

FUNDAMENTALS OF NANOELECTRONICS

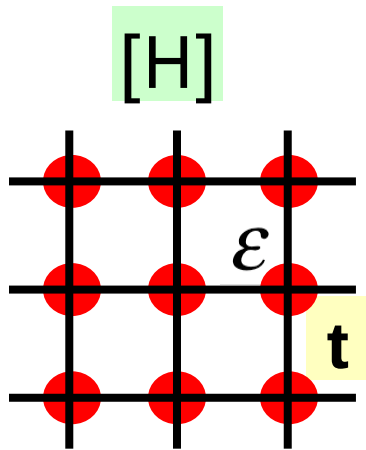
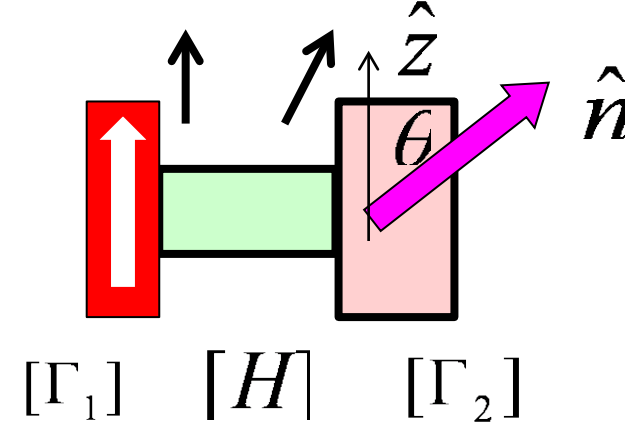
B. Quantum Transport

1. Schrodinger Equation
2. Contact-ing Schrodinger
3. More Examples

4. Spin Transport →

- 4.1. Introduction
- 4.2. Magnetic contacts
- 4.3. Rotating contacts
- 4.4. Vectors and spinors
- 4.5. Spin-orbit coupling
- 4.6. Spin Hamiltonian
- 4.7. Spin density/current
- 4.8. Spin voltage
- 4.9. Spin circuits
- 4.10. Summing up ..

4.6a Spin Hamiltonian



B3.2

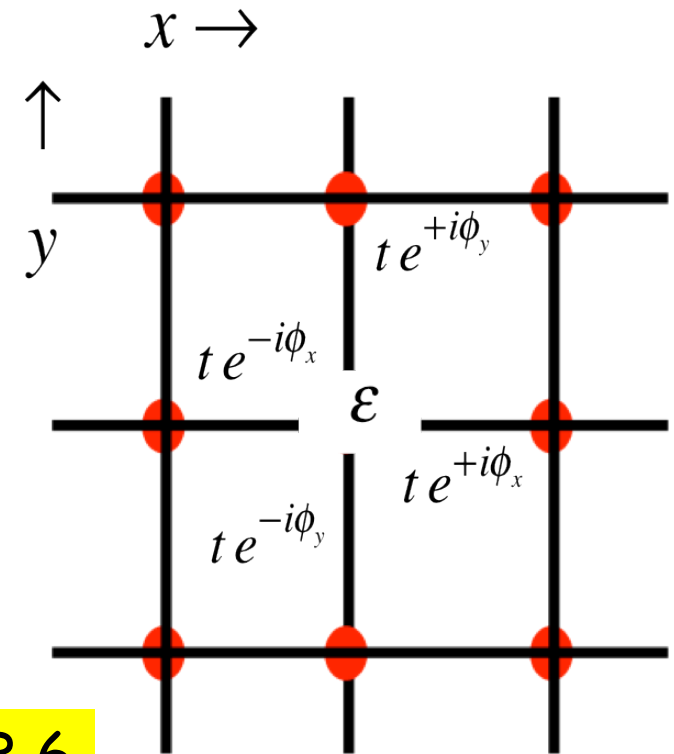
$$H = E_c + \frac{\vec{p} \cdot \vec{p}}{2m}$$

$$t = -\frac{\hbar^2}{2ma^2}$$

$$\epsilon = E_c + 4t$$

$$\phi_x = \frac{qA_x a}{\hbar}$$

$$\phi_y = \frac{qA_y a}{\hbar}$$

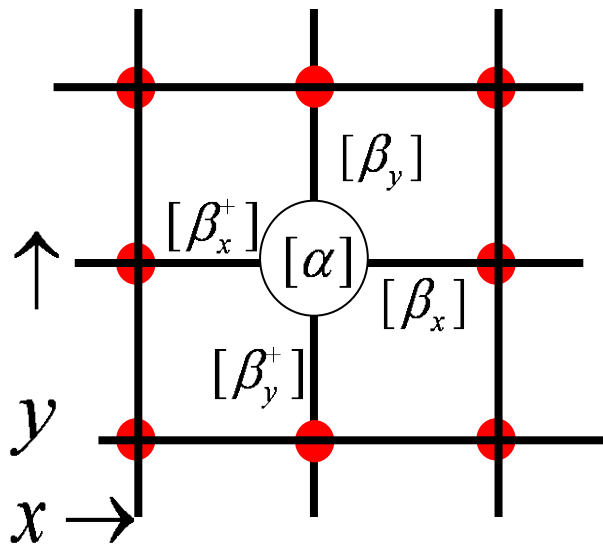


B 3.6

$$H = E_c + \frac{(\vec{p} + q\vec{A}) \cdot (\vec{p} + q\vec{A})}{2m}$$

$$p_x = -i\hbar \frac{\partial}{\partial x}$$

4.6b Spin Hamiltonian



$$[\alpha] = \varepsilon [I]$$

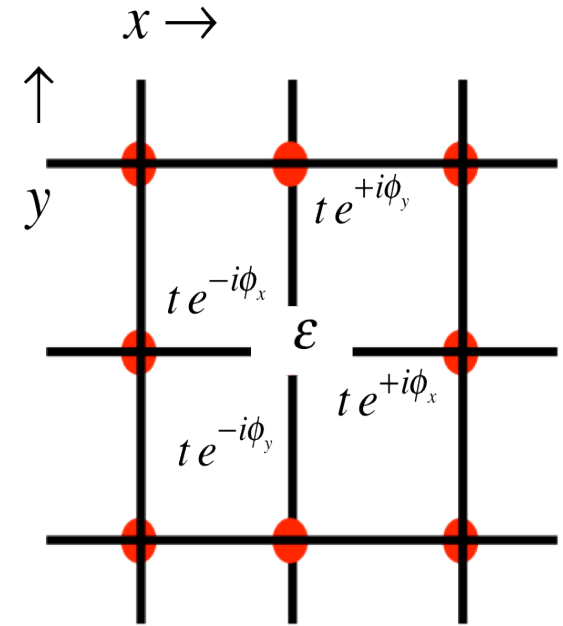
$$[\beta_x] = t e^{+i\phi_x} [I]$$

$$[\beta_x^+] = t e^{-i\phi_x} [I]$$

⋮

B 4.5

$$H = \left(E_c + \frac{(\vec{p} + q\vec{A}) \cdot (\vec{p} + q\vec{A})}{2m} \right) I$$



B 3.6

$$H = E_c + \frac{(\vec{p} + q\vec{A}) \cdot (\vec{p} + q\vec{A})}{2m}$$

$$I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, \quad \sigma_z = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}, \quad \sigma_x = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}, \quad \sigma_y = \begin{bmatrix} 0 & -i \\ +i & 0 \end{bmatrix}$$

$$p_x = -i\hbar \frac{\partial}{\partial x}$$

4.6c Spin Hamiltonian

$$h(\vec{k}) = \sum_m H_{nm} e^{+i\vec{k} \cdot (\vec{r}_m - \vec{r}_n)}$$

$$= \varepsilon \mathbf{I} + t e^{+i\phi_x} e^{+ik_x a} \mathbf{I} + t e^{-i\phi_x} e^{-ik_x a} \mathbf{I} \\ + t e^{+i\phi_y} e^{+ik_y a} \mathbf{I} + t e^{-i\phi_y} e^{-ik_y a} \mathbf{I}$$

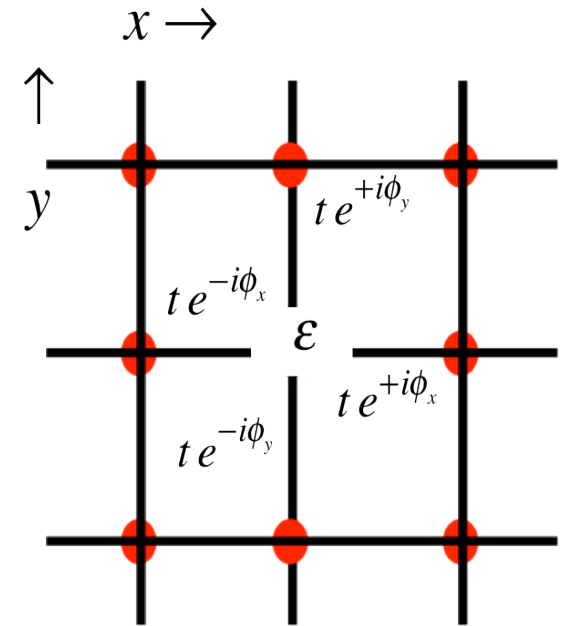
$$= \varepsilon + 2t \cos(k_x a + \phi_x) + 2t \cos(k_y a + \phi_y) \mathbf{I}$$

$$E_c + \frac{\hbar^2}{2m} \left(\left(k_x + \frac{qA_x}{\hbar} \right)^2 + \left(k_y + \frac{qA_y}{\hbar} \right)^2 \right) \mathbf{I}$$

$$E_c + \frac{1}{2m} \left((p_x + qA_x)^2 + (p_y + qA_y)^2 \right) \mathbf{I}$$

$$H = E_c + \frac{(\vec{p} + q\vec{A}) \cdot (\vec{p} + q\vec{A})}{2m} \mathbf{I}$$

$$p_x = -i\hbar \frac{\partial}{\partial x}$$



B 3.6

4.6d Spin Hamiltonian

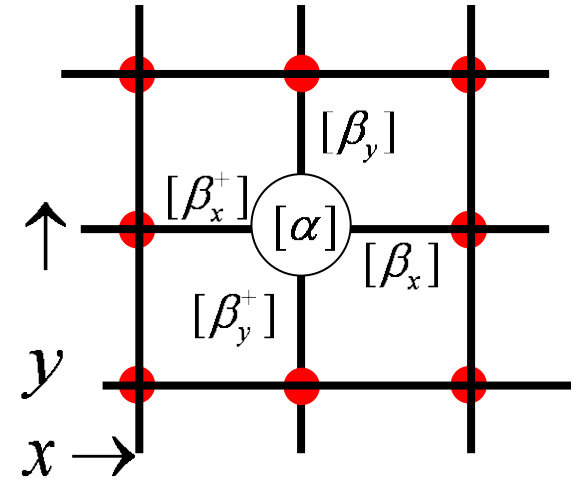
$$[\alpha] = \varepsilon [I] + \frac{g}{2} \mu_B \vec{\sigma} \cdot \vec{B}$$

$$[\beta_x] = t e^{+i\phi_x} [I] + \frac{i\eta}{2a} \sigma_y$$

$$[\beta_x^+] = t e^{-i\phi_x} [I] - \frac{i\eta}{2a} \sigma_y$$

$$[\beta_y] = t e^{+i\phi_y} [I] - \frac{i\eta}{2a} \sigma_x$$

$$[\beta_y^+] = t e^{-i\phi_y} [I] + \frac{i\eta}{2a} \sigma_x$$



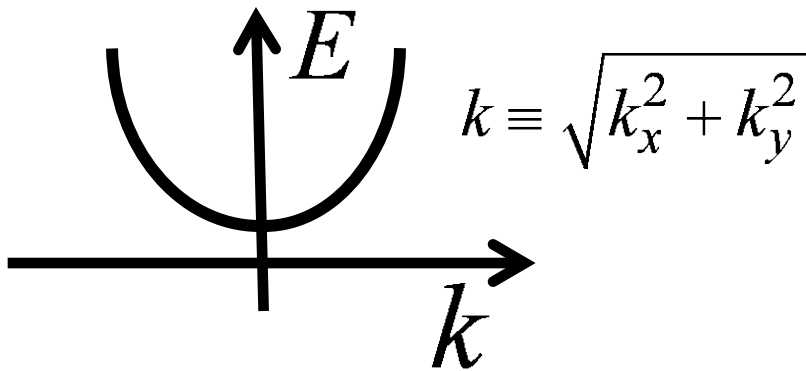
$$\approx \frac{\eta}{a} (\sigma_x \sin k_y a - \sigma_y \sin k_x a)$$

$$\frac{\eta}{2ia} \sigma_x (e^{+ik_y a} - e^{-ik_y a}) - \frac{\eta}{2ia} \sigma_y (e^{+ik_x a} - e^{-ik_x a})$$

$$H = \left(E_c + \frac{(\vec{p} + q\vec{A}) \cdot (\vec{p} + q\vec{A})}{2m} \right) I + \frac{g}{2} \mu_B \vec{\sigma} \cdot \vec{B} - \frac{\eta}{\hbar} \vec{\sigma} \cdot (\hat{z} \times \vec{p})$$

$$\eta (\sigma_x k_y - \sigma_y k_x)$$

4.6e Spin Hamiltonian



$$\vec{\sigma} \cdot \vec{V} \rightarrow \begin{matrix} \swarrow \text{spin} \parallel \vec{V} \\ \pm V \\ \nwarrow \text{spin} \parallel -\vec{V} \end{matrix}$$

$$\text{spin} \parallel -(\hat{z} \times \vec{k})$$

$$E = E_c + \frac{\hbar^2 k^2}{2m} \pm \eta k$$

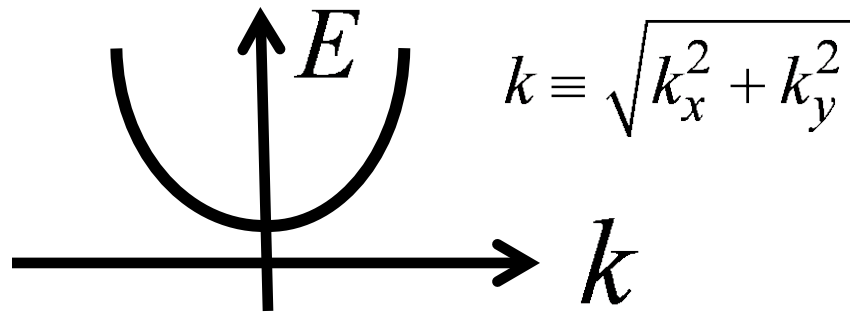
$$\text{spin} \parallel (\hat{z} \times \vec{k})$$

$$\rightarrow \left(E_c + \frac{p^2}{2m} \right) I - \frac{\eta}{\hbar} \vec{\sigma} \cdot (\hat{z} \times \vec{p})$$

Zero magnetic field

$$H = \left(E_c + \frac{(\vec{p} + q\vec{A}) \cdot (\vec{p} + q\vec{A})}{2m} \right) I + \frac{g}{2} \mu_B \vec{\sigma} \cdot \vec{B} - \frac{\eta}{\hbar} \vec{\sigma} \cdot (\hat{z} \times \vec{p})$$

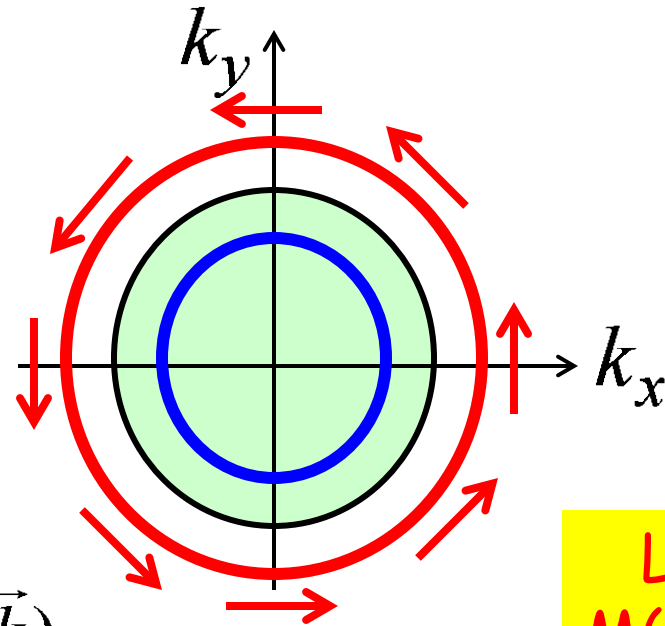
4.6f Spin Hamiltonian



$$\text{spin} \parallel -(\hat{z} \times \vec{k})$$

$$E = E_c + \frac{\hbar^2 k^2}{2m} \pm \eta k$$

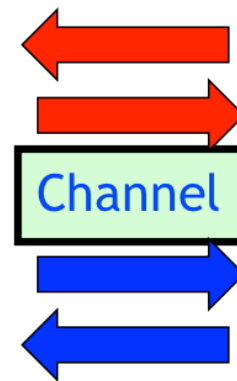
$$\text{spin} \parallel (\hat{z} \times \vec{k})$$



Larger
 $M(E)$, $D(E)$

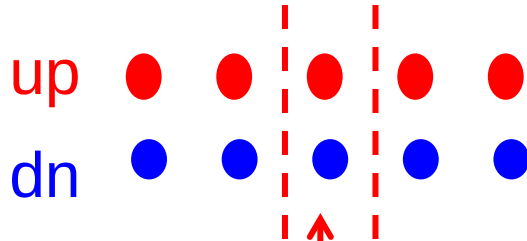
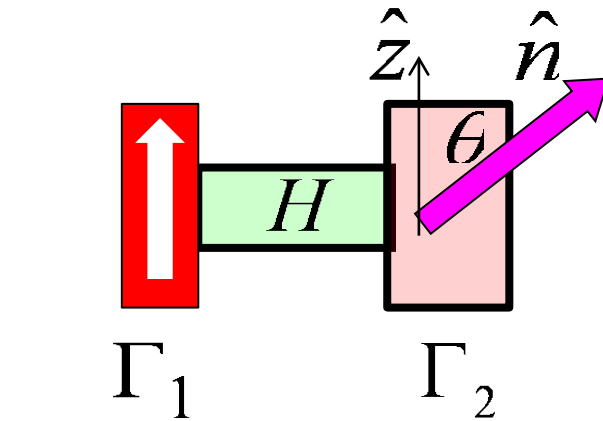
$$\pm \frac{g}{2} \mu_B \vec{\sigma} \cdot \vec{B}$$

100
100
Magnetic
50
50



100	50
100	100
Normal	Spin-orbit
100	50
100	100

Coming up next ..



$$[N, \vec{S}]$$

$$\frac{G^n}{2\pi} = \frac{1}{2} \begin{bmatrix} N + S_z & S_x - iS_y \\ S_x + iS_y & N - S_z \end{bmatrix}$$

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