B. Quantum Transport

1. Schrodinger Equation
2. Contact-ing Schrodinger
3. More Examples
4. Spin Transport

4.1. Introduction

4.2. Magnetic contacts

4.3. Rotating contacts

4.4. Vectors and spinors

4.5. Spin-orbit coupling

4.6. Spin Hamiltonian

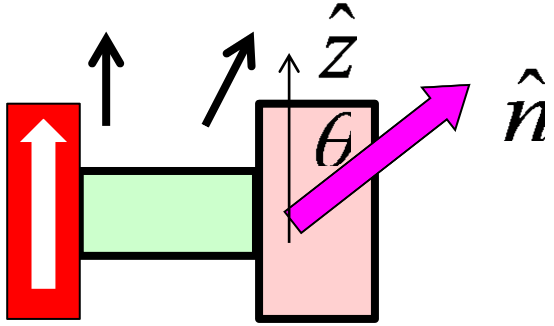
4.7. Spin density/current

4.8. Spin voltage

4.9. Spin circuits

4.10. Summing up ..

4.5a Spin-orbit coupling



$[\Gamma_1]$ $[H]$ $[\Gamma_2]$

Electrons in atoms

$$H = \frac{\vec{p}.\vec{p}}{2m_0} \; I$$

$$+ \frac{q\hbar}{4m_0^2c^2} \vec{\sigma}.\,[\vec{F} \times \vec{p}]$$

Spin-orbit coupling

Electrons in solids

$$H = \left(E_c + \frac{\vec{p}.\vec{p}}{2m} \right) \; I$$

$$- \eta \; \vec{\sigma}.(\hat{z} \times \vec{p})$$

$$I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, \quad \sigma_z = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}, \quad \sigma_x = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}, \quad \sigma_y = \begin{bmatrix} 0 & -i \\ +i & 0 \end{bmatrix} \qquad p_x = -i\hbar \frac{\partial}{\partial x}$$

Relativistic

4.5b Spin-orbit coupling

Non-relativistic

$$E^2 = m_0^2 c^4 + p^2 c^2 \quad \rightarrow \quad H = ??$$

$$E = \frac{p^2}{2m_0} \quad \rightarrow \quad H = \frac{\vec{p}.\vec{p}}{2m_0}$$

$$H = \begin{bmatrix} m_0 c^2 I & c\vec{\sigma}.\vec{p} \\ c\vec{\sigma}.\vec{p} & -m_0 c^2 I \end{bmatrix}$$

$$H^2 = \begin{bmatrix} m_0 c^2 I & c\vec{\sigma}.\vec{p} \\ c\vec{\sigma}.\vec{p} & -m_0 c^2 I \end{bmatrix} \begin{bmatrix} m_0 c^2 I & c\vec{\sigma}.\vec{p} \\ c\vec{\sigma}.\vec{p} & -m_0 c^2 I \end{bmatrix}$$

$$= (m_0^2 c^4 + c^2 p^2) \begin{bmatrix} I & Z \\ Z & I \end{bmatrix}$$

$$= (m_0^2 c^4 + p^2 c^2) \quad I_4$$

$$Z \equiv \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

$$p_x = -i\hbar \frac{\partial}{\partial x}$$

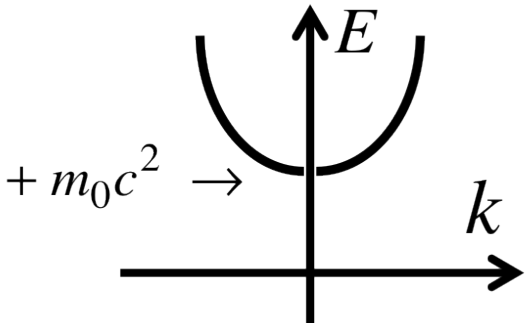
$$[\vec{\sigma}.\vec{p}][\vec{\sigma}.\vec{p}] = p^2[I] \leftarrow [\vec{\sigma}.\vec{V}_1][\vec{\sigma}.\vec{V}_2] = (\vec{V}_1.\vec{V}_2)\, I + i\vec{\sigma}.(\vec{V}_1 \times \vec{V}_2)$$

Dirac Equation

4.5c Spin-orbit coupling

$$H = \begin{bmatrix} m_0c^2 I & c\vec{\sigma}.\vec{p} \\ c\vec{\sigma}.\vec{p} & -m_0c^2 I \end{bmatrix}$$

$$E = \pm\sqrt{m_0^2c^4 + c^2\hbar^2k^2}$$



$$E\begin{Bmatrix} \psi \\ \phi \end{Bmatrix} = \begin{bmatrix} (m_0c^2 + U)\,I & c\vec{\sigma}.\vec{p} \\ c\vec{\sigma}.\vec{p} & (-m_0c^2 + U)\,I \end{bmatrix}\begin{Bmatrix} \psi \\ \phi \end{Bmatrix}$$

$$c^2\,[\vec{\sigma}.\vec{p}]\;\;[(E + m_0c^2 - U)I]^{-1}\;\;[\vec{\sigma}.\vec{p}]$$

$$E\{\psi\} = [H + \Sigma]\,\{\psi\}$$

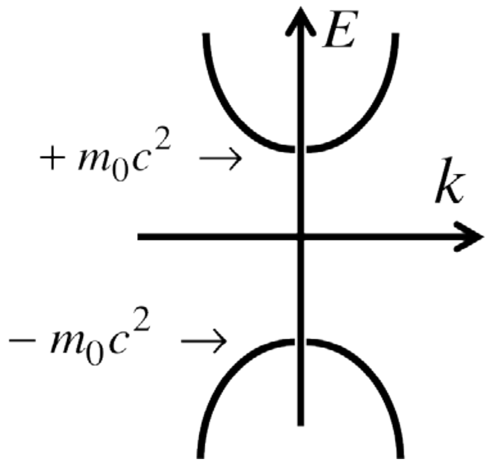
B3.3 $\Sigma = \tau\,[EI - H_R]^{-1}\,\tau^+$

$$E\begin{Bmatrix} \psi \\ \phi \end{Bmatrix} = \begin{bmatrix} H & \tau \\ \tau^+ & H_R \end{bmatrix}\begin{Bmatrix} \psi \\ \phi \end{Bmatrix}$$

4.5d Spin-orbit coupling

Dirac H

$$H = \begin{bmatrix} m_0 c^2 I & c\,\vec{\sigma}.\vec{p} \\ c\,\vec{\sigma}.\vec{p} & -m_0 c^2 I \end{bmatrix}$$



Reduced H

$$(m_0 c^2 + U) I \; + \; c^2 \, [\vec{\sigma}.\vec{p}] \; [(E + m_0 c^2 - U) I]^{-1} \, [\vec{\sigma}.\vec{p}]$$

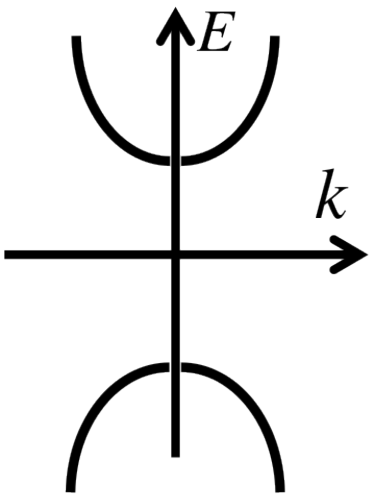
$$\approx (m_0 c^2 + U) I \; + \; \frac{[\vec{\sigma}.\vec{p}] \; [\vec{\sigma}.\vec{p}]}{2 m_0}$$

$$\frac{p^2}{2 m_0} I \; + \; \frac{q \hbar}{4 m_0^2 c^2} \vec{\sigma}. \, [\vec{F} \times \vec{p}]$$

4.5e Spin-orbit coupling

Dirac Equation

$$H = \begin{bmatrix} (m_0 c^2 + U) I & c\vec{\sigma}.\vec{p} \\ c\vec{\sigma}.\vec{p} & (-m_0 c^2 + U) I \end{bmatrix}$$



Reduced H

$$\frac{p^2}{2m_0} I$$

$$\approx U I + \frac{[\vec{\sigma}.\vec{p}]\ [\vec{\sigma}.\vec{p}]}{2m_0}$$

$$+ \frac{q\hbar}{4 m_0^2 c^2} \vec{\sigma}.\ [\vec{F} \times \vec{p}]$$

B 3.6

$$\frac{[\vec{\sigma}.(\vec{p} + q\vec{A})][\vec{\sigma}.(\vec{p} + q\vec{A})]}{2m_0}$$

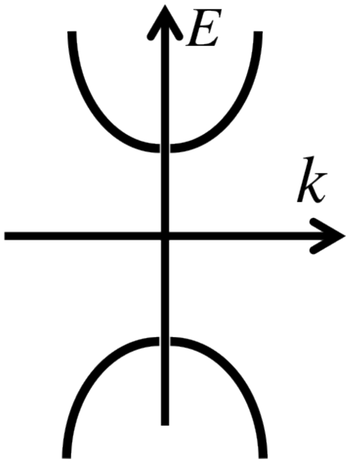
$$\vec{B}_{eff} = \frac{[\vec{F} \times \vec{p}]}{2 m_0 c^2}$$

$$\frac{(\vec{p} + q\vec{A}).(\vec{p} + q\vec{A})}{2m_0} I + \mu_B \vec{\sigma}.\vec{B}$$

4.5f Spin-orbit coupling

Dirac Equation

$$H=\begin{bmatrix}(m_0c^2+U)I & c\vec{\sigma}.(\vec{p}+q\vec{A})\\ c\vec{\sigma}.(\vec{p}+q\vec{A}) & (-m_0c^2+U)I\end{bmatrix}$$



Reduced H

$$H=\left(U+\frac{(\vec{p}+q\vec{A}).(\vec{p}+q\vec{A})}{2m_0}\right)I$$

$$+\mu_B\,\vec{\sigma}.\vec{B}\;+\;\frac{q\hbar}{4m_0^2c^2}\vec{\sigma}.\,[\vec{F}\times\vec{p}]$$

$$H=\left(E_c+\frac{(\vec{p}+q\vec{A}).(\vec{p}+q\vec{A})}{2m}\right)I$$

$$+\;\frac{g}{2}\mu_B\,\vec{\sigma}.\vec{B}\;-\;\eta\vec{\sigma}.\,(\hat{z}\times\vec{p})$$

Zero external magnetic field $\rightarrow\left(E_c+\dfrac{p^2}{2m}\right)I\;-\;\eta\vec{\sigma}.\,(\hat{z}\times\vec{p})$

Coming up next ..

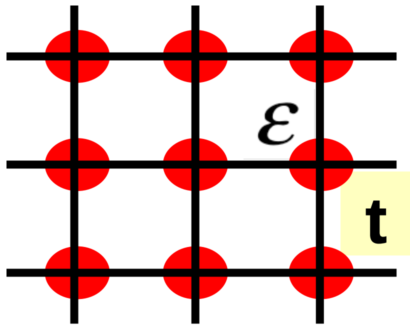
B 3.2

$$H = E_c + \frac{\vec{p}.\vec{p}}{2m}$$

B 3.6

$$H = E_c + \frac{(\vec{p}+q\vec{A}).(\vec{p}+q\vec{A})}{2m}$$

[H]



$$H = \left(E_c + \frac{(\vec{p}+q\vec{A}).(\vec{p}+q\vec{A})}{2m} \right) I + \frac{g}{2}\mu_B \vec{\sigma}.\vec{B} - \eta \vec{\sigma}.(\hat{z}\times\vec{p})$$