B. Quantum Transport

1. Schrodinger Equation
2. Contact-ing Schrodinger

More Examples

4. Spin Transport

3.1. Introduction

3.2. Quantum Point Contact

3.3. Self-Energy

3.4. Surface Green's Function

3.5. Graphene

3.6. Magnetic Field

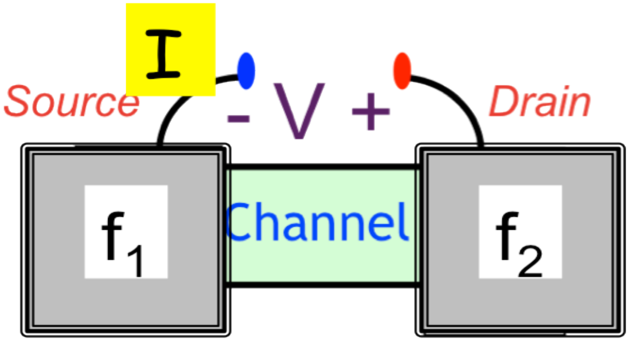
3.7. Golden Rule

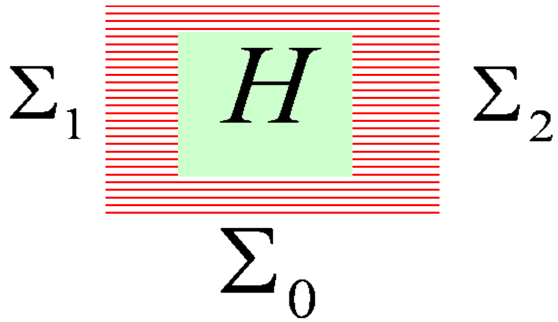
3.8. Inelastic Scattering

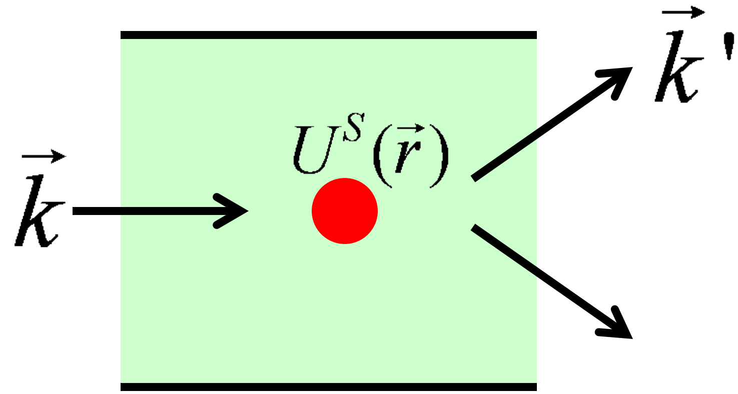
3.9. Can NEGF Include Everything?

3.10. Summing up ..

3.7a Fermi's golden rule







$$\Sigma = \tau\, g\, \tau^+$$

$$\Gamma = i\left[\Sigma - \Sigma^+\right] \rightarrow \frac{\hbar}{\text{"}lifetime\text{"}}$$

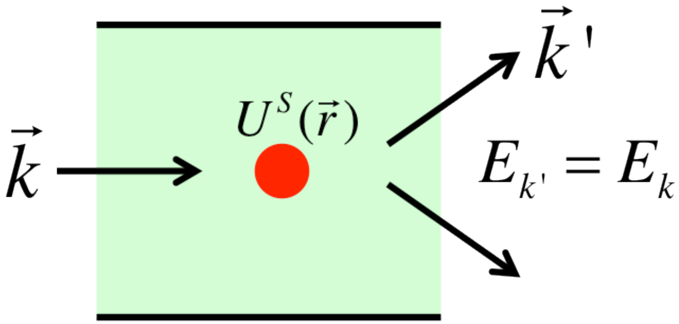
$$= \tau\, i\left[g - g^+\right]\, \tau^+$$

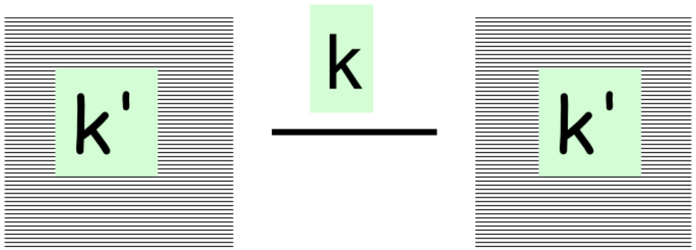
$$= \tau\, a\ \tau^+$$

$$S_{k,k'} = \frac{2\pi}{\hbar}\left|\tau_{k,k'}\right|^2\ \delta(E_k - E_{k'})$$

$$\tau_{k,k'} = \int d\vec{r}\ \phi^*_{\vec{k}}(\vec{r})\, U^S(\vec{r})\, \phi_{\vec{k}'}(\vec{r})$$

3.7b Fermi's golden rule





$$\vec{\beta} \equiv \vec{k} - \vec{k}\,'$$

$$\Gamma_{k,k}(E) = \sum_{k'} \tau_{k,k'}\, a_{k',k'}(E)\, \tau^{+}_{k',k}$$

$$= \sum_{k'} \tau_{k,k'}\, \tau^{*}_{k,k'}\, \underbrace{a_{k',k'}(E)}$$

$$= \hbar \sum_{k'} \frac{1}{\hbar} \left|\tau_{k,k'}\right|^2 2\pi\, \delta(E_k - E_{k'})$$

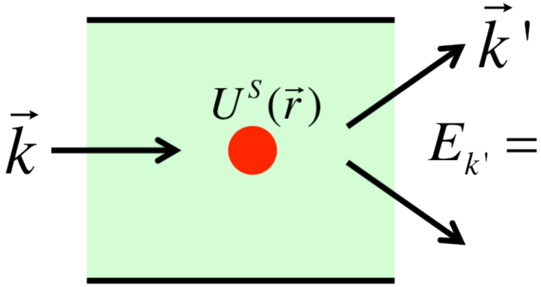
$S_{k,\,k'}$

$$\Sigma = \tau\, g\, \tau^{+}$$

$$\Gamma = \tau\, a\, \tau^{+}$$

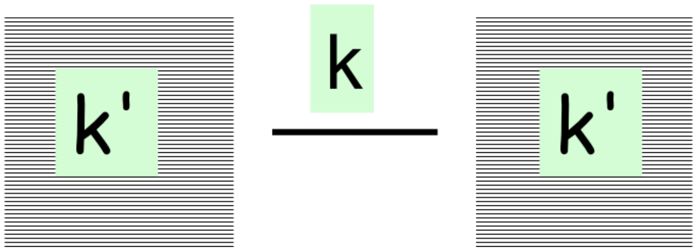
$$\rightarrow \frac{\hbar}{\text{"lifetime"}}$$

3.7c Fermi's golden rule



$$E_{k'} = E_k$$

$$\vec{\beta} \equiv \vec{k} - \vec{k}\,'$$



$$\frac{\Gamma_{k,k}(E)}{\hbar} = \sum_{k'} S_{k,k'}$$

$$S_{k,k'} = \frac{2\pi}{\hbar}\left|\tau_{k,k'}\right|^2 \delta(E_k - E_{k'})$$

Part B, Lecture 1.3

$$= \frac{e^{-i\vec{k}.\vec{r}}}{\sqrt{V}} \qquad = \frac{e^{i\vec{k}\,'.\vec{r}}}{\sqrt{V}}$$

$$\tau_{k,k'} = \int d\vec{r}\ \phi_{\vec{k}}^{*}(\vec{r})\,U^S(\vec{r})\,\phi_{\vec{k}\,'}(\vec{r})$$

$$= \frac{1}{V}\int d\vec{r}\ U^S(\vec{r})\,e^{-i(\vec{k}-\vec{k}\,').\vec{r}}$$

$$\rightarrow \tilde{U}^S(\vec{\beta})$$

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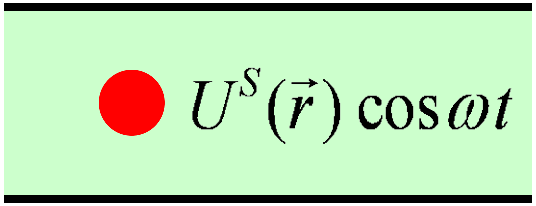
3.7d Fermi's golden rule

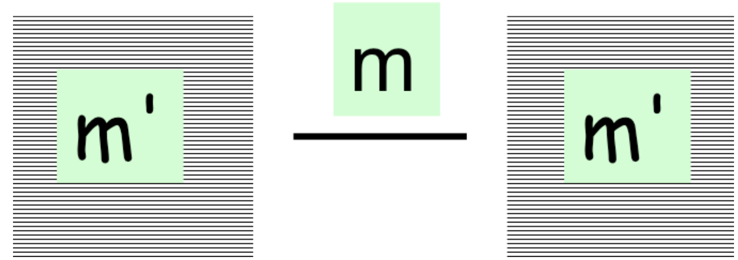


$$Absorption: \quad E_m + \hbar\omega = E_{m'}$$

$$S_{m,m'} = \frac{2\pi}{\hbar}\left|\tau_{m,m'}\right|^2 N_\omega \times \delta(E_m - E_{m'} + \hbar\omega)$$

$$+ \quad \frac{2\pi}{\hbar}\left|\tau_{m,m'}\right|^2 (N_\omega + 1) \times \delta(E_m - E_{m'} - \hbar\omega)$$

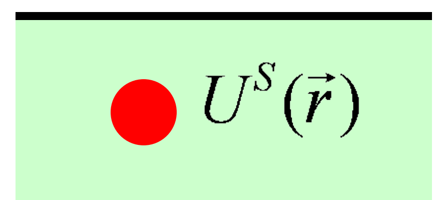
$$Emission: \quad E_m - \hbar\omega = E_{m'}$$



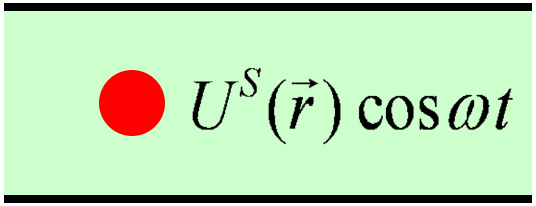
$$\frac{\Gamma_{m,m}(E)}{\hbar} = \sum_{m'} S_{m,m'}$$

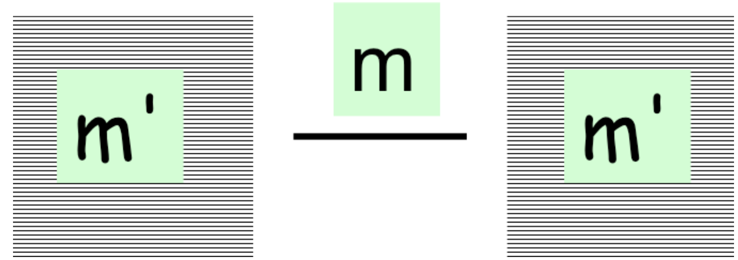
$$S_{m,m'} = \frac{2\pi}{\hbar}\left|\tau_{m,m'}\right|^2 \delta(E_m - E_{m'})$$

$$\tau_{m,m'} = \int d\vec{r}\, \phi_m^*(\vec{r})\, U^S(\vec{r})\, \phi_{m'}(\vec{r})$$



3.7e Fermi's golden rule





Absorption $E_m + \hbar\omega = E_{m'}$

$$S_{m,m'} = \frac{2\pi}{\hbar}\left|\tau_{m,m'}\right|^2 N_\omega \times \delta(E_m - E_{m'} + \hbar\omega)$$

$$+ \quad \frac{2\pi}{\hbar}\left|\tau_{m,m'}\right|^2 (N_\omega + 1) \times \delta(E_m - E_{m'} - \hbar\omega)$$

Emission $E_m - \hbar\omega = E_{m'}$

$$i\hbar\frac{\partial\psi}{\partial t} = H\psi + U^S\psi$$

$$\phi_m(\vec{r})\, e^{-iE_m t/\hbar}$$

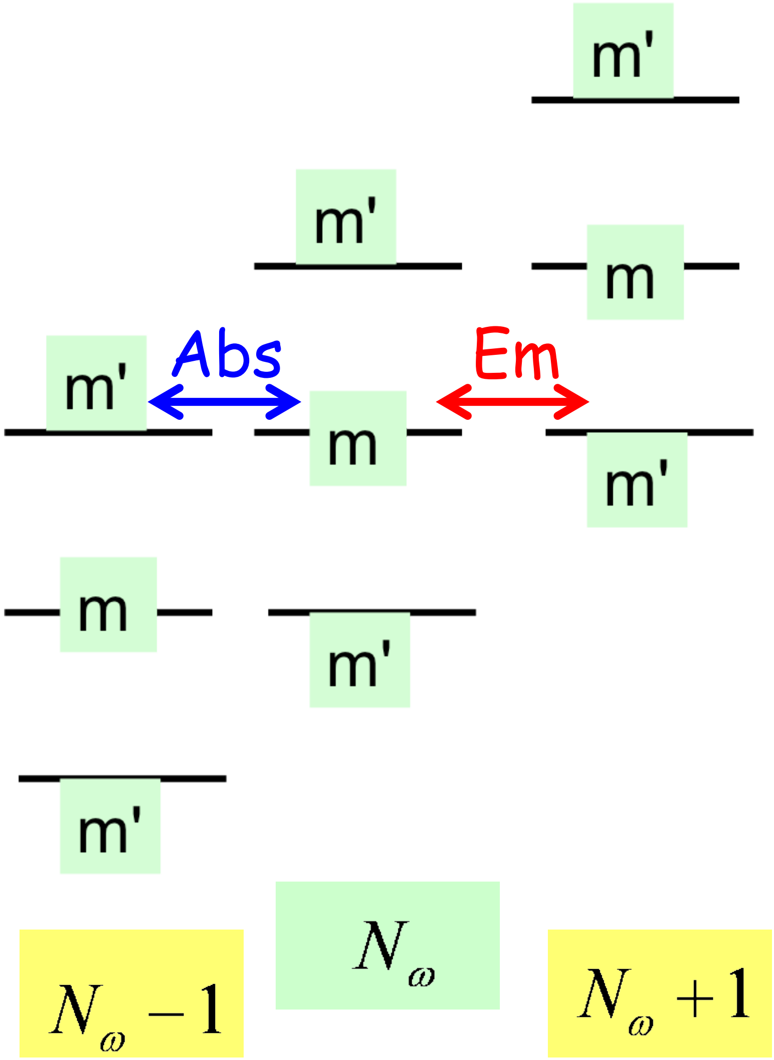
$$e^{+i\omega t} + e^{-i\omega t}$$

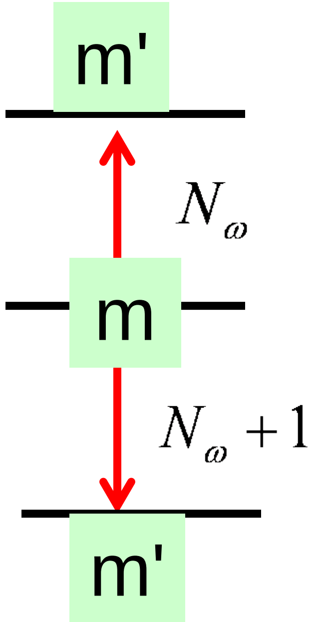
$$\sim\ e^{-i(E_m - \hbar\omega)t/\hbar} \qquad \sim\ e^{-i(E_m + \hbar\omega)t/\hbar}$$

Emission **Absorption**

3.7f Fermi's golden rule

$$S_{m,m'} = \frac{2\pi}{\hbar} \left| \tau_{m,m'} \right|^2 \delta(E_m - E_{m'})$$

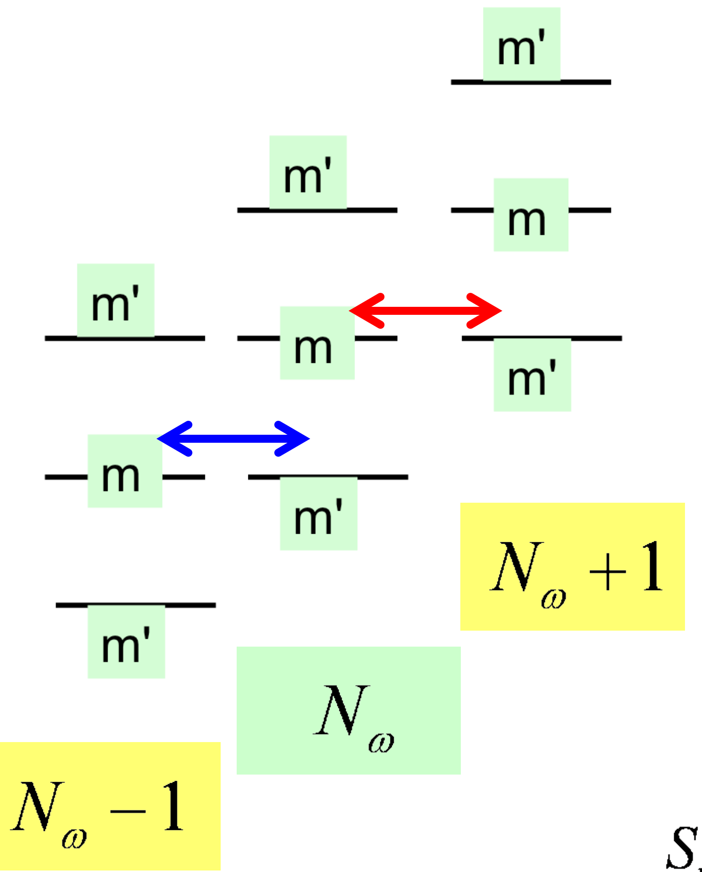




$$S_{m,m'} = \frac{2\pi}{\hbar} \left| \tau_{m,m'} \right|^2 \times \begin{pmatrix} (N+1)\,\delta(E_m - E_{m'} - \hbar\omega) \\ + N\,\delta(E_m - E_{m'} + \hbar\omega) \end{pmatrix}$$

3.7g Fermi's golden rule

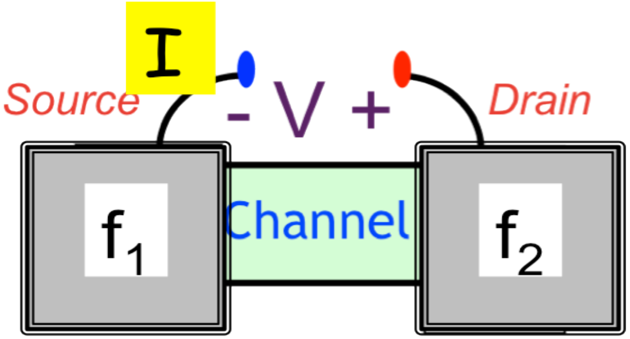
$$S_{m,m'} = \frac{2\pi}{\hbar}\left|\tau_{m,m'}\right|^2 \delta(E_m - E_{m'})$$

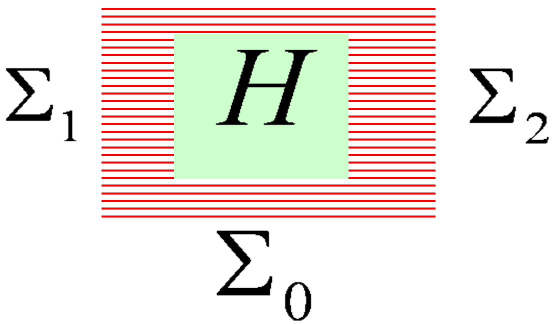


$$S_{m,m'} = \frac{2\pi}{\hbar}\left|\tau_{m,m'}\right|^2 \left|\tau_{N,N+1}\right|^2 \times \delta(E_m + N\hbar\omega - E_{m'} - \overline{N+1}\,\hbar\omega)$$

$$+ \frac{2\pi}{\hbar}\left|\tau_{m,m'}\right|^2 \left|\tau_{N,N-1}\right|^2 \times \delta(E_m + N\hbar\omega - E_{m'} - \overline{N-1}\,\hbar\omega)$$

$$S_{m,m'} = \frac{2\pi}{\hbar}\left|\tau_{m,m'}\right|^2 \times \begin{pmatrix} (N+1)\,\delta(E_m - E_{m'} - \hbar\omega) \\ +N\,\delta(E_m - E_{m'} + \hbar\omega) \end{pmatrix}$$





$$\Sigma = \tau\, g\, \tau^{+}$$

$$\Gamma = \tau\, a\, \tau^{+}$$

$$S_{m,m'} = \frac{2\pi}{\hbar}\left|\tau_{m,m'}\right|^{2}\, \delta(E_m - E_{m'})$$

$$S_{m,m'} = \frac{2\pi}{\hbar}\left|\tau_{m,m'}\right|^{2} \times \begin{pmatrix} (N+1)\,\delta(E_m - E_{m'} - \hbar\omega) \\ +N\,\delta(E_m - E_{m'} + \hbar\omega) \end{pmatrix}$$

Coming up next ..