

FUNDAMENTALS OF NANOELECTRONICS

B. Quantum Transport

1. Schrodinger Equation
2. Contact-ing Schrodinger

More Examples



4. Spin Transport

3.1. Introduction

3.2. Quantum Point Contact

3.3. Self-Energy

3.4. Surface Green's Function

3.5. Graphene

3.6. Magnetic Field

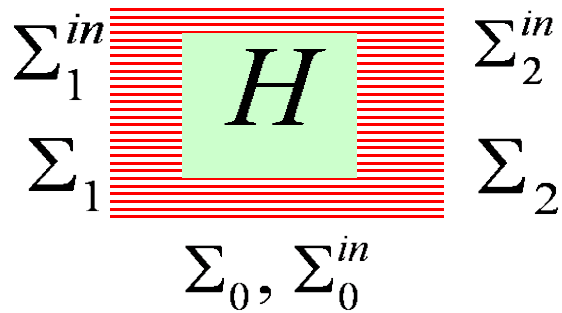
3.7. Golden Rule

3.8. Inelastic Scattering

3.9. Can NEGF Include
Everything?

3.10. Summing up ..

3.9a Can NEGF Include Everything?



$$\Sigma_1^{in} \quad \Sigma_1$$

$$\Sigma_2^{in} \quad \Sigma_2$$

$$\Sigma_0, \Sigma_0^{in}$$

*One-electron
Schrodinger
equation*

$$[\Sigma_0^{in}(E)]_{ij} = \int_{-\infty}^{+\infty} \frac{d(\hbar\omega)}{2\pi} D_{im;jn}(\hbar\omega) [G^n(E + \hbar\omega)]_{mn}$$

$$[\Gamma_0(E)]_{ij} = \int_{-\infty}^{+\infty} \frac{d(\hbar\omega)}{2\pi} \sum_{m,n} D_{im;jn}(\hbar\omega) \times$$

$$\left[G^p(E - \hbar\omega) + G^n(E + \hbar\omega) \right]_{mn}$$

$$\Sigma_0(E) = -\frac{i}{2} \Gamma_0(E) \otimes \left(\delta(E) + \frac{i}{\pi E} \right)$$

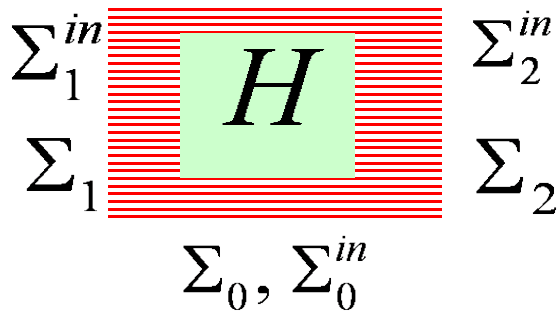
$$i\hbar \frac{\partial \psi}{\partial t} = H\psi + U^R \psi$$

$$D_{im;jn} = \left\langle \tau_{im} \tau_{jn}^* \right\rangle$$

$$\tau_{im} = \int d\vec{r} \phi_i^* U^R \phi_m$$

*Standard Approach
Many-body
Perturbation Theory
(MBPT)*

3.9b Can NEGF Include Everything?

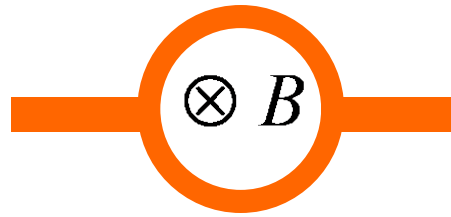


Electron-electron
Interaction ?

*One-electron
Schrodinger
equation*

Conductivity \rightarrow *Momentum
Relaxation Time*

Oscillatory \rightarrow
Magnetoresistance



Conductance Fluctuations
Weak Localization

*Phase
Relaxation Time*

$$i\hbar \frac{\partial \psi}{\partial t} = H\psi + U^R \psi$$

$$D_{im;jn} = \langle \tau_{im} \tau_{jn}^* \rangle$$

$$\tau_{im} = \int d\vec{r} \phi_i^* U^R \phi_m$$

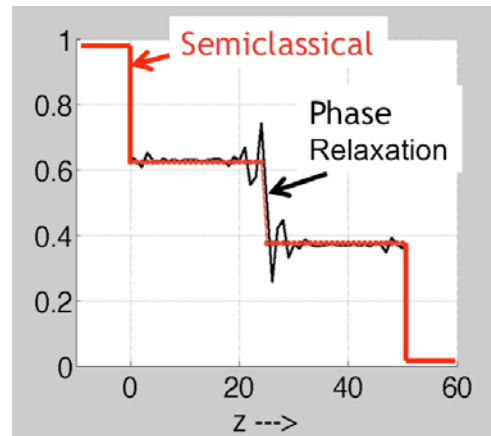
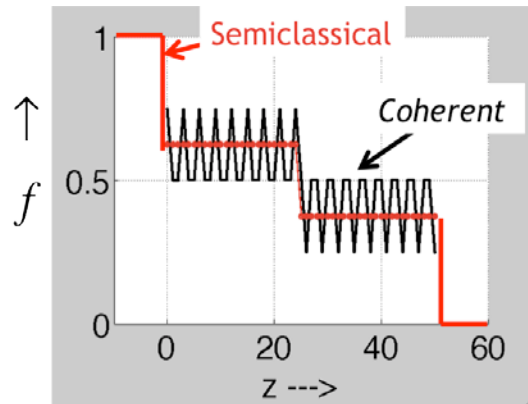
*Standard Approach
Many-body
Perturbation Theory
(MBPT)*



3.9c Can NEGF Include Everything?

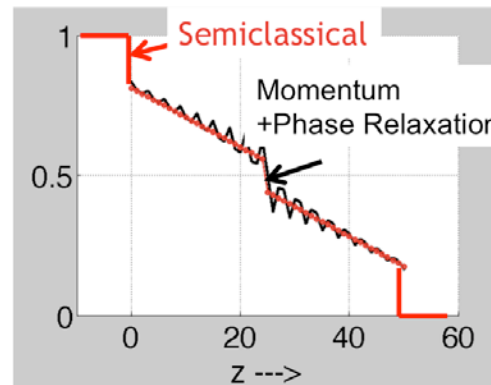
One-electron
Schrodinger
equation

$$D = d_0 \begin{bmatrix} 1 & 1 & 1 & \dots \\ 1 & 1 & 1 & \dots \\ 1 & 1 & 1 & \dots \\ \dots & \dots & \dots & \dots \end{bmatrix}$$



Part B, Lecture 2.9

$$D = d_0 \begin{bmatrix} 1 & 0 & 0 & \dots \\ 0 & 1 & 0 & \dots \\ 0 & 0 & 1 & \dots \\ \dots & \dots & \dots & \dots \end{bmatrix}$$



$$i\hbar \frac{\partial \psi}{\partial t} = H\psi + U^R\psi$$

$$D_{im;jn} = \langle \tau_{im} \tau_{jn}^* \rangle$$

$$\rightarrow \langle \tau_{ii} \tau_{jj}^* \rangle \delta_{im} \delta_{jn}$$

Standard Approach
Many-body
Perturbation Theory
(MBPT)

$$\begin{array}{ccc} \Sigma_1^{in} & \boxed{H + U} & \Sigma_2^{in} \\ \Sigma_1 & & \Sigma_2 \\ & \Sigma_0, \Sigma_0^{in} & \end{array}$$

3.9d Can NEGF Include Everything?

Electron-electron
Interaction ?

One-electron
Schrodinger
equation

$$U(\vec{r}, \vec{r}') = \delta(\vec{r} - \vec{r}') \int d\vec{r}'' \frac{q^2}{4\pi\epsilon|\vec{r} - \vec{r}''|} \left[\int dE \frac{G^n(\vec{r}'', \vec{r}''; E)}{2\pi} \right. \\ \left. - \int dE \frac{q^2}{4\pi\epsilon|\vec{r} - \vec{r}''|} G^{ns}(\vec{r}, \vec{r}'; E) \right]$$

Poisson Eq

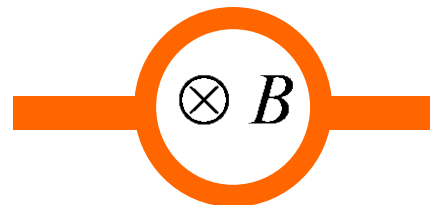
$$\nabla^2 U = \frac{q^2}{\epsilon} n$$

Exchange Correction

Oscillatory →
Magnetoresistance

Conductance Fluctuations
Weak Localization

Phase
Relaxation Time



$$i\hbar \frac{\partial \psi}{\partial t} = H\psi + U^R\psi$$

$$D_{im;jn} = \langle \tau_{im} \tau_{jn}^* \rangle$$

$$\tau_{im} = \int d\vec{r} \phi_i^* U^R \phi_m$$

Standard Approach
Many-body
Perturbation Theory
(MBPT)

$$\begin{array}{ccc} \Sigma_1^{in} & \boxed{H + U} & \Sigma_2^{in} \\ \Sigma_1 & & \Sigma_2 \\ & \Sigma_0, \Sigma_0^{in} & \end{array}$$

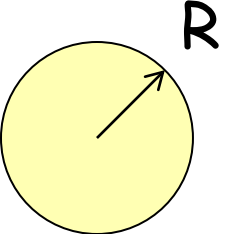
$$U \approx U_0 N$$



Single - electron
charging energy

$$U_0 \sim \frac{q^2}{4\pi\epsilon_0 R}$$

→ 1.6 eV for $R = 1 \text{ nm}$



3.9e Can NEGF Include Everything?

Electron-electron
Interaction ?

*Requires
non-perturbative
methods*

If $U_0 \gg kT, \Gamma$

Single - electron
charging effects
also known as
Coulomb blockade

$$\frac{1}{1-x} \approx x + x^2 + \dots$$

Standard Approach
Many-body
Perturbation Theory
(MBPT)

Coming up next ..

$$G^n = G^R \Sigma^{in} G^A$$

◆ Some applications may require non-perturbative approach

Standard Approach
Many-body
Perturbation Theory (MBPT)

$$\begin{aligned}\Sigma &= \Sigma_1 + \Sigma_2 + \Sigma_0 \\ \Sigma^{in} &= \Sigma_1^{in} + \Sigma_2^{in} + \Sigma_0^{in}\end{aligned}$$

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