FUNDAMENTALS OF NANOELECTRONICS

B. Quantum Transport

1. Schrodinger Equation

Contact-ing Schrodinger

3. Advanced Examples

4. Spin Transport

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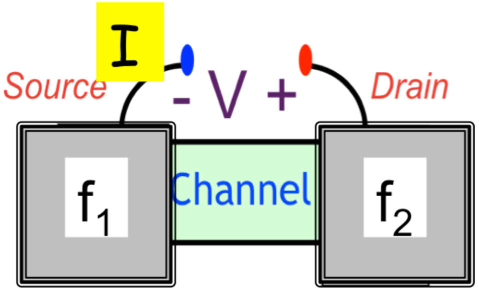
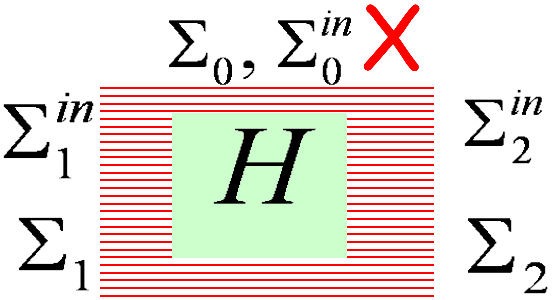
http://nanohub.org/groups/lnebook

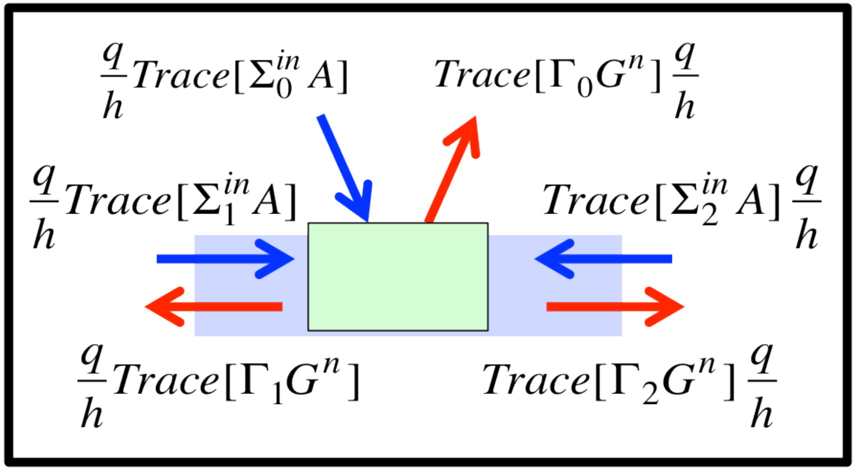
2.7a Transmission



$$G^R = [EI - H - \Sigma]^{-1}$$

$$G^n = G^R \, \Sigma^{in} \, G^A$$





$$\Sigma = \Sigma_1 + \Sigma_2 + \Sigma_0$$

$$\Sigma^{in} = \Sigma_1^{in} + \Sigma_2^{in} + \Sigma_0^{in}$$

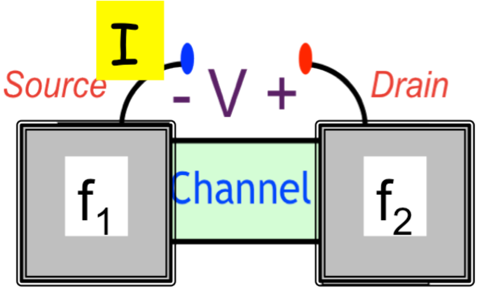
$$\Sigma_1^{in} = \Gamma_1 \, f_1(E)$$

$$\Sigma_2^{in} = \Gamma_2 \, f_2(E)$$

$$I_1 = \frac{q}{h} Trace\,[\Sigma_1^{in} A - \Gamma_1 G^n]$$

$$= \frac{q}{h} Trace\,[\Gamma_1 (A f_1 - G^n)]$$

2.7b Transmission



$$G^R = [EI - H - \Sigma]^{-1}$$

$$G^n = G^R \Sigma^{in} G^A = G^R(\Gamma_1 f_1 + \Gamma_2 f_2) G^A$$

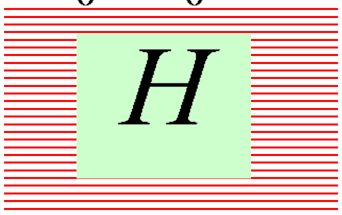
$$A = G^R \Gamma G^A$$

$$= G^R(\Gamma_1 + \Gamma_2) G^A$$

$$A\, f_1 = G^R(\Gamma_1 f_1 + \Gamma_2 f_1) G^A$$

Σ_0, Σ_0^{in} X

Σ_1^{in} Σ_2^{in}



Σ_1 Σ_2

$$I_1 = \frac{q}{h}\int_{-\infty}^{+\infty} dE\ Trace[\Gamma_1 G^R \Gamma_2 G^A]\ (f_1 - f_2)$$

$$\Sigma = \Sigma_1 + \Sigma_2$$

$$\Sigma^{in} = \Sigma_1^{in} + \Sigma_2^{in}$$

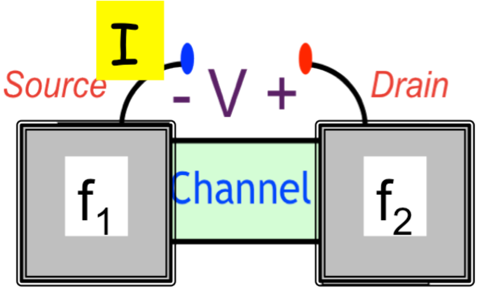
$$= \frac{q}{h} Trace[\Gamma_1 \overbrace{(A\, f_1 - G^n)}^{G^R \Gamma_2 G^A (f_1 - f_2)}]$$

$$\Sigma_1^{in} = \Gamma_1 f_1(E)$$

$$\Sigma_2^{in} = \Gamma_2 f_2(E)$$

$$I_1 = \frac{q}{h} Trace[\Sigma_1^{in} A - \Gamma_1 G^n]$$

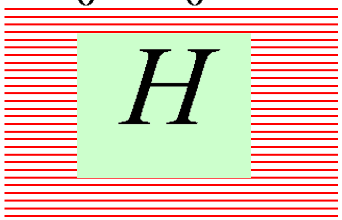
2.7c Transmission



$$G^R = [EI - H - \Sigma]^{-1}$$

$$G^n = G^R \Sigma^{in} G^A = G^R (\Gamma_1 f_1 + \Gamma_2 f_2 + \cdots) G^A$$

$$\Sigma_0, \Sigma_0^{in}$$



$$\Sigma_1^{in} \quad \Sigma_1 \quad \Sigma_2^{in} \quad \Sigma_2$$

$$A = G^R \Gamma G^A = G^R (\Gamma_1 + \Gamma_2 + \cdots) G^A$$

$$A f_p = G^R (\Gamma_1 f_p + \Gamma_2 f_p + \cdots) G^A$$

$$A f_p - G^n$$

$$\sum_q G^R \Gamma_q G^A \; (f_p - f_q)$$

$$\Sigma = \Sigma_1 + \Sigma_2 + \cdots$$

$$\Sigma^{in} = \Sigma_1^{in} + \Sigma_2^{in} + \cdots$$

$$\Sigma_p^{in} = \Gamma_p f_p(E)$$

$$\boxed{I_p = \frac{q}{h} \sum_q \overbrace{Trace[\Gamma_p G^R \Gamma_q G^A]}^{\bar{T}_{pq}(E)} \; (f_p - f_q)}$$

$$= \frac{q}{h} Trace[\Gamma_p \overbrace{(A f_p - G^n)}\,]$$

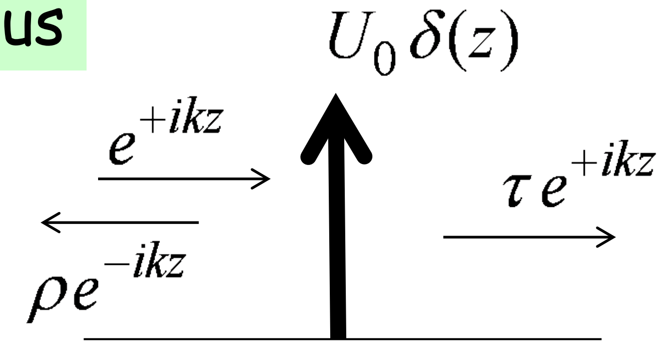
$$I_p = \frac{q}{h} Trace[\Sigma_p^{in} A - \Gamma_p G^n]$$

2.7d Transmission

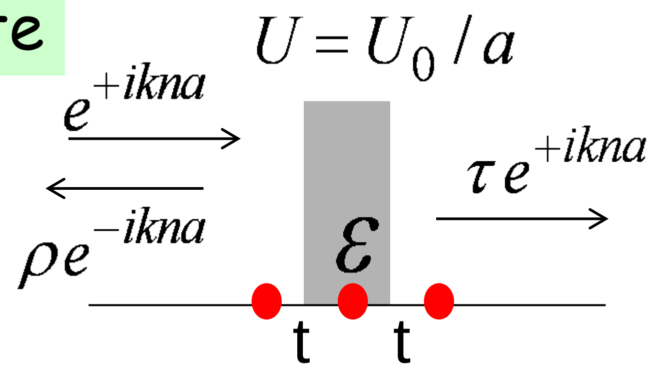
$$\bar{T}(E) = Trace\,[\Gamma_1 G^R \Gamma_2 G^A]$$

$$= \frac{(\hbar \nu)^2}{U_0^2 + (\hbar \nu)^2}$$

Continuous



Discrete



2.7e Transmission

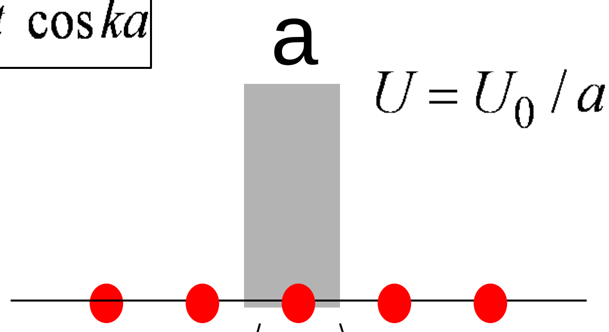
$$G^R = \frac{1}{E - \varepsilon - U - 2t\, e^{ika}}$$

$$= \frac{1}{E - \varepsilon - U - 2t(\cos ka + i \sin ka)}$$

$$= \frac{1}{-U - 2it \sin ka}$$

$$= -\frac{1}{-U + i\hbar\nu / a}$$

$$\boxed{\begin{aligned} \frac{\hbar \nu}{a} &= -2t \sin ka \\ E &= \varepsilon + 2t \cos ka \end{aligned}}$$



$$\Sigma_1 = t\, e^{ika} \qquad H = \varepsilon + U \qquad \Sigma_2 = t\, e^{ika}$$

$$\Gamma_1 = -2t \sin ka = \hbar \nu / a \qquad \Gamma_2 = -2t \sin ka = \hbar \nu / a$$

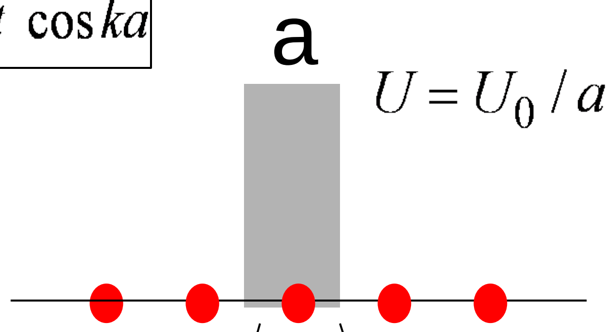
$$\bar{T}(E) = Trace\,[\Gamma_1 G^R \Gamma_2 G^A]$$

2.7f Transmission

$$G^R = -\frac{1}{-U + i\hbar\nu / a}$$

$$\frac{\hbar\nu}{a} = -2t\sin ka$$

$$E = \varepsilon + 2t\cos ka$$



$$\bar{T}(E) = Trace\,[\Gamma_1 G^R \Gamma_2 G^A]$$

$$= \frac{\hbar\nu}{a}\,\frac{1}{-U + i\hbar\nu / a}\,\frac{\hbar\nu}{a}\,\frac{1}{-U - i\hbar\nu / a}$$

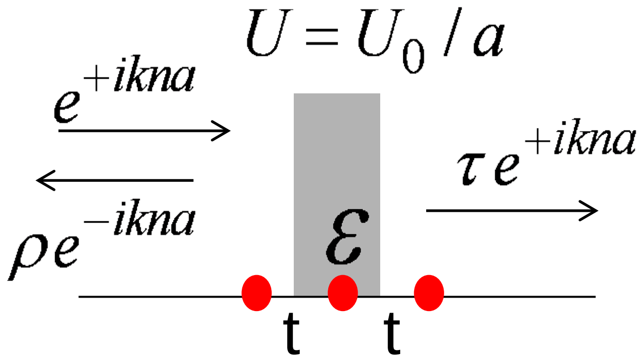
$$= \frac{(\hbar\nu / a)^2}{U^2 + (\hbar\nu / a)^2}$$

$$= \frac{(\hbar\nu)^2}{U_0^2 + (\hbar\nu)^2}$$

$$\Sigma_1 = t\,e^{ika} \qquad H = \varepsilon + U \qquad \Sigma_2 = t\,e^{ika}$$

$$\Gamma_1 = -2t\sin ka = \hbar\nu / a \qquad \Gamma_2 = -2t\sin ka = \hbar\nu / a$$

2.7g Transmission

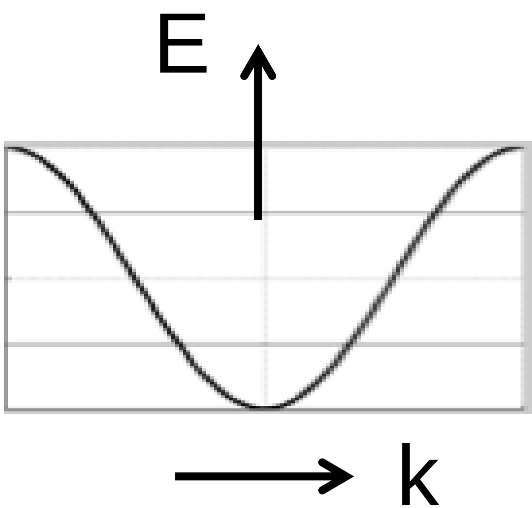


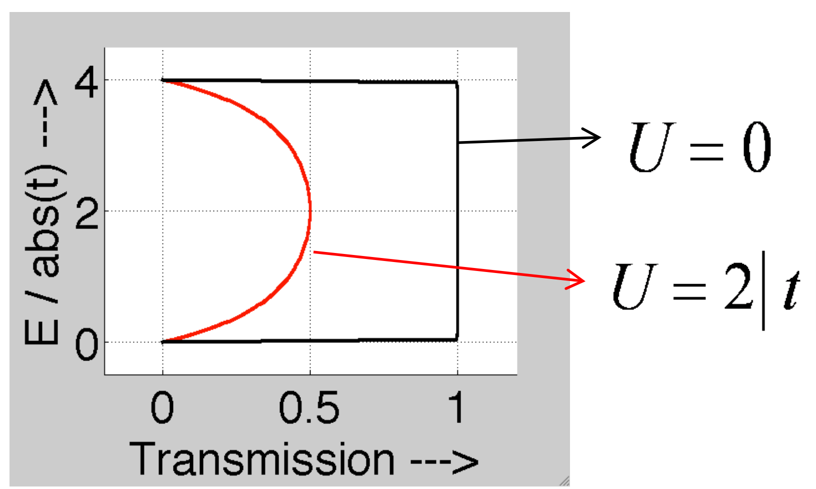
$$\frac{\hbar \nu}{a} = -2t \sin ka$$

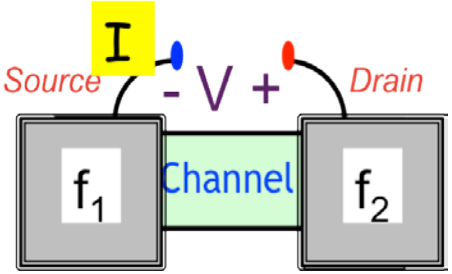
$$E = \varepsilon + 2t \cos ka$$

$$\bar{T}(E) = Trace\,[\Gamma_1 G^R \Gamma_2 G^A]$$

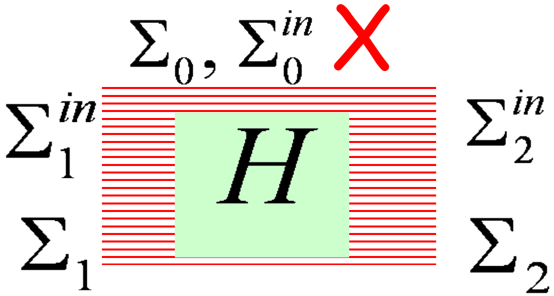
$$= \frac{(\hbar \nu)^2}{U_0^2 + (\hbar \nu)^2}$$







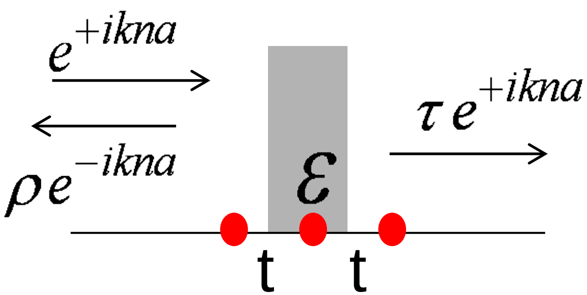
$$I_1 = \frac{q}{h}\, Trace\,[\Sigma_1^{in} A - \Gamma_1 G^n]$$



$$\Sigma_1^{in} = \Gamma_1\, f_1(E)$$

$$\Sigma_2^{in} = \Gamma_2\, f_2(E)$$

$$I_1 = \frac{q}{h}\int_{-\infty}^{+\infty} dE\, \underbrace{Trace\,[\Gamma_1 G^R \Gamma_2 G^A]}_{\bar{T}(E)}\, (f_1 - f_2)$$



Coming up next ..