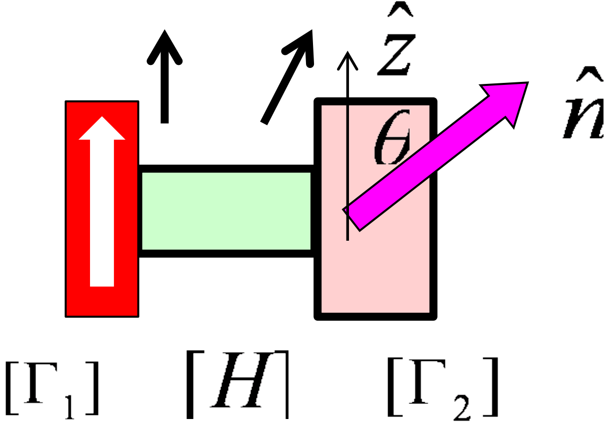
B. Quantum Transport

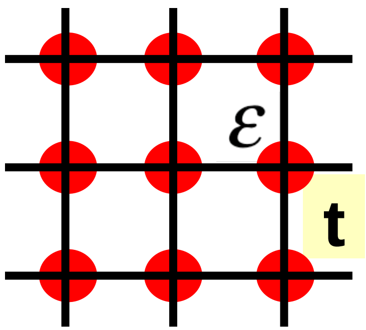
1. Schrodinger Equation
2. Contact-ing Schrodinger
3. More Examples
4. Spin Transport

Supriyo Datta

4.6a Spin Hamiltonian



[H]

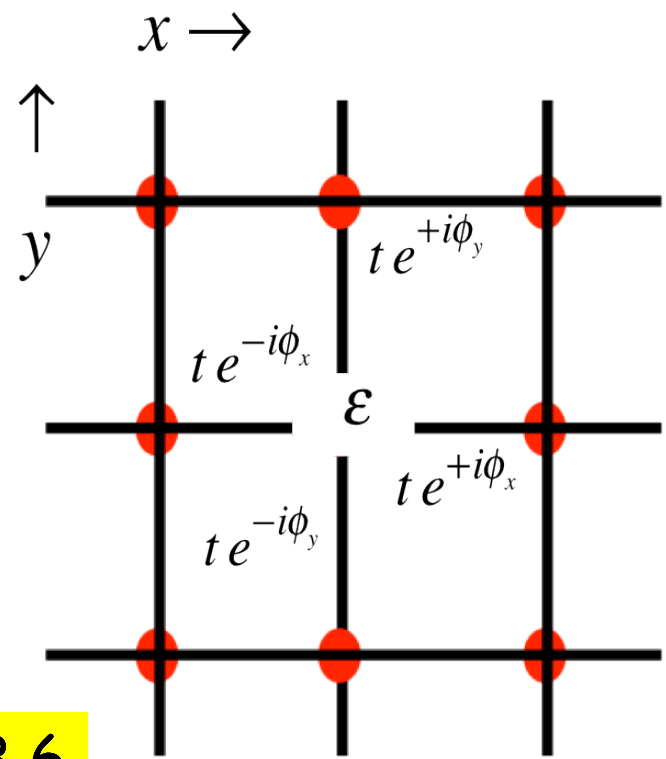


$$t = -\frac{\hbar^2}{2ma^2}$$

$$\varepsilon = E_c + 4t$$

$$\phi_x = \frac{qA_x a}{\hbar}$$

$$\phi_y = \frac{qA_y a}{\hbar}$$



B 3.6

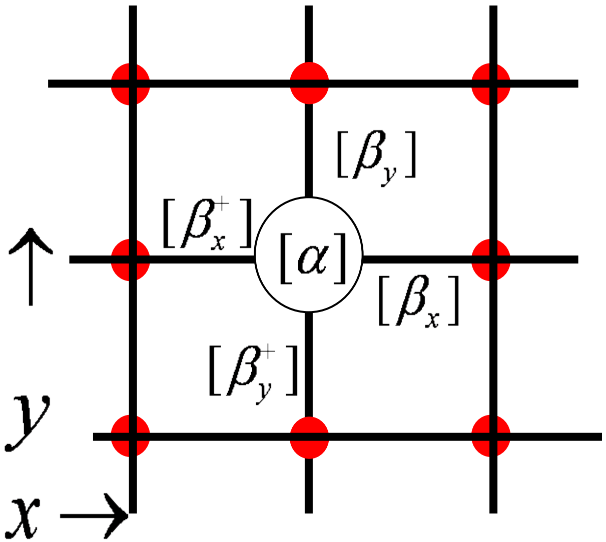
$$H = E_c + \frac{(\vec{p} + q\vec{A}).(\vec{p} + q\vec{A})}{2m}$$

B3.2

$$H = E_c + \frac{\vec{p}.\vec{p}}{2m}$$

$$p_x = -i\hbar\frac{\partial}{\partial x}$$

4.6b Spin Hamiltonian



$$[\alpha] = \varepsilon\,[I]$$

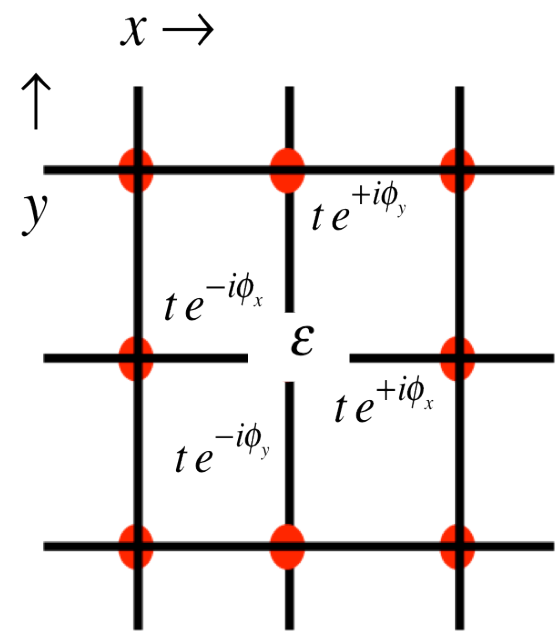
$$[\beta_x] = t\,e^{+i\phi_x}\,[I]$$

$$[\beta_x^+] = t\,e^{-i\phi_x}\,[I]$$

$$\vdots$$

B 4.5

$$H = \left(E_c + \frac{(\vec{p}+q\vec{A}).(\vec{p}+q\vec{A})}{2m} \right) I$$



B 3.6

$$H = E_c + \frac{(\vec{p}+q\vec{A}).(\vec{p}+q\vec{A})}{2m}$$

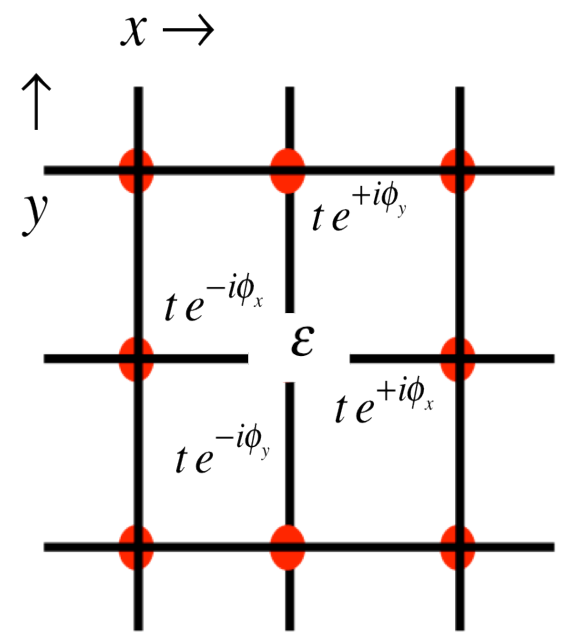
$$I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, \quad \sigma_z = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}, \quad \sigma_x = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}, \quad \sigma_y = \begin{bmatrix} 0 & -i \\ +i & 0 \end{bmatrix} \qquad p_x = -i\hbar \frac{\partial}{\partial x}$$

4.6c Spin Hamiltonian

$$h(\vec{k}) = \sum_m H_{nm}\, e^{+i\vec{k}.(\vec{r}_m - \vec{r}_n)}$$

$$= \varepsilon\, I + t\, e^{+i\phi_x} e^{+ik_x a}\, I + t\, e^{-i\phi_x} e^{-ik_x a}\, I + t\, e^{+i\phi_y} e^{+ik_y a}\, I + t\, e^{-i\phi_y} e^{-ik_y a}\, I$$

$$= \varepsilon + 2t\cos(k_x a + \phi_x) + 2t\cos(k_y a + \phi_y)\ I$$



B 3.6

$$I\ E_c + \frac{\hbar^2}{2m}\left((k_x + \frac{qA_x}{\hbar})^2 + (k_y + \frac{qA_y}{\hbar})^2 \right) I$$

$$I\ E_c + \frac{1}{2m}\left((p_x + qA_x)^2 + (p_y + qA_y)^2 \right) I$$

$$H = E_c\, I + \frac{(\vec{p} + q\vec{A}).(\vec{p} + q\vec{A})}{2m}\, I$$

$$p_x = -i\hbar \frac{\partial}{\partial x}$$

4.6d Spin Hamiltonian

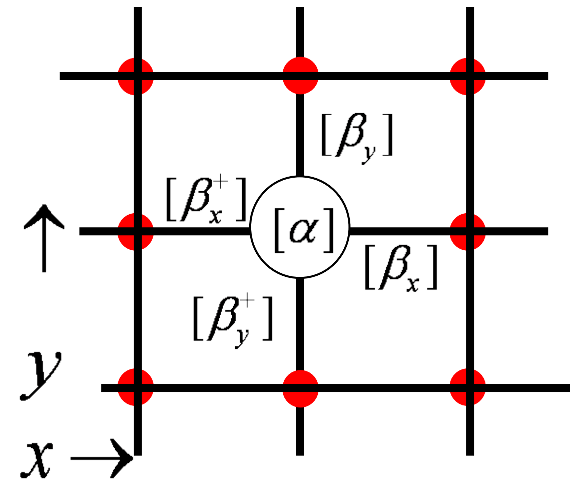
$$[\alpha] = \varepsilon\,[I] \quad + \frac{g}{2}\mu_B\,\vec{\sigma}.\vec{B}$$

$$[\beta_x] = t\,e^{+i\phi_x}\,[I] \;+ \frac{i\,\eta}{2\,a}\sigma_y$$

$$[\beta_x^+] = t\,e^{-i\phi_x}\,[I] \;\; - \frac{i\,\eta}{2\,a}\sigma_y$$

$$[\beta_y] = t\,e^{+i\phi_y}\,[I] \;- \frac{i\,\eta}{2\,a}\sigma_x$$

$$[\beta_y^+] = t\,e^{-i\phi_y}\,[I] \;+ \frac{i\,\eta}{2\,a}\sigma_x$$



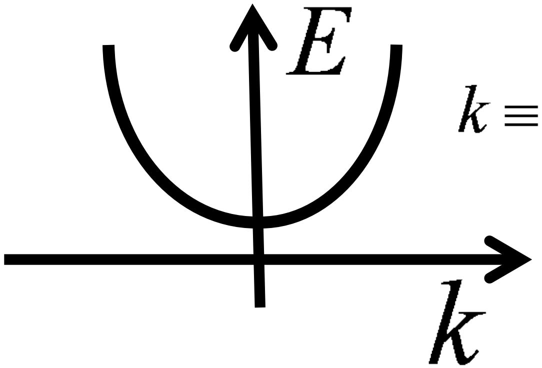
$$\frac{\eta}{2i\,a}\sigma_x\,(e^{+ik_ya} - e^{-ik_ya}) - \frac{\eta}{2i\,a}\sigma_y\,(e^{+ik_xa} - e^{-ik_xa})$$

$$\approx \frac{\eta}{a}(\sigma_x \sin k_y a - \sigma_y \sin k_x a)$$

$$\eta(\sigma_x\,k_y - \sigma_y\,k_x)$$

$$H = \left(E_c + \frac{(\vec{p}+q\vec{A}).(\vec{p}+q\vec{A})}{2m}\right) I \; + \; \frac{g}{2}\mu_B\,\vec{\sigma}.\vec{B} \; - \; \frac{\eta}{\hbar}\vec{\sigma}.\,(\hat{z}\times\vec{p})$$

4.6e Spin Hamiltonian



$$k \equiv \sqrt{k_x^2 + k_y^2}$$

$$\vec{\sigma}.\vec{V} \rightarrow \pm V$$

$spin \,||\, \vec{V}$

$spin \,||\, -\vec{V}$

$spin \,||\, -(\hat{z} \times \vec{k})$

$$E = E_c + \frac{\hbar^2 k^2}{2m} \pm \eta k$$

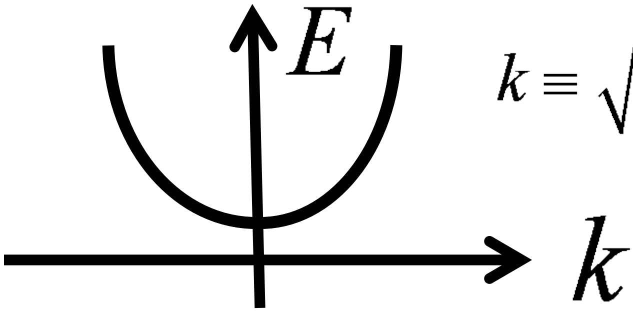
$spin \,||\, (\hat{z} \times \vec{k})$

$$\rightarrow \left(E_c + \frac{p^2}{2m} \right) I - \frac{\eta}{\hbar} \vec{\sigma}.\,(\hat{z} \times \vec{p})$$

Zero magnetic field

$$H = \left(E_c + \frac{(\vec{p} + q\vec{A}).(\vec{p} + q\vec{A})}{2m} \right) I + \frac{g}{2} \mu_B \vec{\sigma}.\vec{B} - \frac{\eta}{\hbar} \vec{\sigma}.\,(\hat{z} \times \vec{p})$$

4.6f Spin Hamiltonian

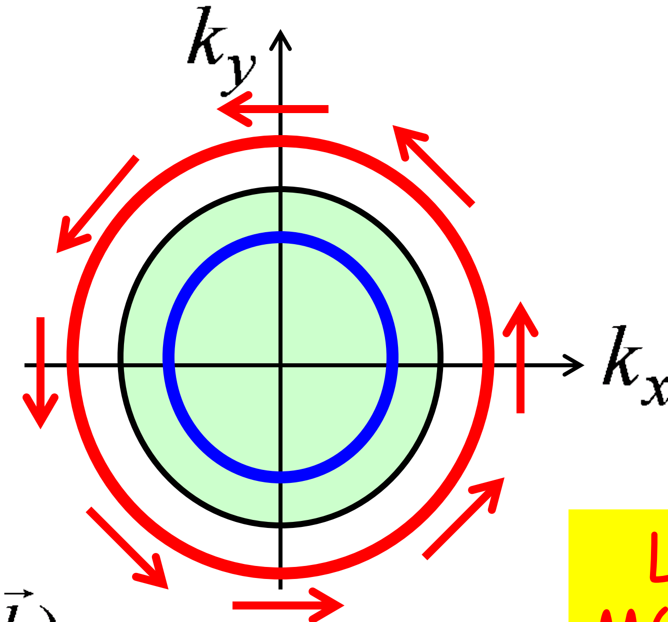


$$k \equiv \sqrt{k_x^2 + k_y^2}$$

$$spin \,\|\, -(\hat{z} \times \vec{k})$$

$$E = E_c + \frac{\hbar^2 k^2}{2m} \pm \eta k$$

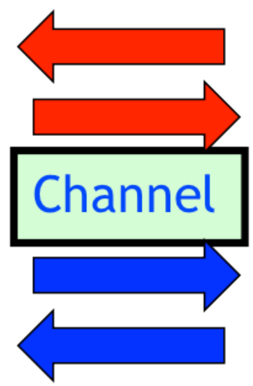
$$spin \,\|\, (\hat{z} \times \vec{k})$$



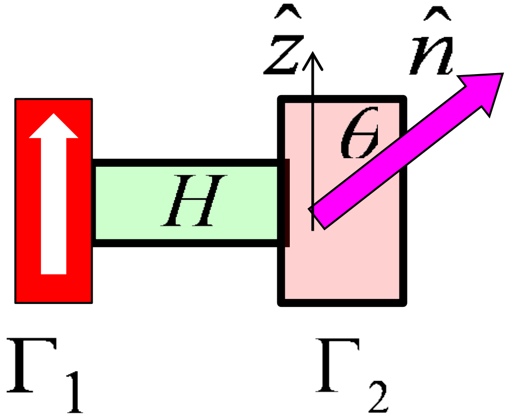
Larger M(E), D(E)

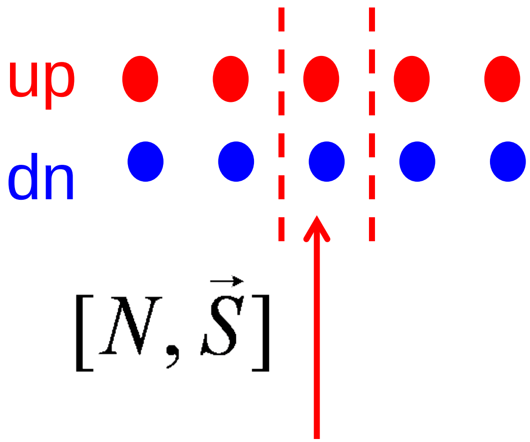
$$\pm \frac{g}{2} \mu_B \vec{\sigma} . \vec{B}$$

100
100
Magnetic
50
50



100	50
100	100
Normal	Spin-orbit
100	50
100	100





$$[N, \vec{S}]$$

$$\frac{G^n}{2\pi} = \frac{1}{2}\begin{bmatrix} N + S_z & S_x - iS_y \\ S_x + iS_y & N - S_z \end{bmatrix}$$

Coming up next ..