**Fundamentals of Nanotransistors**
**Unit 2 Homework SOLUTIONS**
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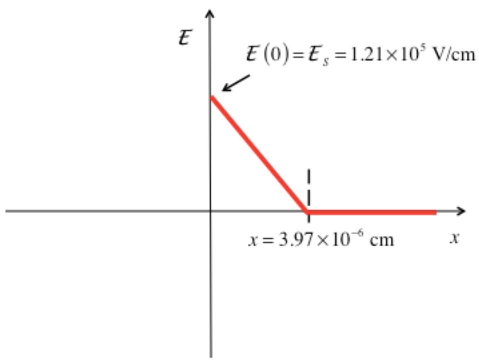
1) This problem concerns an MOS capacitor that consists of a metal electrode, a 2 nm thick silicon dioxide layer with $\kappa_{ox} = 3.9$, and a silicon substrate ($\kappa_S = 11.8$) that begins at $x = 0$. The electric field in the semiconductor is shown below. Assume that there is no charge at the oxide-Si interface, and no charge in the oxide.




Answer the following questions.

1a) What is the surface potential, $\psi_S$? (Assume that the potential is zero deep in the semiconductor.

**Solution:**

$$\psi_S = \frac{1}{2}\mathcal{E}_S W = 0.5\times\left(1.21\times10^5\right)\times\left(3.97\times10^{-6}\right) = 0.24$$

$$\boxed{\psi_S = +0.24\text{ V}}$$

The sign is positive, because the electric field > 0, so the potential at the surface must be greater than the potential in the bulk.

1b) What is the doping density in the semiconductor?

**Solution:**
Use the Poisson equation in the depletion approximation:

$$\frac{d\mathcal{E}}{dx} = \frac{\rho}{\kappa_S\epsilon_0} = -\frac{qN_A}{\kappa_S\epsilon_0}$$

$$\frac{d\mathcal{E}}{dx} = \frac{0-\mathcal{E}_S}{W} = -\frac{\mathcal{E}_S}{W}$$

Putting the above two equations together, we find

$$N_A = \frac{\kappa_S \varepsilon_0}{qW} \mathcal{E}_S$$

$$N_A = \frac{11.8 \times 8.854 \times 10^{-14}}{\left(1.6 \times 10^{-19}\right)\left(3.97 \times 10^{-6}\right)} 1.21 \times 10^{5} = 1.99 \times 10^{17}$$

$$\boxed{N_A = 1.99 \times 10^{17} \text{ cm}^{-3}}$$

1c) What is the electric field in the oxide?

**Solution:**
The D-field is continuous because there is no charge at the oxide-Si interface. So:

$$\kappa_{ox} \varepsilon_0 \mathcal{E}_{ox} = \kappa_S \varepsilon_0 \mathcal{E}_S \qquad \mathcal{E}_{ox} = \frac{\kappa_S}{\kappa_{ox}} \mathcal{E}_S = \frac{11.8}{3.9} \mathcal{E}_S = 3.03 \mathcal{E}_S$$

$$\mathcal{E}_{ox} = 3.03 \times 1.21 \times 10^{5} = 3.67 \times 10^{5}$$

$$\boxed{\mathcal{E}_{ox} = 3.67 \times 10^{5} \text{ V/cm}}$$

1d) What is the voltage drop across the oxide?

**Solution:**
The electric field in the oxide is constant because there is no charge in the oxide, so:

$$\Delta V_{ox} = \mathcal{E}_{ox} t_{ox}$$

$$\Delta V_{ox} = \left(3.67 \times 10^{5}\right) \times 2 \times 10^{-7} = 0.073$$

$$\boxed{\Delta V_{ox} = 0.073 \text{ V}}$$

1e) What is the electrostatic potential in the metal gate?

**Solution:**

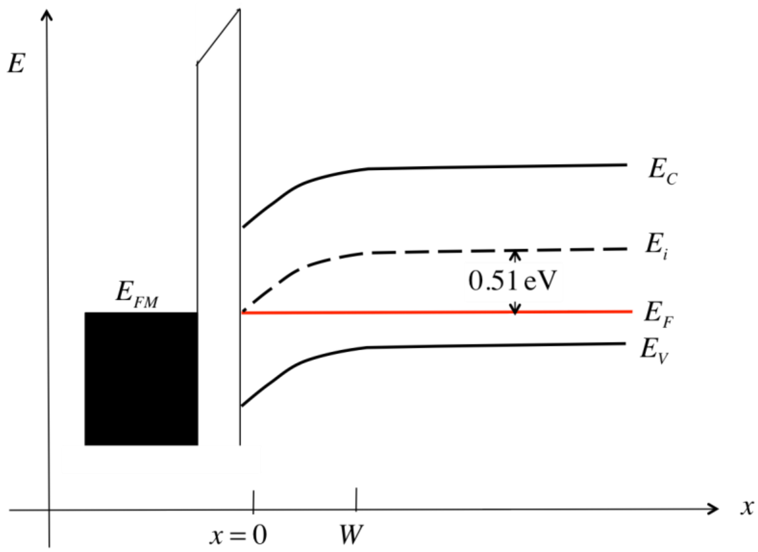
Add the volt drop across the semiconductor and the volt drop across the oxide:

$$V_{metal} = \Delta V_{ox} + \psi_S = 0.073 + 0.24 = 0.31 \text{ V}$$

$$\boxed{V_{metal} = 0.31 \text{ V}}$$

## Unit 2 HW Solutions (continued)

2) The energy band diagram for an MOS capacitor is sketched below. Assume $T$ = 300K and an oxide thickness of $t_{ox} = 1.1$ nm, and an intrinsic carrier concentration of $n_i = 1 \times 10^{10}$ cm$^{-3}$. Answer the following questions using the depletion approximation as needed. (Note that $E_F = E_i$ at the oxide-silicon interface.)



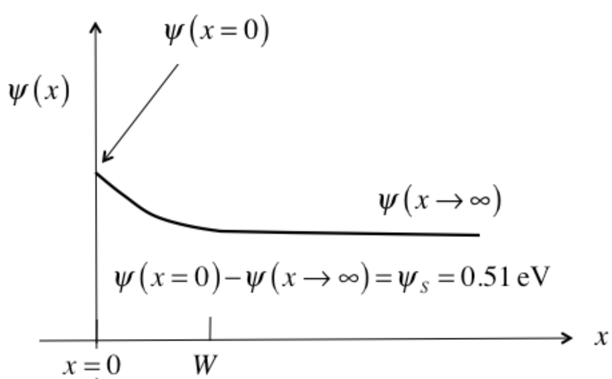

2a) What is the surface potential?

**Solution:**

$$\boxed{\psi_S = \psi(x=0) - \psi(x \to \infty) = 0.51\ \text{V}}$$

2b) Sketch the electrostatic potential vs. position inside the semiconductor. Label the surface potential on your sketch.

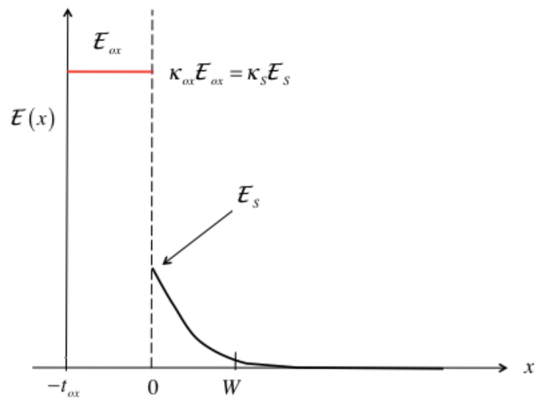
**Solution:**



## Unit 2 HW Solutions (continued)

2c) Sketch the electric field vs. position inside the oxide and semiconductor.

**Solution:**



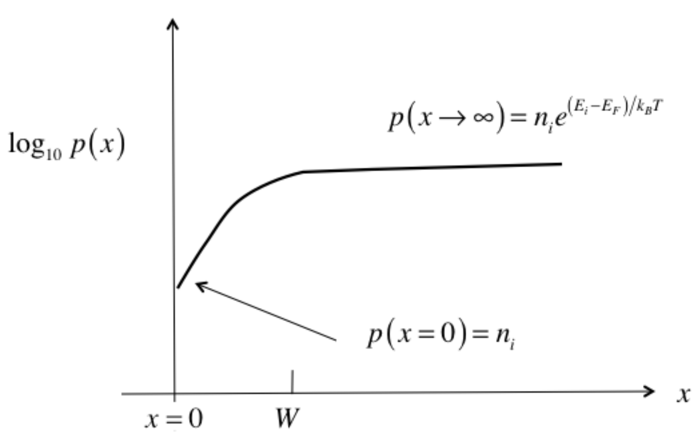

**Note:** We assume that there is no charge in the oxide and no charge at the oxide-Si interface.

2d) Do equilibrium conditions apply inside the semiconductor? Explain

**Solution:**
YES. The Fermi level is constant, but even if the Fermi level in the metal does not align with the Fermi level in the semiconductor (as is the case when a gate voltage is applied) the oxide insures that no current flows, so the metal and semiconductor are two separate systems in equilibrium with possibly different Fermi levels. (Note: We assume that light is not shining on the semiconductor.)

2e) Sketch the hole concentration vs. position inside the semiconductor.

**Solution:**



**Unit 2 HW Solutions (continued)**

2f) What is the hole concentration in the bulk?

**Solution:**

$$p(x \to \infty) = n_i e^{(E_i - E_F)/k_B T} = 10^{10} e^{0.51/0.026} = 3.3 \times 10^{18} \text{ cm}^{-3}$$

$$\boxed{p(x \to \infty) = N_A = 3.3 \times 10^{18} \text{ cm}^{-3}}$$

2g) What is the hole concentration at the surface?

**Solution:**

$$p(x = 0) = n_i e^{(E_i - E_F)/k_B T} = 10^{10} e^{0} = 1 \times 10^{10}$$

$$\boxed{p(x = 0) = 10^{10} \text{ cm}^{-3}}$$

2h) What is the gate voltage?

**Solution:**
The Fermi level in the metal aligns with the Fermi level in the semiconductor, so the gate voltage must be **zero.** (**Note:** The fact that there is a volt drop across the oxide and the semiconductor with $V_G = 0$ indicates that there is a workfunction difference between the metal and the semiconductor.)

2i) What is the voltage drop across the oxide?

**Solution:**
The electric field at the surface of the semiconductor is given by:

$$\mathcal{E}_S = \sqrt{\frac{2qN_A\psi_S}{\kappa_S\epsilon_0}}$$

We find the electric field in the oxide from:

$$\kappa_{ox}\mathcal{E}_{ox} = \kappa_S\mathcal{E}_S$$

so

$$\mathcal{E}_{ox} = \frac{\kappa_S}{\kappa_{ox}}\sqrt{\frac{2qN_A\psi_S}{\kappa_S\epsilon_0}}$$

The volt drop across the oxide is:

$$\Delta V_{ox} = t_{ox}\mathcal{E}_{ox} = t_{ox}\frac{\kappa_S}{\kappa_{ox}}\sqrt{\frac{2qN_A\psi_S}{\kappa_S\epsilon_0}}.$$

Alternatively we can write this as

$$\Delta V_{ox} = \frac{t_{ox}}{\kappa_{ox}\epsilon_0}\sqrt{2q\kappa_S\epsilon_0 N_A\psi_S} = -\frac{Q_B(\psi_S)}{C_{ox}},$$

where

$Q_B(\psi_S)$ is the depletion charge in C/cm² in the semiconductor and
$C_{ox}$ is the oxide capacitance in F/cm²

Putting in numbers:

$$C_{ox} = \frac{\kappa_{ox}\epsilon_0}{t_{ox}} = \frac{3.9\times 8.854\times 10^{-14}}{1.1\times 10^{-7}} = 3.1\times 10^{-6}\ \text{F/cm}^2$$

We found the doping density in 2f) and the surface potential in 2a), so

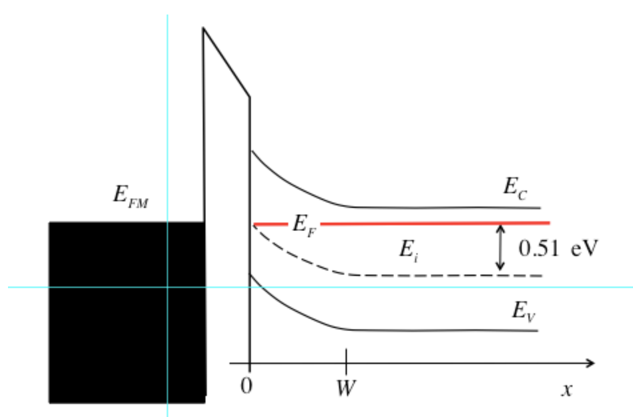
$$Q_D = -\sqrt{2q\kappa_S\epsilon_0 N_A\psi_S} = -\sqrt{2\times 1.6\times 10^{-19}\times 11.8\times 8.854\times 10^{-14}\times 3.3\times 10^{18}\times 0.51}$$

$$Q_D = -7.5\times 10^{-7}\ \text{C/cm}^2$$

$$\Delta V_{ox} = -\frac{Q_D(\psi_S)}{C_{ox}} = \frac{7.5\times 10^{-7}}{3.1\times 10^{-6}} = 0.24$$

$$\boxed{\Delta V_{ox} = 0.24\ \text{V}}$$

**Question:** If the substrate were n-type, as in the figure below, could you repeat this problem?



## Unit 2 HW Solutions (continued)

3) Consider an MOS capacitor with a gate oxide 1.2 nm thick. The Si substrate doping is $N_A = 10^{18}$ cm$^{-3}$. The gate voltage is selected so that the sheet density of electrons in the inversion layer is $n_S = 10^{13}$ cm$^{-2}$. Assume room temperature and that $|Q_n| = qn_S \approx \sqrt{2\kappa_S \epsilon_0 k_B T\, n_i^2/N_A}\; e^{+q\psi_s/2k_BT}$ C/cm$^2$. Answer the following questions.

3a) What is the surface potential? Compare it with $2\psi_F$.

**Solution:**

$$|Q_i| = qn_S \approx \sqrt{2\epsilon_S k_B T\, n_i^2/N_A}\; e^{+q\psi_s/2k_BT}$$

$$\psi_S = \frac{2k_BT}{q}\ln\left(\frac{qn_S}{\sqrt{2\kappa_S\epsilon_0 k_B T\, n_i^2/N_A}}\right)$$

Putting in numbers: $\boxed{\psi_S = 1.11\ \mathrm{V}}$

Now compare this to $2\psi_F$:

$$2\psi_F = 2\times 0.026\ln\left(\frac{10^{18}}{10^{10}}\right) = 0.958$$

$$\psi_S - 2\psi_F = 1.11 - 0.96 = 0.15$$

$$\boxed{\psi_S - 2\psi_F = 5.8\left(k_BT/q\right)}$$

The actual surface potential under strong inversion is about 6 $k_BT/q$ bigger than $2\psi_F$.

3b) How much does the **surface potential** need to increase to double the inversion layer density?

**Solution:**

From: $\psi_S = \dfrac{2k_BT}{q}\ln\left(\dfrac{qn_S}{\sqrt{2\kappa_{Si}\epsilon_0 k_B T\, n_i^2/N_A}}\right)$ with $n_S = 2\times 10^{13}$, we find

$\psi_S = 1.14$ so

$$\boxed{\Delta\psi_S = 0.03\ \mathrm{V}}$$

**Unit 2 HW Solutions (continued)**

3c) How much does the **gate voltage** need to increase to double the inversion layer density?

**Solution:**

$$\Delta V_G = \Delta\psi_S + \Delta V_{ox}$$

$$\Delta V_{ox} \approx -\frac{\Delta Q_n}{C_{ox}}$$

$$C_{ox} = \frac{\kappa_{ox}\varepsilon_0}{t_{ox}} = 2.88 \times 10^{-6}$$

$$\Delta\psi_{ox} \approx -\frac{\Delta Q_n}{C_{ox}} = \frac{q \times 10^{13}}{2.88 \times 10^{-6}} = 0.56 \text{ V}$$

$$\Delta V_G = \Delta\psi_S + \Delta V_{ox} = 0.03 + 0.56 = 0.59 \text{ V}$$

$$\boxed{\Delta V_G = 0.59}$$

3d) Explain in words why it is difficult to increase the surface potential for an MOS capacitor above threshold.

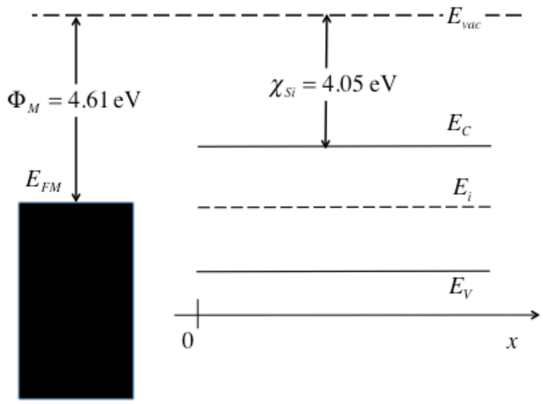
**Solution:**

It takes only a small amount of additional band bending in the semiconductor to double the inversion layer charge because the inversion layer charge increases exponentially with surface potential.

The increased inversion layer charge leads to a large increase in the electric field in the oxide, which increases the volt drop in the oxide and as a result, the gate voltage.

So above threshold, it take a large increase in the gate voltage to increase the surface potential a little bit because most of the increase in gate voltage is lost in the volt drop across the oxide.

4) Gate workfunctions must be chosen to produce the desired threshold voltage. If there is a difference between the metal gate and the semiconductor, then the flatband voltage is not zero. Consider the "mid-gap" workfunction shown below and assume that the semiconductor is silicon at room temperature with $N_C = 3.23 \times 10^{19}$ cm$^{-3}$ and $N_V = 1.83 \times 10^{19}$ cm$^{-3}$.




4a) If the semiconductor is n-type with $N_D = 10^{18}$ cm$^{-3}$, then what is the flatband voltage? Explain the sign of the flatband voltage.

**Solution:**

$$n_0 = 10^{18} = N_C e^{(E_F - E_C)/k_BT} \Longrightarrow (E_F - E_C) = k_BT \ln\left(\frac{n_0}{N_C}\right) = 0.026 \ln\left(\frac{10^{18}}{3.23 \times 10^{19}}\right)$$

$$(E_F - E_C) = -0.09 \text{ eV}$$

$$\Phi_S = \chi_{Si} + (E_C - E_F) = 4.05 + 0.09 = 4.14$$

$$V_{FB} = (\Phi_M - \Phi_S)/q = (4.61 - 4.14) = 0.47$$

$$\boxed{V_{FB} = +0.47 \text{ V}}$$

**Explanation:** The Fermi level of the metal (before coming into contact with the oxide and semiconductor) is **below** that of the n-type semiconductor, so when brought together, a negative built-in voltage (with respect to the semiconductor) must develop on the metal gate to align the Fermi levels. To undo this negative built-in potential, a positive flatband voltage must develop.

4b) If the semiconductor is p-type with $N_A = 10^{18}$ cm$^{-3}$, then what is the flatband voltage? Explain the sign of the flatband voltage.

**Solution:**

$$p_0 = 10^{18} = N_V e^{(E_V - E_F)/k_BT} \Longrightarrow (E_V - E_F) = k_BT \ln\left(\frac{p_0}{N_V}\right) = 0.026 \ln\left(\frac{10^{18}}{1.83 \times 10^{19}}\right)$$

$$(E_V - E_F) = -0.076 \text{ eV}$$

$$\Phi_S = \chi_{Si} + E_G - \left(E_F - E_V\right) = 4.05 + 1.12 - 0.076 = 5.09$$
$$V_{FB} = \left(\Phi_M - \Phi_S\right)/q = \left(4.61 - 5.09\right) = -0.48$$
$$\boxed{V_{FB} = -0.48 \text{ V}}$$

**Explanation:** The Fermi level of the metal (before coming into contact with the oxide and semiconductor) is **above** that of the p-type semiconductor, so when brought together, a positive built-in voltage (with respect to the semiconductor) must develop on the metal gate to align the Fermi levels. To undo this positive built-in potential, a negative flatband voltage must develop. Note that even though the Fermi level of the isolated metal was exactly in the middle of the gap, the flatband voltages for the two cases are not exactly equal in magnitude because the conduction and valence band effective densities of states are a little different.

5) Assume an MOS capacitor on a p-type Si substrate with the following parameters:

$N_A = 2.7 \times 10^{18}\ \text{cm}^{-3}$ for the bulk doping   Oxide thickness: $t_{ox} = 1.1\ \text{nm}$ $\kappa_{ox} = 3.9$
$Q_F = 0$ (no charge at the oxide-Si interface)   $T = 300\ \text{K}$   $V_G = 1\ \text{V}$

Also assume that the structure is ideal with no metal-semiconductor workfunction difference. When working MOS problems, it is easiest to assume a surface potential, and then calculate the gate voltage that produced it, but in practice, we only have access to the gate terminal. In this problem, you are given the gate voltage and will be asked to find the surface potential. You may assume the depletion approximation, but you must check to be sure that it is valid.

Find the surface potential, $\psi_S$.

**Solution:**

$$V_G = V_{FB} + \psi_S + \Delta V_{ox} = V_{FB} + \psi_S - \frac{Q_S\left(\psi_S\right)}{C_{ox}} = V_{FB} + \psi_S + \frac{\sqrt{2q\kappa_S\varepsilon_0 N_A}}{C_{ox}}\sqrt{\psi_S}$$

We are assuming here that all of the charge in the semiconductor is depletion charge - no inversion charge. This assumption will be checked later. We are told that there is no metal-semiconductor workfunction difference and no charge at the oxide-Si interface, so $V_{FB} = 0$. We find:

$$V_G = \psi_S + b\sqrt{\psi_S} \qquad b = \frac{\sqrt{2q\kappa_S\varepsilon_0 N_A}}{C_{ox}} = 0.303$$

$\psi_S + b\sqrt{\psi_S} - V_G = 0$ is a quadratic equation for $\sqrt{\psi_S}$

**Unit 2 HW Solutions (continued)**

$\sqrt{\psi_S} = \dfrac{-b \pm \sqrt{b^2 + 4V_G}}{2}$ (take positive sign. Since there is a positive gate voltage, we should get a positive surface potential.)

Putting in numbers, we find:

$\psi_S = 0.74 \text{ V}$

Is this greater than $2\psi_F$?

$$\psi_F = \frac{k_B T}{q} \ln\left(\frac{N_A}{n_i}\right) = 0.026 \times \ln\left(\frac{2.7 \times 10^{18}}{10^{10}}\right) = 0.505$$

$2\psi_F = 1.01\text{V}$

Our use of the depletion approximation for $Q_S(\psi_S)$ in the first equation is justified, and

$\boxed{\psi_S = 0.74 \text{ V}}$