

4.7. Law of Equilibrium

Consider a system with two one-electron energy levels having the same energy ϵ in equilibrium with an electrochemical potential μ : $x = \frac{\epsilon - \mu}{kT}$

4.7a If we assume that the interaction energy $U_0 = 0$, the average number of electrons in the system (both levels added together) at equilibrium is given by

(a) $\langle N \rangle = \frac{2}{1 + 2e^x}$

(b) $\langle N \rangle = \frac{2}{1 + e^x}$

(c) $\langle N \rangle = \frac{1}{1 + 2e^x}$

(d) $\langle N \rangle = \frac{1}{2 + e^x}$

(e) None of the above

$$1 = p_{00} + p_{01} + p_{10} + p_{11} = \frac{1 + 2e^{-x} + e^{-2x}}{Z} \rightarrow Z = (1 + e^{-x})^2$$

$$\langle N \rangle = p_{01} + p_{10} + 2p_{11} = \frac{2e^{-x} + 2e^{-2x}}{Z} = \frac{2e^{-x}}{1 + e^{-x}} = \frac{2}{1 + e^x}$$

4.7b If we assume that the interaction energy U_0 is extremely large so that the probability of having two electrons is zero, then the average number of electrons in the system at equilibrium is given by

(a) $\langle N \rangle = \frac{2}{2 + e^x}$

(b) $\langle N \rangle = \frac{2}{1 + e^x}$

(c) $\langle N \rangle = \frac{1}{1 + 2e^x}$

(d) $\langle N \rangle = \frac{1}{2 + e^x}$

(e) None of the above

$$1 = p_{00} + p_{01} + p_{10} = \frac{1 + 2e^{-x}}{Z} \rightarrow Z = 1 + 2e^{-x}$$

$$\langle N \rangle = p_{01} + p_{10} = \frac{2e^{-x}}{Z} = \frac{2e^{-x}}{1 + 2e^{-x}} = \frac{2}{2 + e^x}$$