = 0 K. Hopefully this will deepen your understanding of the concepts discussed in Unit 3.

1) Consider a modern, silicon, N-channel MOSFET with the following parameters:

$$L = 20 \text{ nm}$$
$$C_{inv} = 2.8\ \mu\text{C}/\text{cm}^2$$
$$\mu_n = 200\ \text{cm}^2/\text{V-s}$$
$$R_S = R_D = 100\ \Omega - \mu\text{m}$$
$$V_{DD} = 0.7\ \text{V}$$
$$T = 300\ \text{K}$$
$$m^* = 0.19 m_0$$

Compute the **ballistic mobility** and compare its value to the scattering limited, bulk mobility, $\mu_n = 200\ \text{cm}^2/\text{V-s}$. You may assume non-degenerate (Maxwell-Boltzmann) statistics for electrons. (In the last problem in this HW assignment, we will re-visit this problem assuming fully degenerate carrier statistics.)

2) To compute the current in an N-channel MOSFET, we would begin with the Landauer expression,

$$I = (2q/h)\int_{E_1}^{E_2} \mathcal{T}(E) M(E)(f_1 - f_2)\, dE\,,$$

Assume that contact one is grounded and that a positive voltage (not necessarily small) has been applied to contact 2. Assume 2D electrons, T = 0 K, that the transmission, $\mathcal{T}_0$, is independent of energy, and answer the following questions.

Unit 3 HW (continued)

2a) Determine the limits of integration, E_1 and E_2, for the integral in the Landauer expression.

2b) Evaluate the integral to obtain an expression for the drain current of an N-channel MOSFET at T = 0 K.

We will see later in this HW assignment that this equation can be used to compute the current of a ballistic MOSFET.

3) In the Landauer approach, current is proportional to the number of channels in the Fermi window:

$$\langle M \rangle \equiv \int M(E)\left(-\frac{\partial f_0}{\partial E}\right)dE$$

At T = 0 K, this becomes

$$\langle M \rangle = M(E_F)$$

Compute the number of channels for a MOSFET under high gate bias with an electron density of $n_S = 10^{13}$ cm^{-3}. Assume a silicon MOSFET with $m^* = 0.19m_0$, $g_V = 2$, T = 0 K, and that $W = 100$ nm. Assume a small drain bias so that $E_{F1} \approx E_{F2} = E_F$.

4) In Lecture 3.3, we showed that for small drain biases, current is proportional to voltage, $I = GV$, and that at T = 0 K:

$$G(T = 0\ \mathrm{K}) = \frac{2q^2}{h}\mathcal{T}(E_F)M(E_F)$$

Show that the solution to problem 2) simplifies to this expected result for small voltages.

5) In Lecture 3.3, we showed that for non-degenerate, Maxwell-Boltzmann, carrier statistics,

$I_{ON}^{ball} = qn_S\upsilon_T$ where

$$\upsilon_T = \sqrt{\frac{2k_BT}{\pi m^*}}$$

is the nondegenerate uni-directional thermal velocity.

Use the results of problem 2) to determine the ballistic on-current for T = 0 K.

Unit 3 HW (continued)

6) The ballistic injection velocity is an important quantity for a MOSFET. Compare the value of the ballistic injection velocity for a typical Si MOSFET computed assuming nondegenerate statistics to the value computed assuming fully degenerate (T = 0 K) statistics. Assume the following parameters:

$$T = 300 \text{ K}$$
$$m^* = 0.19 m_0$$
$$g_V = 2$$
$$n_S = 10^{13} \text{ cm}^{-2}$$

7) In Lecture 3.4, we derived the following expression for the ballistic channel conductance assuming non-degenerate carrier statistics

$$G_{CH} = W\, Q_n\left(V_{GS}, V_{DS}\right) \frac{\upsilon_T}{2\left(k_B T / q\right)} \quad (*)$$

Derive the corresponding expression for the fully degenerate (T = 0 K) case.

8) The ballistic model we developed in Lecture 3.4,

$$I_{DS} = W\left|Q_n\left(V_{GS}, V_{DS}\right)\right| \upsilon_T \left(\frac{1 - e^{-qV_{DS}/k_B T}}{1 + e^{-qV_{DS}/k_B T}}\right)$$

assumed non-degenerate carrier statistics. In this case, the drain current saturates when $V_{DS} >> k_B T$. Answer the following questions about the T = 0 K drain saturation voltage.

8a) Derive an expression for V_{DSAT}.

8b) Compare nondegenerate and degenerate drain saturation voltages for a silicon MOSFET biased at a gate voltage so that $n_S = 10^{13} \text{ cm}^{-3}$.

9) Now let's re-visit prob. 1) but this time assuming fully degenerate conditions. Answer the following questions.

9a) Derive an expression for the ballistic mobility at T = 0 K.

9b) Numerically evaluate the ballistic mobility for the MOSFET of prob. 1) and compare your answer to the result obtained by assuming nondegenerate conditions.