

FUNDAMENTALS OF NANOELECTRONICS

B. Quantum Transport

1. Schrodinger Equation
2. Contact-ing Schrodinger
3. More Examples

4. Spin Transport →

4.1. Introduction

4.2. Magnetic contacts

4.3. Rotating contacts

4.4. Vectors and spinors

4.5. Spin-orbit coupling

4.6. Spin Hamiltonian

4.7. Spin density/current

4.8. Spin voltage

4.9. Spin circuits

4.10. Summing up ..

4.2a Magnetic contacts

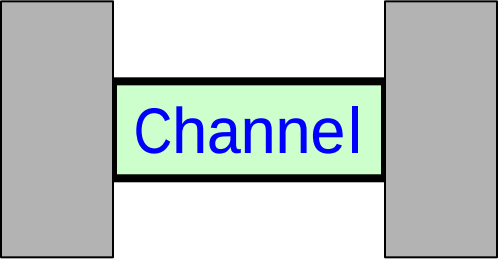


Diagram illustrating a quantum transport system with two electrodes (left and right) and a central channel. The electrodes are represented by gray rectangles, and the channel is a green rectangle labeled "Channel". The system is coupled to two reservoirs, Σ_1 and Σ_2 , which are represented by red dots. The coupling is shown by red lines connecting the electrodes to the reservoirs.

$$\Sigma_1 = -\frac{i}{2}[\gamma_1]$$

$$\Sigma_2 = -\frac{i}{2}[\gamma_2]$$

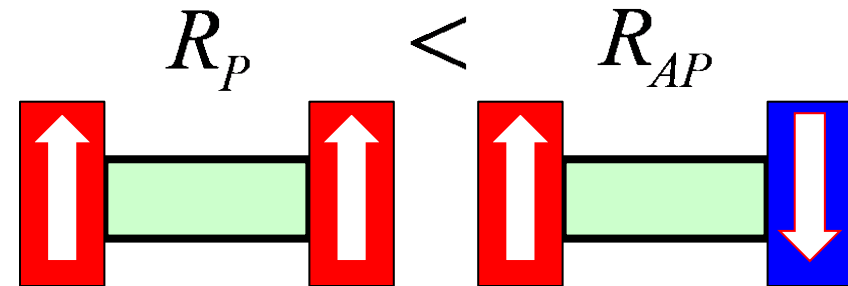
$$\Gamma_1 = [\gamma_1]$$

$$\Gamma_2 = [\gamma_2]$$

$$H = [\varepsilon]$$

$$G^R = \left[E - \varepsilon + \frac{i}{2}(\gamma_1 + \gamma_2) \right]^{-1}$$

$$\rightarrow G^A = \left[E - \varepsilon - \frac{i}{2}(\gamma_1 + \gamma_2) \right]^{-1}$$



$$\Gamma = i[\Sigma - \Sigma^+]$$

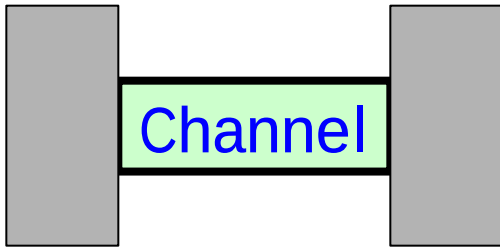
$$G^R = [EI - H - \Sigma_1 - \Sigma_2]^{-1}$$

$$\bar{T}(E) = \text{Trace}[\Gamma_1 G^R \Gamma_2 G^A]$$

$$= \frac{\gamma_1 \gamma_2}{(E - \varepsilon)^2 + \left(\frac{\gamma_1 + \gamma_2}{2} \right)^2} \times 2$$

4.2b Magnetic contacts

$[\Sigma_1]$ $[H]$ $[\Sigma_2]$



up ● ● ● ● ●
 dn ● ● ● ● ●

$$\Sigma_1 = -\frac{i}{2} \begin{bmatrix} \gamma_{1u} & 0 \\ 0 & \gamma_{1d} \end{bmatrix}$$

$$\Gamma_1 = \begin{bmatrix} \gamma_{1u} & 0 \\ 0 & \gamma_{1d} \end{bmatrix}$$

$$H = \begin{bmatrix} \varepsilon & 0 \\ 0 & \varepsilon \end{bmatrix}$$

$$\Sigma_2 = -\frac{i}{2} \begin{bmatrix} \gamma_{2u} & 0 \\ 0 & \gamma_{2d} \end{bmatrix}$$

$$\Gamma_2 = \begin{bmatrix} \gamma_{2u} & 0 \\ 0 & \gamma_{2d} \end{bmatrix}$$

$$\Gamma = i[\Sigma - \Sigma^+]$$

$$G^R = [EI - H - \Sigma_1 - \Sigma_2]^{-1}$$

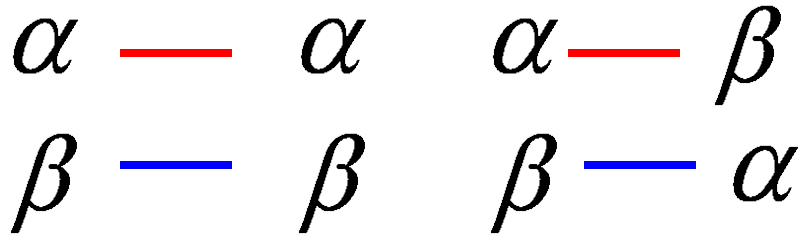
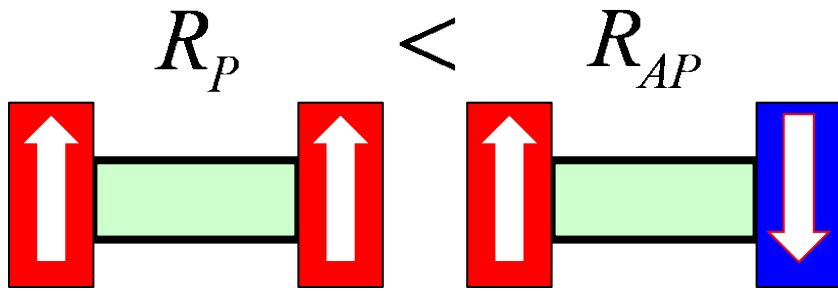
$$\bar{T}(E) = \text{Trace}[\Gamma_1 G^R \Gamma_2 G^A]$$

$$= \frac{\gamma_{1u}\gamma_{2u}}{(E - \varepsilon)^2 + \left(\frac{\gamma_{1u} + \gamma_{2u}}{2}\right)^2} + \frac{\gamma_{1d}\gamma_{2d}}{(E - \varepsilon)^2 + \left(\frac{\gamma_{1d} + \gamma_{2d}}{2}\right)^2}$$

1 Level

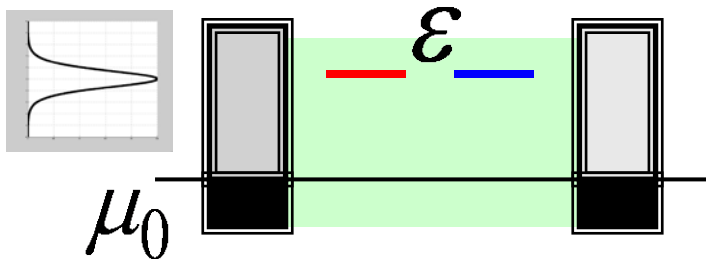
$$\frac{\gamma_1 \gamma_2}{(E - \varepsilon)^2 + \left(\frac{\gamma_1 + \gamma_2}{2}\right)^2}$$

4.2c Magnetic contacts



$$T_P \approx \frac{\alpha^2 + \beta^2}{(\mu_0 - \varepsilon)^2}$$

$$T_{AP} \approx \frac{2\alpha\beta}{(\mu_0 - \varepsilon)^2}$$



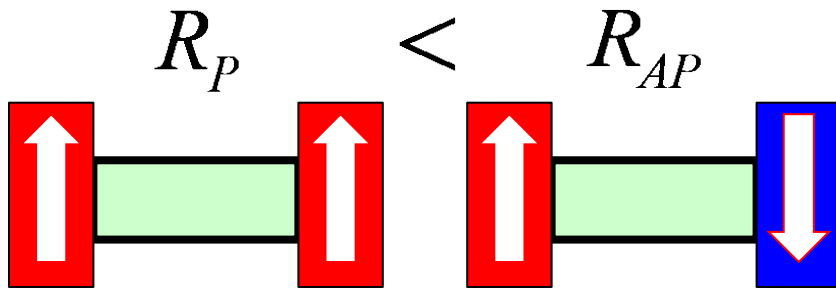
$$\Gamma = i[\Sigma - \Sigma^+]$$

$$G^R = [EI - H - \Sigma_1 - \Sigma_2]^{-1}$$

$$\bar{T}(E) = \text{Trace}[\Gamma_1 G^R \Gamma_2 G^A]$$

$$\begin{aligned} &= \frac{\gamma_{1u}\gamma_{2u}}{(E - \varepsilon)^2 + \left(\frac{\gamma_{1u} + \gamma_{2u}}{2}\right)^2} \\ &\quad + \frac{\gamma_{1d}\gamma_{2d}}{(E - \varepsilon)^2 + \left(\frac{\gamma_{1d} + \gamma_{2d}}{2}\right)^2} \\ &\approx \frac{\gamma_{1u}\gamma_{2u} + \gamma_{1d}\gamma_{2d}}{(\mu_0 - \varepsilon)^2} \end{aligned}$$

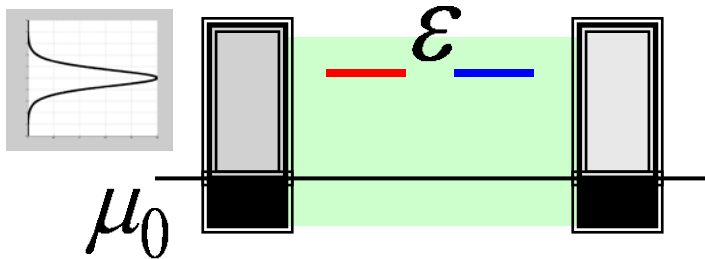
4.2d Magnetic contacts



α — α α — β
 β — β β — α

$$T_P \approx \frac{\alpha^2 + \beta^2}{(\mu_0 - \varepsilon)^2}$$

$$T_{AP} \approx \frac{2\alpha\beta}{(\mu_0 - \varepsilon)^2}$$



$$TMR = \frac{R_{AP}}{R_P} - 1 = \frac{T_P}{T_{AP}} - 1$$

$$= \frac{\alpha^2 + \beta^2}{2\alpha\beta} - 1$$

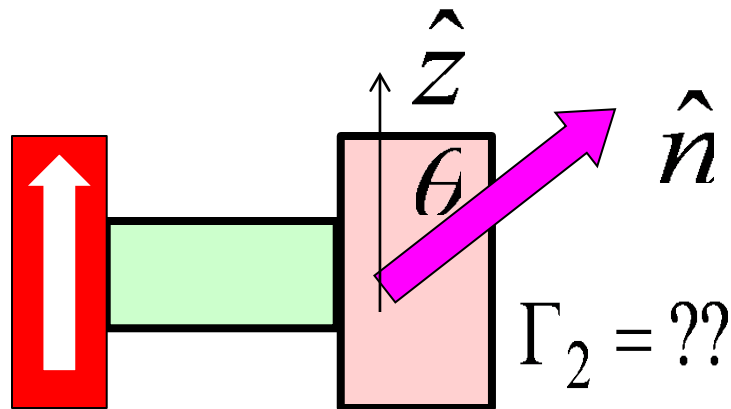
$$= 2 \frac{(\alpha - \beta)^2}{(\alpha + \beta)^2 - (\alpha - \beta)^2}$$

$$= \frac{2P^2}{1 - P^2}, \quad P \equiv \frac{\alpha - \beta}{\alpha + \beta}$$

Coming up next ..

$$\Gamma_1 = \begin{bmatrix} \alpha & 0 \\ 0 & \beta \end{bmatrix} \quad \begin{array}{c} \uparrow \\ \uparrow \end{array} \quad H = \begin{bmatrix} \varepsilon & 0 \\ 0 & \varepsilon \end{bmatrix} \quad \Gamma_2 = \begin{bmatrix} \alpha & 0 \\ 0 & \beta \end{bmatrix}$$

$$\Gamma_1 = \begin{bmatrix} \alpha & 0 \\ 0 & \beta \end{bmatrix} \quad \begin{array}{c} \uparrow \\ \uparrow \end{array} \quad \begin{array}{c} \downarrow \\ \downarrow \end{array} \quad \Gamma_2 = \begin{bmatrix} \beta & 0 \\ 0 & \alpha \end{bmatrix}$$



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