FUNDAMENTALS OF NANOELECTRONICS

Basic Concepts

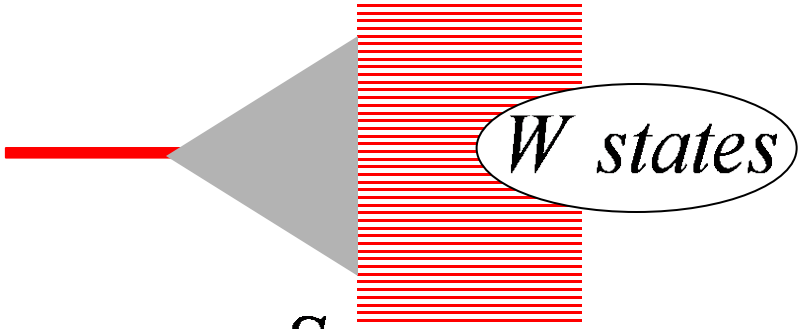
1. The New Perspective
2. Energy Band Model
3. What and Where is the Voltage?

Heat & Electricity:
Second Law & Information

4.1. Introduction
4.2. Seebeck Coefficient
4.3. Heat Current
4.4. One-level Device
4.5. Second Law
4.6. Entropy
4.7. Law of Equilibrium
4.8. Shannon Entropy
4.9. Fuel Value of Information
4.10. Summing up ..

System | Contact

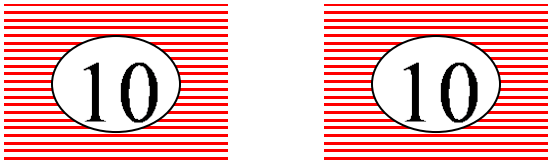
4.8a Shannon Entropy



$$\frac{S}{k} = \ell n\, W$$

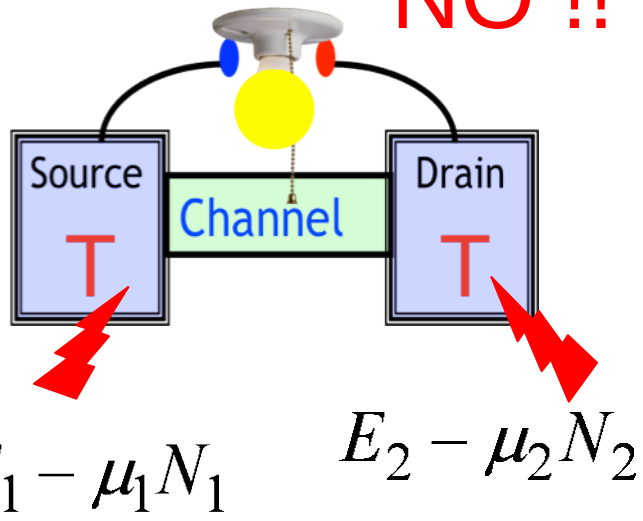
- 4.5. Second Law
- 4.6. Entropy
- 4.7. Law of Equilibrium

$$\ell n\,(10 \times 10) = \ell n\, 10 + \ell n\, 10$$



Entropy is additive

NO !!



4.8b Shannon Entropy

$$\ell n\, n! \approx n\ell n n - n$$

$$n_i = n\,\tilde{p}_i$$

$$\sum_i n_i = n$$

$$\frac{S}{k} = \ell n\, W$$

$$= \ell n \frac{n!}{n_1! n_2! \cdots}$$

$$= \ell n n! - \ell n n_1! - \ell n n_2! - \cdots$$

$$= n\ell n n - n_1 \ell n n_1 - n_2 \ell n n_2 - \cdots$$

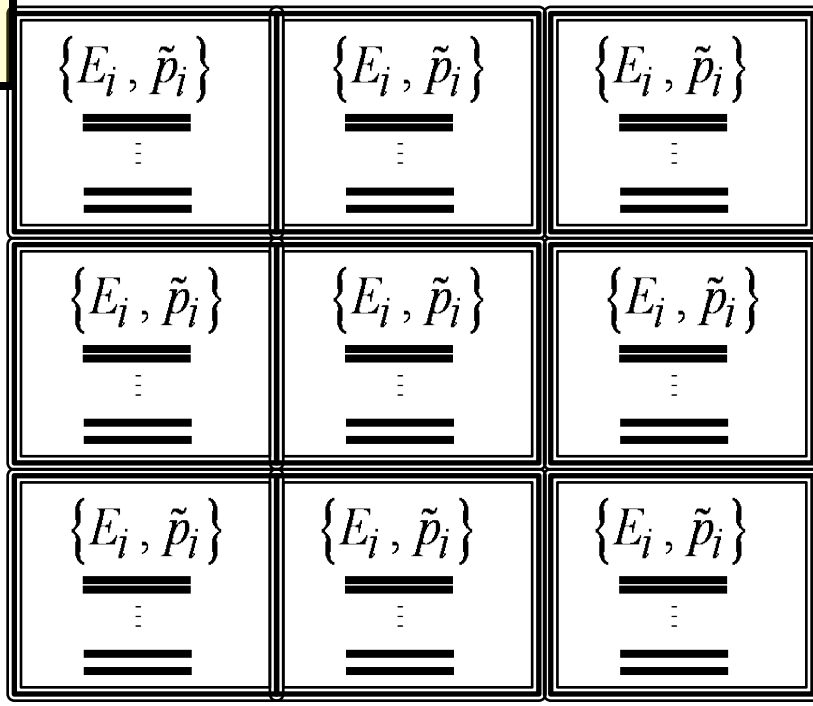
$$-(n - n_1 - n_2 - \cdots)$$

$$= n\ell n n - n_1 \ell n n - n_2 \ell n n - \cdots$$

$$- n_1 \ell n\, \tilde{p}_1 - n_2 \ell n\, \tilde{p}_2 - \cdots$$

$$= - n\,\tilde{p}_1 \ell n\, \tilde{p}_1 - n\,\tilde{p}_2 \ell n\, \tilde{p}_2 - \cdots$$

n identical systems



Entropy is additive

$$\frac{S}{k} = -n \sum_i \tilde{p}_i \ell n\, \tilde{p}_i$$

4.8c Shannon Entropy

Equilibrium

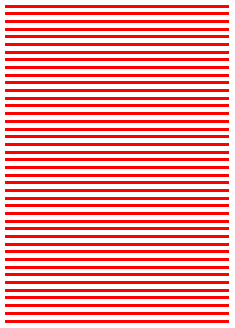
$$p_i = \frac{1}{Z} e^{-(E_i - \mu N_i)/kT}$$

$$\rightarrow \quad \ell n\, p_i = \ell n \frac{1}{Z} - \frac{E_i - \mu N_i}{kT}$$

$$dS = -k \sum_i d\tilde{p}_i \, \ell n\, \tilde{p}_i + \tilde{p}_i \frac{d\tilde{p}_i}{\tilde{p}_i}$$

$$= -k \sum_i dp_i \, \ell n \frac{1}{Z} + \sum_i dp_i \frac{E_i - \mu N_i}{T}$$

$$\rightarrow \quad dS = \frac{1}{T} dE - \frac{\mu}{T} dN$$



$$\{E_i, N_i, \tilde{p}_i\}$$

$$S = -k \sum_i \tilde{p}_i \, \ell n\, \tilde{p}_i$$

$$E = \sum_i E_i \, \tilde{p}_i$$

$$N = \sum_i N_i \, \tilde{p}_i$$

$$0 \le \tilde{p}_i \le 1$$

NOT necessarily equilibrium

4.8d Shannon Entropy

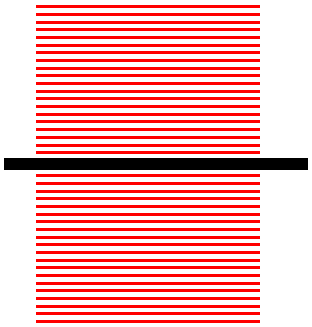
$S = 0$

$E = 0$

$S = nk\ell n2$

Equilibrium

$\{E_i, N_i, \tilde{p}_i\}$

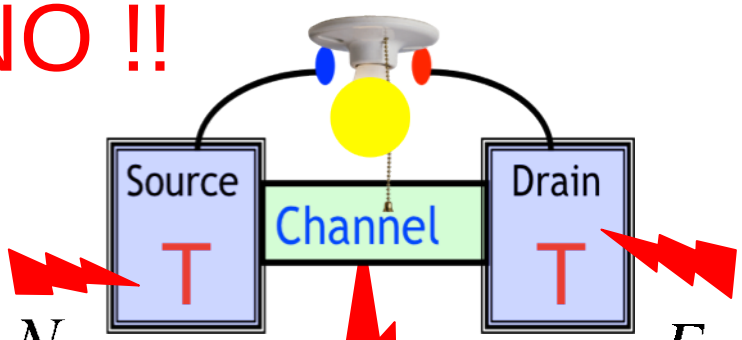


$$S = -k\left(\frac{1}{2}\ell n\frac{1}{2}+\frac{1}{2}\ell n\frac{1}{2}\right) = k\ell n2 \longleftarrow S = -k\sum_i \tilde{p}_i \ell n\,\tilde{p}_i$$

$$H = -\sum_i \tilde{p}_i \ell n\,\tilde{p}_i \rightarrow \ell n2$$

Information Content

X NO !!



$E_1 - \mu_1 N_1$ $\qquad$ $E_2 - \mu_2 N_2$

Can extract energy $nkT\,\ell n2$ without violating Second law

$S = 0$ $\qquad E = 0$

$S = nk\,\ell n2$

$$(E_1 - \mu_1 N_1) + (E_2 - \mu_2 N_2) \leq T\Delta S$$

$$-\Delta S + \frac{E_1 - \mu_1 N_1}{T} + \frac{E_2 - \mu_2 N_2}{T} \leq 0$$

Second Law

Coming up next ..