Fundamentals of Nanotransistors
Unit 3 Homework SOLUTIONS

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In the Unit 3 lectures, I largely used Maxwell-Boltzmann (nondegenerate) statistics for carriers, because it simplified the calculations but still illustrated the key points. In practice, one would need to include Fermi-Dirac statistics, and that complicates things by bringing in Fermi-Dirac integrals, but the fully degenerate (T = 0 K) case is just as easy to solve as the nondegenerate case, and it is also illustrative of more general principles. In this homework assignment, you will re-do many of the calculations done in the lectures, but this time assuming T = 0 K. Hopefully this will deepen your understanding of the concepts discussed in Unit 3.

1) Consider a modern, silicon, N-channel MOSFET with the following parameters:

$$L = 20 \text{ nm}$$
$$C_{inv} = 2.8\ \mu F/\text{cm}^2$$
$$\mu_n = 200\ \text{cm}^2/\text{V-s}$$
$$R_S = R_D = 100\ \Omega - \mu\text{m}$$
$$V_{DD} = 0.7\ \text{V}$$
$$T = 300\ \text{K}$$
$$m^* = 0.19 m_0$$

Compute the **ballistic mobility** and compare its value to the scattering limited, bulk mobility, $\mu_n = 200\ \text{cm}^2/\text{V-s}$. You may assume non-degenerate (Maxwell-Boltzmann) statistics for electrons. (In the last problem in this HW assignment, we will re-visit this problem assuming fully degenerate carrier statistics.)

Solution:
According to Lecture 3.1, the ballistic mobility is given by

$$\mu_B \equiv \frac{\upsilon_T L}{2\left(k_B T/q\right)}$$

Using the given effective mass and temperature, we find

$$\upsilon_T = \sqrt{\frac{2k_B T}{\pi m^*}} = 1.23\times 10^7 \text{ cm/s}$$

Unit 3 HW Solutions (continued)

$$\mu_B \equiv \frac{\upsilon_T L}{2(k_B T/q)} = \frac{(1.23\times 10^7)\times 20\times 10^{-7}}{2\times 0.026} = 473\ \text{cm}^2/\text{V-s}$$

$$\boxed{\mu_B = 473\ \text{cm}^2/\text{V-s} > \mu_n}$$

We will learn in Unit 4 that when $\mu_B >> \mu_n$, the transistor operates in the diffusive (scattering dominated) limit, and when $\mu_B << \mu_n$, the transistor operates in the ballistic (no scattering) limit. This transistor, which is typical of modern N-channel Si MOSFETs operates in the quasi-ballistic regime that we will discuss in Unit 4.

2) To compute the current in an N-channel MOSFET, we would begin with the Landauer expression,

$$I = (2q/h)\int_{E_1}^{E_2} \mathcal{T}(E) M(E)(f_1 - f_2)\,dE,$$

Assume that contact one is grounded and that a positive voltage (not necessarily small) has been applied to contact 2. Assume 2D electrons, T = 0 K, that the transmission, $\mathcal{T}_0$, is independent of energy, and answer the following questions.

2a) Determine the limits of integration, E_1 and E_2, for the integral in the Landauer expression.

2b) Evaluate the integral to obtain an expression for the drain current of an N-channel MOSFET at T = 0 K.

We will see later in this HW assignment that this equation can be used to compute the current of a ballistic MOSFET.

Solution 2a):

Because of the bias on contact 2 (the drain), the Fermi level is lowered

$$E_{F2} = E_{F1} - qV$$

so

$$E_{F1} > E_{F2}$$

For $f_1(E)$ at T = 0 K:

$$E < E_{F1}: \quad f_1(E) = 1$$

$$E > E_{F1}: \quad f_1(E) = 0$$

For $f_2(E)$ at $T = 0$ K:

$$E < E_{F2}: \quad f_2(E) = 1$$
$$E > E_{F2}: \quad f_2(E) = 0$$

The quantify, $(f_1 - f_2)$ is non-zero only in the energy range $E_{F2} < E < E_{F1}$ where $f_1(E) = 1$ and $f_2(E) = 0$. We conclude that the limits of integration should be

$$\boxed{\begin{array}{l} E_1 = E_{F2} \\ E_2 = E_{F1} \end{array}}$$

$$I = (2q/h)\int_{E_{F2}}^{E_{F1}} \mathcal{T}(E) M(E)(f_1 - f_2)\, dE$$

Solution 2b):

Using the results of 2a), assuming a constant transmission, and that $f_1(E) = 1$ and $f_2(E) = 0$ in the energy range of interest, we find:

$$I = (2q/h)\mathcal{T}_0 \int_{E_{F2}}^{E_{F1}} M(E)\, dE$$

In Lecture 3.2, we learned that

$$M(E) = WM_{2D}(E) = Wg_v \frac{\sqrt{2m^*(E - E_C)}}{\pi\hbar} \ \text{m}^{-1}$$

(In Lecture 3.2, the bottom the conduction band was taken as $E = 0$, i.e., we assumed that $E_C = 0$ in Lecture 3.2.)

The current becomes

$$I = (2q/h) W g_v \frac{\sqrt{2m^*}}{\pi\hbar} \mathcal{T}_0 \int_{E_{F2}}^{E_{F1}} (E - E_C)^{1/2}\, dE$$

$$I = \frac{2}{3}(2q/h) W g_v \frac{\sqrt{2m^*}}{\pi\hbar} \mathcal{T}_0 \left\{ (E_{F1} - E_C)^{3/2} - (E_{F2} - E_C)^{3/2} \right\}$$

Note that there are no channels below E_C (except for the valence band channels that we are ignoring), so when

$$E_{F1} < E_C$$
$$I = 0$$

and when

$E_{F1} > E_C$ and $E_{F2} = E_{F1} - qV < E_C$

There are no channels for electrons injected from contact 2, so

$$I = \frac{2}{3}(2q/h)Wg_v \frac{\sqrt{2m^*}}{\pi\hbar}\mathcal{T}_0\left(E_{F1} - E_C\right)^{3/2}$$

Alternatively, the above equation is valid when $E_{F1} > E_C \quad qV > E_{F1} - E_C$.

The complete drain current expression is

$$\boxed{\begin{aligned} & E_{F1} < E_C \quad qV \geq 0: \\ & I = 0 \\ \\ & E_{F1} > E_C \quad qV < E_{F1} - E_C: \\ & I = \frac{2}{3}(2q/h)Wg_v \frac{\sqrt{2m^*}}{\pi\hbar}\mathcal{T}_0\left\{\left(E_{F1} - E_C\right)^{3/2} - \left(E_{F2} - E_C\right)^{3/2}\right\} \\ \\ & E_{F1} > E_C \quad qV > E_{F1} - E_C: \\ & I = \frac{2}{3}(2q/h)Wg_v \frac{\sqrt{2m^*}}{\pi\hbar}\mathcal{T}_0\left(E_{F1} - E_C\right)^{3/2} \end{aligned}}$$

3) In the Landauer approach, current is proportional to the number of channels in the Fermi window:

$$\langle M \rangle \equiv \int M(E)\left(-\frac{\partial f_0}{\partial E}\right)dE$$

At T = 0 K, this becomes

$$\langle M \rangle = M\left(E_F\right)$$

Compute the number of channels for a MOSFET under high gate bias with an electron density of $n_S = 10^{13}\ \text{cm}^{-3}$. Assume a silicon MOSFET with $m^* = 0.19 m_0$, $g_V = 2$, T = 0 K, and that $W = 100$ nm. Assume a small drain bias so that $E_{F1} \approx E_{F2} = E_F$.

Solution:

For parabolic energy bands, we have seen that: $M_{2D}(E) = g_v \dfrac{\sqrt{2m^*\left(E - E_C\right)}}{\pi\hbar}$,so

$$\langle M(E)\rangle = WM_{2D}(E_F) = Wg_v \frac{\sqrt{2m^*(E_F - E_C)}}{\pi\hbar} \qquad (*)$$

and we only need to find E_F. We are given the carrier density, and we can find the Fermi level from the carrier density,

$$n_S = \int_{E_C}^{\infty} D_{2D}(E) f_0(E)\, dE = \int_{E_C}^{E_F} \left(g_v \frac{m^*}{\pi\hbar^2} \right)(1)\, dE = g_v \frac{m^*}{\pi\hbar^2}(E_F - E_C)$$

$$(E_F - E_C) = \frac{n_S}{\left(g_v m^* / \pi\hbar^2\right)}.$$

Now use this result for the Fermi level in the expression for modes (*):

$$M(E_F) = Wg_v \frac{\sqrt{2m^* n_S / \left(g_v m^* / \pi\hbar^2\right)}}{\pi\hbar} = W\sqrt{2g_v n_S / \pi}$$

Putting in numbers:

$$M(E_F) = W\sqrt{2g_v n_S/\pi} = 100\times 10^{-7}\sqrt{4\times 10^{13}/\pi} = 35.7$$

$$\boxed{M(E_F) = 35}$$

This number is rather small, so it is probably better to counts modes. There can't be a fraction of a mode, so we truncate 35.7 to 35.

4) In Lecture 3.3, we showed that for small drain biases, current is proportional to voltage, $I = GV$, and that at T = 0 K:

$$G(T = 0\,\mathrm{K}) = \frac{2q^2}{h}\mathcal{T}(E_F)M(E_F)$$

Show that the solution to problem 2) simplifies to this expected result for small voltages.

Solution:

Begin with the solution to problem 2):

$$I = \frac{2}{3}(2q/h)Wg_v \frac{\sqrt{2m^*}}{\pi\hbar}\mathcal{T}_0\left\{(E_{F1} - E_C)^{3/2} - (E_{F2} - E_C)^{3/2}\right\}$$

$$I = \frac{2}{3}(2q/h)Wg_v \frac{\sqrt{2m^*}}{\pi\hbar}\mathcal{T}_0\left\{(E_{F1} - E_C)^{3/2} - (E_{F1} - qV - E_C)^{3/2}\right\} \quad (*)$$

Now let's expand $(E_{F1} - qV - E_C)^{3/2}$ for small voltages.

$$f(V)=\left(E_{F1}-qV-E_C\right)^{3/2}\approx f(V=0)+\left.\frac{df}{dV}\right|_{V=0}V+...$$

Ignoring higher order terms in the Taylor series expansions, we find

$$\left(E_{F1}-qV-E_C\right)^{3/2}\approx\left(E_{F1}-E_C\right)^{3/2}+\left.\frac{d\left(E_{F1}-qV-E_C\right)^{3/2}}{dV}\right|_{V=0}V+...$$

$$\left(E_{F1}-qV-E_C\right)^{3/2}\approx\left(E_{F1}-E_C\right)^{3/2}-\frac{3}{2}\left(E_{F1}-E_C\right)^{1/2}qV+...$$

$$\left(E_{F1}-E_C\right)^{3/2}-\left(E_{F1}-qV-E_C\right)^{3/2}\approx\left(E_{F1}-E_C\right)^{3/2}-\left\{\left(E_{F1}-E_C\right)^{3/2}-\frac{3}{2}\left(E_{F1}-E_C\right)^{1/2}qV\right\}$$

$$\left(E_{F1}-E_C\right)^{3/2}-\left(E_{F1}-qV-E_C\right)^{3/2}\approx\frac{3}{2}q\left(E_{F1}-E_C\right)^{1/2}V \qquad (**)$$

Now use (**) in (*) to find

$$I=\frac{2}{3}(2q/h)Wg_v\frac{\sqrt{2m^*}}{\pi\hbar}\mathcal{T}_0\left\{\frac{3}{2}q\left(E_{F1}-E_C\right)^{1/2}V\right\}$$

$$I=\left(2q^2/h\right)\mathcal{T}_0\left\{Wg_v\frac{\sqrt{2m^*\left(E_{F1}-E_C\right)}}{\pi\hbar}\right\}V$$

We recognize the term in curly brackets as $M\left(E_{F1}\right)$, so (*) can be written as

$$\boxed{I=\left(2q^2/h\right)\mathcal{T}_0M\left(E_{F1}\right)V=GV}$$

which is the expected result.

5) In Lecture 3.3, we showed that for non-degenerate, Maxwell-Boltzmann, carrier statistics,

$I_{ON}^{ball}=qn_S\upsilon_T$ where

$$\upsilon_T=\sqrt{\frac{2k_BT}{\pi m^*}}$$

is the nondegenerate uni-directional thermal velocity.

Use the results of problem 2) to determine the ballistic on-current for T = 0 K.

Unit 3 HW Solutions (continued)

Solution:

Begin with the current for large drain bias:

$$E_{F1} > E_C \quad qV > E_{F1} - E_C$$

$$I = \frac{2}{3}(2q/h) W g_v \frac{\sqrt{2m^*}}{\pi\hbar} \mathcal{T}_0 \left(E_{F1} - E_C\right)^{3/2}$$

Assume ballistic transport $\mathcal{T}_0 = 1$:

$$I_{ON}^{ball} = \frac{2}{3}(2q/h) W g_v \frac{\sqrt{2m^*}}{\pi\hbar} \left(E_{F1} - E_C\right)^{3/2} \quad (*)$$

Now we need to relate this to the sheet carrier density, n_S:

$$n_S = \int_{E_C}^{\infty} \frac{D_{2D}}{2}(E) f_0(E)\, dE = \int_{E_C}^{E_F} \left(g_v \frac{m^*}{2\pi\hbar^2}\right)(1)\, dE = g_v \frac{m^*}{2\pi\hbar^2}\left(E_F - E_C\right) \quad (**)$$

(As discussed in Lecture 3.3, we divide the density of states by 2 because half the states can be filled by contact 1 and half by contact 2, but under high bias, the Fermi level in so low in contact 2, so that no states are filled by contact 2.)

From (*) and (**), we find:

$$\frac{I_{ON}^{ball}}{q n_S} = \frac{2}{3}\frac{(2q/h)}{q} \frac{W g_v \dfrac{\sqrt{2m^*}}{\pi\hbar}\left(E_{F1} - E_C\right)^{3/2}}{g_v \dfrac{m^*}{2\pi\hbar^2}\left(E_{F1} - E_C\right)} \quad (***)$$

$$= \frac{2}{3} \times \frac{2}{h} W\, 2\hbar \sqrt{\frac{2\left(E_{F1} - E_C\right)}{m^*}}$$

Now for parabolic energy bands, velocity and energy are related by

$$\frac{1}{2} m^* \upsilon(E)^2 = \left(E - E_C\right)$$

$$\upsilon(E) = \sqrt{\frac{2\left(E - E_C\right)}{m^*}}$$

At the Fermi energy, the velocity is

$$\upsilon(E_F) = \sqrt{\frac{2\left(E_{F1} - E_C\right)}{m^*}} = \upsilon_F \quad (****)$$

where υ_F is the so-called Fermi velocity. Using (****) in (***), we find

$$\frac{I_{ON}^{ball}}{qn_S} = W\frac{2}{3}\left(\frac{2}{\pi}\upsilon_F\right)$$

Unit 3 HW Solutions (continued)

We recognize $(2/\pi)\upsilon_F$ as the average x-directed velocity for electrons at the Fermi level. The factor 2/3 comes from averaging the x-directed velocity for all energies from $E_C < E < E_{F1}$. We conclude

$$\boxed{\begin{aligned} I_{ON}^{ball} &= W\left(qn_S\right)\left\langle\left\langle \upsilon_x^+ \right\rangle\right\rangle \\ \left\langle\left\langle \upsilon_x^+ \right\rangle\right\rangle &= \frac{2}{3}\left(\frac{2}{\pi}\upsilon_F\right) \end{aligned}}$$

The first set of brackets, $\langle\ \rangle$, denotes an average over angle at a given energy, which gives the factor, $2/\pi$. The second set of brackets, $\langle\ \rangle$, denotes an average over energy and gives the factor, 2/3.

Comparing to Lecture 3.3, we see that the nondegenerate and fully degenerate velocities are

$$\upsilon_T = \sqrt{\frac{2k_BT}{\pi m^*}} \rightarrow \frac{2}{3}\left(\frac{2}{\pi}\upsilon_F\right) = \frac{2}{3}\left(\frac{2}{\pi}\sqrt{\frac{2\left(E_{F1}-E_C\right)}{m^*}}\right)$$

6) The ballistic injection velocity is an important quantity for a MOSFET. It is the velocity at the virtual source under high drain bias. Compare the value of the ballistic injection velocity for a typical Si MOSFET computed assuming nondegenerate statistics to the value computed assuming fully degenerate (T = 0 K) statistics. Assume the following parameters:

$$T = 300\text{ K}$$
$$m^* = 0.19m_0$$
$$g_V = 2$$
$$n_S = 10^{13}\text{ cm}^{-2}$$

Solution:

For the nondegenerate case, we use the given effective mass and temperature to find

$$\upsilon_{inj}^{ball} = \upsilon_T = \sqrt{\frac{2k_B T}{\pi m^*}} = 1.23 \times 10^7 \text{ cm/s}$$

For the fully degenerate case, we make the T = 0 approximation even though we are at T = 300 K. (If the semiconductor is strongly degenerate, this is a reasonable approximation.)

Unit 3 HW Solutions (continued)

From the result of prob. 5):

$$\upsilon_{inj}^{ball} = \frac{2}{3}\left(\frac{2}{\pi}\sqrt{\frac{2\left(E_{F1} - E_C\right)}{m^*}}\right) \quad (*)$$

Next, we must determine the location of the Fermi level. Only positive velocity states are occupied under on-current conditions, so only half of the density of states is used:

$$n_S = \int_{E_C}^{\infty} \frac{D_{2D}}{2}\left(E\right) f_0\left(E\right) dE = \int_{E_C}^{E_F} \left(g_v \frac{m^*}{2\pi\hbar^2}\right)\left(1\right) dE = g_v \frac{m^*}{2\pi\hbar^2}\left(E_{F1} - E_C\right)$$

Solve for $\left(E_{F1} - E_C\right)$ to find

$$\left(E_{F1} - E_C\right) = \frac{2\pi\hbar^2}{g_V m^*} n_S,$$

which can be used in (*) to find

$$\upsilon_{inj}^{ball} = \frac{8}{3\sqrt{\pi}} \frac{\hbar}{m^*} \sqrt{n_S / g_V}.$$

Putting in numbers, we find:

$$\upsilon_{inj}^{ball} = \frac{8}{3\sqrt{\pi}} \frac{\hbar}{m^*} \sqrt{n_S / g_V} = 2.05 \times 10^7 \text{ cm/s}$$

$$\boxed{\upsilon_{inj}^{ball}\left(\text{degenerate}\right) > \upsilon_{inj}^{ball}\left(\text{nondegenerate}\right)}$$

In practice, the injection velocity would between these two values because at $n_S = 10^{13} \text{ cm}^{-2}$ the semiconductor is between the non-degenerate and fully degenerate limits. As mentioned in Lecture 3.4, in this case

$$\upsilon_{inj}^{ball} = \left\langle\left\langle \upsilon_x^+ \right\rangle\right\rangle = \upsilon_T \frac{\mathcal{F}_{1/2}\left(\eta_{FS}\right)}{\mathcal{F}_0\left(\eta_{FS}\right)}.$$

When the carrier density is high (which produces a high Fermi energy), then we should also worry about conduction band non-parabolicity and the possibility that upper subbands (with possibly larger effective masses) could be occupied.

7) In Lecture 3.4, we derived the following expression for the ballistic channel conductance assuming non-degenerate carrier statistics

$$G_{CH} = W\,Q_n\left(V_{GS}, V_{DS}\right)\frac{\upsilon_T}{2\left(k_B T/q\right)} \quad (*)$$

Derive the corresponding expression for the fully degenerate (T = 0 K) case. (Remember that ballistic conductance is defined for small drain bias.)

Unit 3 HW Solutions (continued)

Solution:

From the results of problem 4), we have in the ballistic limit

$$I = \left(2q^2/h\right)\left\{Wg_v\frac{\sqrt{2m^*\left(E_{F1}-E_C\right)}}{\pi\hbar}\right\}V$$

so

$$G = \left(2q^2/h\right)\left\{Wg_v\frac{\sqrt{2m^*\left(E_{F1}-E_C\right)}}{\pi\hbar}\right\}$$

We also know that

$$n_S = \int_{E_C}^{\infty} D_{2D}\left(E\right)f_0\left(E\right)dE = \int_{E_C}^{E_F}\left(g_v\frac{m^*}{\pi\hbar^2}\right)\left(1\right)dE = g_v\frac{m^*}{\pi\hbar^2}\left(E_F-E_C\right)$$

Combining these two results, we find

$$\frac{G}{n_S} = \frac{\left(2q^2/h\right)}{g_v\frac{m^*}{\pi\hbar^2}\left(E_{F1}-E_C\right)}\left\{Wg_v\frac{\sqrt{2m^*\left(E_{F1}-E_C\right)}}{\pi\hbar}\right\}$$

$$= Wq^2\frac{1}{\pi}\sqrt{\frac{2\left(E_{F1}-E_C\right)}{m^*}}\Bigg/\left(E_{F1}-E_C\right)$$

which can be re-written as

$$G = W\left(qn_S\right)\frac{1}{2\left(E_{F1}-E_C\right)/q}\left\{\frac{2}{\pi}\sqrt{\frac{2\left(E_{F1}-E_C\right)}{m^*}}\right\}$$

we recognize the term in curly brackets as the average x-directed velocity as the Fermi energy

$$\left\langle \upsilon\left(E_F\right)_x^+\right\rangle = \frac{2}{\pi}\sqrt{\frac{2\left(E_{F1}-E_C\right)}{m^*}} = \frac{2}{\pi}\upsilon_F$$

so the final answer is

$$\boxed{\begin{aligned}&G = WQ_n\left(V_{GS}, V_{DS}\right)\frac{1}{2\left(E_{F1} - E_C\right)/q}\left\langle \upsilon_x^+\left(E_{F1}\right)\right\rangle \\ &Q_n\left(V_{GS}, V_{DS}\right) = qn_S \\ &\left\langle \upsilon_x^+\left(E_{F1}\right)\right\rangle = \frac{2}{\pi}\upsilon_F\end{aligned}}$$

Unit 3 HW Solutions (continued)

Comparing the fully degenerate case to the non-degenerate case, (*), we see that

$$2\left(k_B T/q\right) \rightarrow 2\left(E_F - E_C\right)/q$$

$$\upsilon_T \rightarrow \left\langle \upsilon_x^+\left(E_F\right)\right\rangle = \frac{2}{\pi}\upsilon_F$$

8) The ballistic model we developed in Lecture 3.4,

$$I_{DS} = W\left|Q_n\left(V_{GS}, V_{DS}\right)\right|\upsilon_T\left(\frac{1 - e^{-qV_{DS}/k_B T}}{1 + e^{-qV_{DS}/k_B T}}\right)$$

assumed non-degenerate carrier statistics. In this case, the drain current saturates when $V_{DS} >> k_B T$. Answer the following questions about the T = 0 K drain saturation voltage.

8a) Derive an expression for V_{DSAT}.

8b) Compare nondegenerate and degenerate drain saturation voltages for a silicon MOSFET biased at a gate voltage so that $n_S = 10^{13}\ \text{cm}^{-3}$.

Solution 8a):

From the solution to problem 2b), we found

$$I = \frac{2}{3}\left(2q/h\right)Wg_v\frac{\sqrt{2m^*}}{\pi\hbar}\mathcal{T}_0\left\{\left(E_{F1} - E_C\right)^{3/2} - \left(E_{F2} - E_C\right)^{3/2}\right\}$$

When

$E_{F1} > E_C$ and $E_{F2} = E_{F1} - qV < E_C$

there are no channels for electrons injected from contact 2, so

$$I = \frac{2}{3}\left(2q/h\right)Wg_v\frac{\sqrt{2m^*}}{\pi\hbar}\mathcal{T}_0\left(E_{F1} - E_C\right)^{3/2}.$$

Alternatively, the above equation is valid when $E_{F1} > E_C \quad qV > E_{F1} - E_C$.

We conclude that

$$V_{DSAT} = \left(E_{F1} - E_C\right)/q \qquad (*)$$

The location of the source Fermi level is determined by the mobile charge density in the channel. For ,$V_D \geq V_{DSAT}$ only positive velocity states are occupied, so

Unit 3 HW Solutions (continued)

$$n_S = \int_{E_C}^{\infty} \frac{D_{2D}}{2}\left(E\right)f_0\left(E\right)dE = \int_{E_C}^{E_{F1}}\left(g_v \frac{m^*}{2\pi\hbar^2}\right)\left(1\right)dE = g_v \frac{m^*}{2\pi\hbar^2}\left(E_{F1} - E_C\right)$$

from which we find:

$$\left(E_{F1} - E_C\right) = \frac{2\pi\hbar^2 n_S}{g_v m^*},$$

which can be used in (*) to find

$$\boxed{V_{DSAT} = \frac{\left(E_{F1} - E_C\right)}{q} = \frac{2\pi\hbar^2 n_S}{q g_v m^*}}$$

Solution 8b):

Putting in the relevant numbers, we find:

$$V_{DSAT} = 2\pi \frac{\hbar}{q}\frac{\hbar n_S}{g_v m^*} = 0.13\ \text{V}$$

Assume that for the non-degenerate case, $V_{DSAT} \approx 3\frac{k_B T}{q} = 0.078\,\text{V}$.

We conclude that the drain saturation voltage is higher when carrier degeneracy is included.

$$\boxed{\begin{aligned} &V_{DSAT}\left(\text{nondegenerate}\right) = 0.08\ \text{V} \\ &V_{DSAT}\left(\text{degenerate}\right) = 0.13\ \text{V}\end{aligned}}$$

9) Now let's re-visit prob. 1) but this time assuming fully degenerate conditions. Answer the following questions.

9a) Derive an expression for the ballistic mobility at $T = 0$ K. (Remember that mobility is defined for small drain bias.)

9b) Numerically evaluate the ballistic mobility for the MOSFET of prob. 1) and compare your answer to the result obtained by assuming nondegenerate conditions.

Unit 3 HW Solutions (continued)

Solution 9a):
In prob. 7), we found the ballistic conductance to be

$$G_B = W(qn_S)\frac{1}{2(E_F - E_C)/q}\left\{\frac{2}{\pi}\sqrt{\frac{2(E_{F1}-E_C)}{m^*}}\right\}$$

Write this as

$$G_B = n_S q \mu_B \frac{W}{L}$$

and equate the two expressions to find

$$n_S q \mu_B \frac{W}{L} = W(qn_S)\frac{1}{2(E_F - E_C)/q}\left\{\frac{2}{\pi}\sqrt{\frac{2(E_{F1}-E_C)}{m^*}}\right\}$$

Solve for the ballistic mobility:

$$\mu_B = \frac{L}{2(E_F - E_C)/q}\left\{\frac{2}{\pi}\sqrt{\frac{2(E_{F1}-E_C)}{m^*}}\right\}$$

which could be written as

$$\boxed{\mu_B = \frac{\langle \upsilon_x^+(E_F)\rangle L}{2(E_F - E_C)/q}}$$

Solution 9b):
Begin with

$$\mu_B = \frac{L}{2(E_F - E_C)/q}\left\{\frac{2}{\pi}\sqrt{\frac{2(E_{F1}-E_C)}{m^*}}\right\} \quad (*)$$

Now solve for the location of the Fermi level in terms of the carrier density

$$n_S = g_v \frac{m^*}{\pi\hbar^2}\left(E_F - E_C\right)$$

$$\left(E_F - E_C\right) = n_S\left(\frac{\pi\hbar^2}{g_V m^*}\right)$$

insert in (*) to find

$$\mu_B = \frac{2qL}{h}\sqrt{\frac{2g_V}{\pi n_S}}$$

Unit 3 HW Solutions (continued)

Now insert *L* = 20 nm, $g_V = 2$, and $n_S = 10^{13}\ \text{cm}^{-2}$ to find

$$\boxed{\mu_B = 345\,\text{cm}^2/\text{V-s}}$$

This should be compared to the value of 473 that was obtained in prob. 1) using nondegenerate carrier statistics. In practice, MOSFETs in the on-state operate between the nondegenerate and fully degenerate limits and the ballistic mobility should be defined in terms of Fermi-Dirac integrals.